# CHBE320 LECTURE XI
# CONTROLLER DESIGN AND PID CONTOLLER TUNING

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# Road Map of the Lecture XI

- **Controller Design and PID Tuning**
  - Performance criteria
  - Trial and error method
  - Continuous cycling method
  - Relay feedback method
  - Tuning relationships
  - Direct Synthesis
  - Internal Model Control (IMC)
  - Effects of modeling error

# CONTROLLER DESIGN

- **Performance criteria for closed-loop systems**
  - **Stable**
  - **Minimal effect of disturbance**
  - **Rapid, smooth response to set point change**
  - **No offset**
  - **No excessive control action**
  - **Robust to plant-model mismatch**

$$\min_{K_c,\tau_I,\tau_D} \int_0^{\infty} (w_1 e^2(\tau) + w_2 \Delta u^2(\tau)) d\tau$$

- **Trade-offs in control problems**
  - **Set point tracking vs. disturbance rejection**
  - **Robustness vs. performance**

# GUIDELINES FOR COMMON CONTROL LOOPS

- **Flow and liquid pressure control**
  - Fast response with no time delay
  - Usually with small high-frequency noise
  - PI controller with intermediate controller gain
    - 0.5< $K_c$ <0.7 and 0.2< $\tau_I$ <0.3min (Fruehauf et al. (1994))
- **Liquid level control**
  - Noisy due to splashing and turbulence
  - High gain PI controller for integrating process
    - Increase in $K_c$ may decrease oscillation (special behavior)
  - Conservative setting for averaging control when it is used for damping the fluctuation of the inlet stream (usually P-control)
    - PI control:

      $K_c = 100\%/\Delta h, \qquad \tau_I = 4V/(K_c Q_{max}) \qquad (\Delta h \equiv \min(h_{max} - h_{sp}, h_{sp} - h_{min}))$
    - Error-squared controller with careful tuning
  - If heat transfer is involved, it becomes much more complicated.

- **Gas pressure control**
  - Usually fast and self regulating
  - PI controller with small integral action (large reset time)
  - D mode is not usually needed.
- **Temperature control**
  - Wide variety of the process nature
  - Usually slow response with time delay
  - Use PID controller to speed up the response
- **Composition control**
  - Similar to temperature control usually with larger noise and more time delay
  - Effectiveness of derivative action is limited
  - Temperature and composition controls are the prime candidates for advance control strategies due to its importance and difficulty of control

# TRIAL AND ERROR TUNING

- **Step1: With P-only controller**
  - **Start with low $K_c$ value and increase it until the response has a sustained oscillation (continuous cycling) for a small set point or load change. ($K_{cu}$)**
  - **Set $K_c = 0.5K_{cu}$.**
- **Step2: Add I mode**
  - **Decrease the reset time until sustained oscillation occurs. ($\tau_{Iu}$)**
  - **Set $\tau_I = 3\tau_{Iu}$ .**
  - **If a further improvement is required, proceed to Step 3.**
- **Step3: Add D mode**
  - **Increase the preact time until sustained oscillation occurs. ($\tau_{Du}$)**
  - **Set $\tau_D = \tau_{Du}/3$.**

**(The sustained oscillation should not be cause by the controller saturation)**

# CONTINUOUS CYCLING METHOD

- **Also called as loop tuning or ultimate gain method**
  - **Increase controller gain until sustained oscillation**
  - **Find ultimate gain ($K_{CU}$) and ultimate period ($P_{CU}$)**
- **Ziegler-Nichols controller setting**
  - **¼ decay ratio (too much oscillatory)**

| Controller | $K_C$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|
| P | $0.5K_{CU}$ | - | - |
| PI | $0.45K_{CU}$ | $P_{CU}/1.2$ | - |
| PID | $0.6K_{CU}$ | $P_{CU}/2$ | $P_{CU}/8$ |

  - **Modified Ziegler-Nichols setting**

| Controller | $K_C$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|
| Original | $0.6K_{CU}$ | $P_{CU}/2$ | $P_{CU}/8$ |
| Some overshoot | $0.33K_{CU}$ | $P_{CU}/2$ | $P_{CU}/3$ |
| No overshoot | $0.2K_{CU}$ | $P_{CU}/2$ | $P_{CU}/3$ |

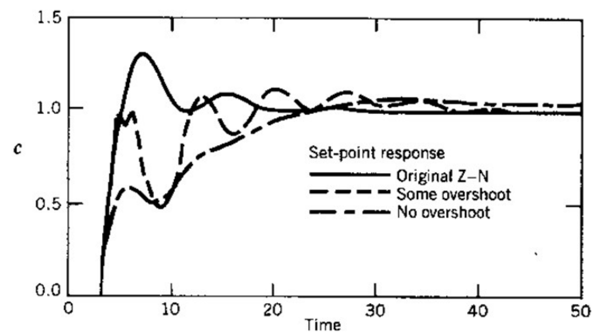
- **Examples**

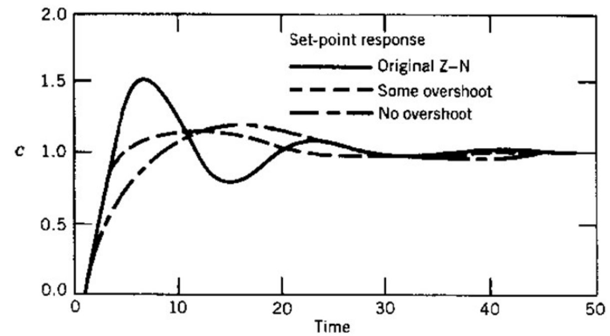
$$G_p(s) = \frac{4e^{-3.5s}}{7s+1} \qquad K_{CU} = 0.95 \quad P_{CU} = 12$$

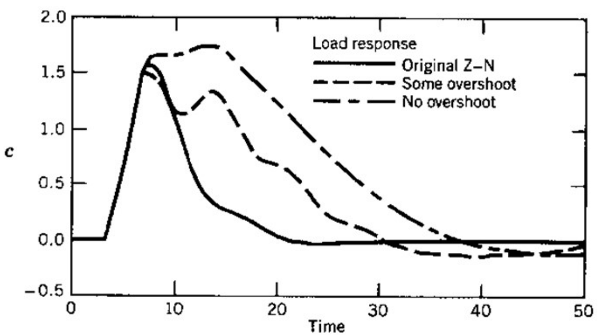
$$G_p(s) = \frac{2e^{-s}}{(10s+1)(5s+1)} \qquad K_{CU} = 7.88 \quad P_{CU} = 11.6$$

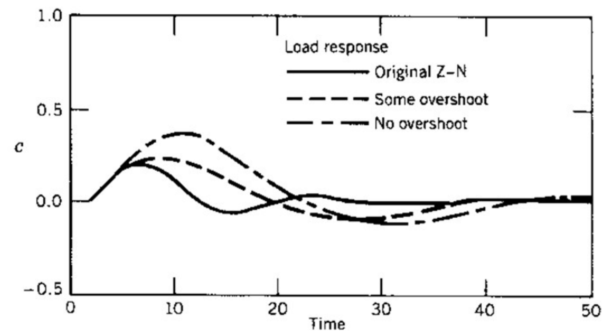
| Controller | $K_C$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|
| Original | 0.57 | 6.0 | *1.5* |
| Some overshoot | 0.31 | 6.0 | *4.0* |
| No overshoot | 0.19 | 6.0 | *4.0* |

| Controller | $K_C$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|
| Original | 4.73 | 5.8 | *1.45* |
| Some overshoot | 2.60 | 5.8 | *3.87* |
| No overshoot | 1.58 | 5.8 | *3.87* |



- **Advantages of continuous cycling method**
  - No a priori information on process required
  - Applicable to all stable processes

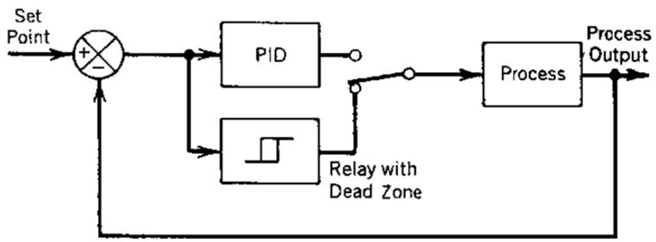
- **Disadvantages of continuous cycling method**
  - Time consuming
  - Loss of product quality and productivity during the tests
  - Continuous cycling may cause the violation of process limitation and safety hazards
  - Not applicable to open-loop unstable process
  - First-order and second-order process without time delay will not oscillate even with very large controller gain

=> Motivates Relay feedback method. (Astrom and Wittenmark)

# RELAY FEEDBACK METHOD

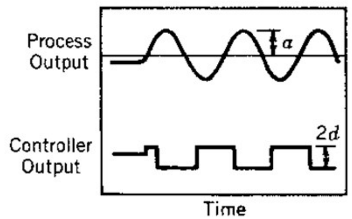
- **Relay feedback controller**
  - **Forces the system to oscillate by a relay controller**
  - **Require a single closed-loop experiment to find the ultimate frequency information**
  - **No *a priori* information on process is required**
  - **Switch relay feedback controller for tuning**
  - **Find $P_{CU}$ and calculate $K_{CU}$**

$$K_{CU} = \frac{4d}{\pi a}$$




  - **User specified parameter: $d$**

    **Decide $d$ in order not to perturb the system too much.**




  - **Use Ziegler-Nichols Tuning rules for PID tuning parameters**

- **Calculation of model parameters from $K_{CU}$ and $P_U$**

  - **Integrator-plus-time-delay model:** $G(s) = \dfrac{Ke^{-\theta s}}{s}$

$$K = \frac{2\pi}{K_{CU}P_U} \quad \theta = P_U/4$$

  - **First-order-plus-time-delay model:** $G(s) = \dfrac{Ke^{-\theta s}}{\tau s + 1}$

$$K = \frac{2\pi}{K_{CU}P_U}$$

$$\tau = \frac{P_U}{2\pi}\tan\frac{\pi(P_U - 2\theta)}{P_U} \quad \text{or} \quad \tau = \frac{P_U}{2\pi}\sqrt{(KK_{CU})^2 - 1}$$

    - **The $\theta$ is decided by visual inspection and $K$ can be calculated using two equations of $\tau$ above.**

# DESIGN RELATIONS FOR PID CONTROLLERS

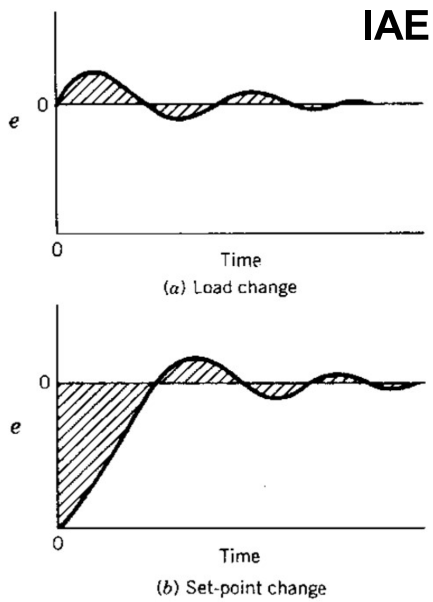
- **Cohen-Coon controller design relations**
  - Empirical relation for ¼ decay ratio for FOPDT model

**Table 12.2 Cohen and Coon Controller Design Relations**

| *Controller* | *Settings* | *Cohen–Coon* |
|---|---|---|
| P | $K_c$ | $\dfrac{1}{K}\dfrac{\tau}{\theta}[1 + \theta/3\tau]$ |
| PI | $K_c$ | $\dfrac{1}{K}\dfrac{\tau}{\theta}[0.9 + \theta/12\tau]$ |
| | $\tau_I$ | $\dfrac{\theta[30 + 3(\theta/\tau)]}{9 + 20(\theta/\tau)}$ |
| PID | $K_c$ | $\dfrac{1}{K}\dfrac{\tau}{\theta}\left[\dfrac{16\tau + 3\theta}{12\tau}\right]$ |
| | $\tau_I$ | $\dfrac{\theta[32 + 6(\theta/\tau)]}{13 + 8(\theta/\tau)}$ |
| | $\tau_D$ | $\dfrac{4\theta}{11 + 2(\theta/\tau)}$ |

- **Design relations based on integral error criteria**
  - ¼ decay ratio is too oscillatory
  - Decay ratio concerns only two peak points of the response
  - IAE: Integral of the Absolute Error

$$\mathrm{IAE} = \int_0^\infty |e(t)|dt$$




  - ISE: Integral of the Square Error

$$\mathrm{ISE} = \int_0^\infty [e(t)]^2 dt$$

    - Large error contributes more
    - Small error contributes less
    - Large penalty for large overshoot
    - Small penalty for small persisting oscillation
  - ITAE: Integral of the Time-weighted Absolute Error

$$\mathrm{ITAE} = \int_0^\infty t|e(t)|dt$$

    - Large penalty for persisting oscillation
    - Small penalty for initial transient response

- **Controller design relation based on ITAE for FOPDT model**

**Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model [6–8][a]**

| *Type of Input* | *Type of Controller* | *Mode* | *A* | *B* |
|---|---|---|---|---|
| Load | PI | P | 0.859 | −0.977 |
| | | I | 0.674 | −0.680 |
| Load | PID | P | 1.357 | −0.947 |
| | | I | 0.842 | −0.738 |
| | | D | 0.381 | 0.995 |
| Set point | PI | P | 0.586 | −0.916 |
| | | I | 1.03[b] | −0.165[b] |
| Set point | PID | P | 0.965 | −0.85 |
| | | I | 0.796[b] | −0.1465[b] |
| | | D | 0.308 | 0.929 |

[a]Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, $\tau/\tau_I$ for the integral mode, and $\tau_D/\tau$ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

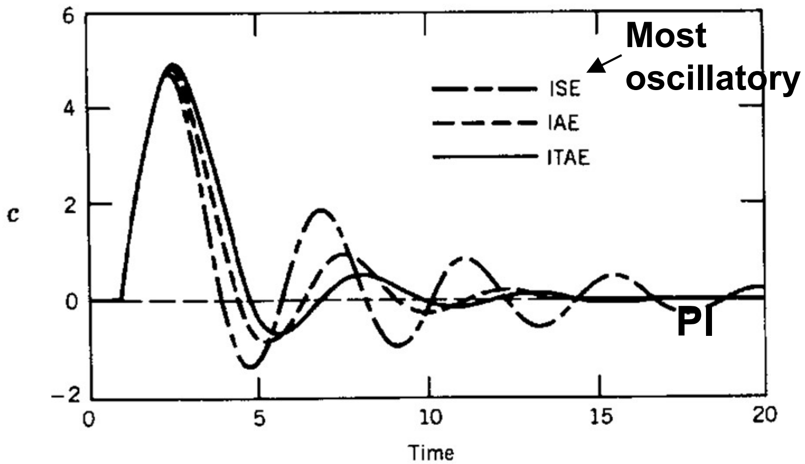
- **Similar design relations based on IAE and ISE for other types of models can be found in literatures.**

- **Example1**

$$G(s) = \frac{10e^{-s}}{2s+1}$$

$$KK_c = (0.859)(1/2)^{-0.977} = 1.69$$
$$\Rightarrow K_c = 0.169$$

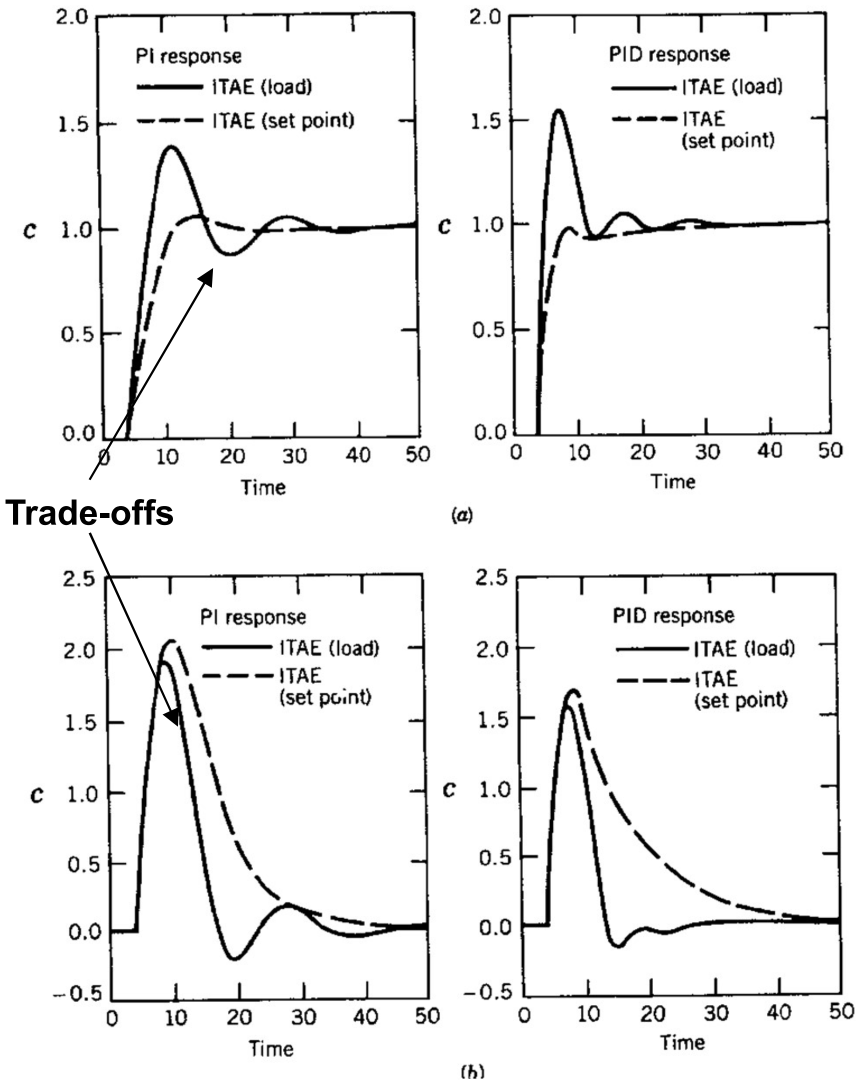
$$\tau/\tau_I = (0.674)(1/2)^{-0.680} = 1.08$$
$$\Rightarrow \tau_I = 1.85$$




| Method | $K_c$ | $\tau_I$ |
|---|---|---|
| **IAE** | **0.195** | **2.02** |
| **ISE** | **0.245** | **2.44** |
| **ITAE** | **0.169** | **1.85** |

**Example2**

$$G(s) = \frac{4e^{-3.5s}}{7s+1}$$



- **Design relations based on process reaction curve**
  - **For the processes who have sigmoidal shape step responses**
    **(Not for underdamped processes)**
  - **Fit the curve with FOPDT model**

$$G(s) = \frac{Ke^{-\theta}}{(\tau s + 1)} \qquad S = K\Delta u/\tau \qquad S^* = S/\Delta u = K/\tau$$

**Table 13.3 Ziegler–Nichols Tuning Relations (Process Reaction Curve Method)**

| *Controller Type* | $K_c$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|
| P | $\frac{1}{\theta S^*}$ | — | — |
| PI | $\frac{0.9}{\theta S^*}$ | $3.33\theta$ | — |
| PID | $\frac{1.2}{\theta S^*}$ | $2\theta$ | $0.5\theta$ |

  - **Very simple**
  - **Inherits all the problems of FOPDT model fitting**

# MISCELLANEOUS TUNING RELATIONS

- **Hägglund and Åström (2002)**

**Table 12.4** PI Controller Settings: Hägglund and Åström (2002)

| $G(s)$ | $K_c$ | $\tau_I$ |
|---|---|---|
| $\dfrac{Ke^{-\theta s}}{s}$ | $\dfrac{0.35}{K\theta}$ | $7\theta$ |
| $\dfrac{Ke^{-\theta s}}{\tau s+1}$ | $\dfrac{0.14}{K}+\dfrac{0.28\tau}{\theta K}$ | $0.33\theta+\dfrac{6.8\theta\tau}{10\theta+\tau}$ |

- **Skogestad (2003)**

**Table 12.5** Controller Settings for $G(s) = Ke^{-\theta s}/(\tau_1 s+1)(\tau_2 s+1)$: Skogestad (2003)

| Conditions | $K_c$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|
| $\tau_1 \le 8\theta$ | $\dfrac{0.5(\tau_1+\tau_2)}{K\theta}$ | $\tau_1+\tau_2$ | $\dfrac{\tau_1\tau_2}{\tau_1+\tau_2}$ |
| $\tau_1 \ge 8\theta$ | $\dfrac{0.5\tau_1}{K\theta}\left(\dfrac{8\theta+\tau_2}{8\theta}\right)$ | $8\theta+\tau_2$ | $\dfrac{8\theta\tau_2}{8\theta+\tau_2}$ |

- **Ziegler-Nichols (1942) and Cohen-Coon (1953) are not recommended since their relations are base on 1/4-decay ratio.**

# CONTROLLERS WITH TWO DEGREES OF FREEDOM

- **Trade-off between set-point tracking and disturbance rejection**
- **Tuning for disturbance rejection is more aggressive.**
- **In general, disturbance rejection is more important. Thus, tune the controller for satisfactory disturbance rejection.**
- **Controllers with two degrees of freedom (Goodwin et al., 2001)**
  - **Strategies to adjust set-point tracking and disturbance rejection independently**

**1. Gradual change in set point (ramp or filtered)**

$$\frac{Y_{sp}^*}{Y_{sp}} = \frac{1}{\tau_f s + 1} \quad \text{(filtered as first order)}$$

**2. Modification of PID control law**

$$p(t) = \bar{p} + K_c(\beta y_{sp} - y_m) + K_c\left(\frac{1}{\tau_I}\int_0^t e(t^*)dt^* - \tau_D\frac{dy_m}{dt}\right) \quad (0 < \beta < 1)$$

- **As b increase, the set-point response becomes faster but more overshoot.**

# DIRECT SYNTHESIS METHOD

- **Analysis: Given $G_c(s)$, what is $y(t)$?**
- **Design: Given $y_d(t)$, what should $G_c(s)$ be?**
- **Derivation**

Let $G_{OL} = K_m G_c G_v G_p \triangleq G_c G$

$$\frac{Y(s)}{R(s)} = \frac{G_{OL}}{1 + G_{OL}} = \frac{G_c G}{1 + G_c G} \quad \Rightarrow \quad G_c = \frac{1}{G}\left(\frac{Y/R}{1 - Y/R}\right)$$

Specify $(Y/R)_d \;\Rightarrow\; G_c = \dfrac{1}{G}\left(\dfrac{(Y/R)_d}{1 - (Y/R)_d}\right)$

- **If $(Y/R)_d = 1$, then it implies perfect control. (infinite gain)**
- **The resulting controller may not be physically realizable**
- **Or, not in PID form and too complicated.**
- **Design with finite settling time:** $(Y/R)_d = \dfrac{1}{\tau_c s + 1}$

# • Examples

## 1. Perfect control ($K_c$ becomes infinite)

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad \text{and} \quad (Y/R)_d = 1$$

$$G_c(s) = \frac{1}{G(s)}\left(\frac{1}{1-1}\right) = \frac{\infty}{G(s)} \quad \text{(infinite gain, unrealizable)}$$

## 2. Finite settling time for 1st-order process

$$G(s) = \frac{K}{(\tau s + 1)} \quad \text{and} \quad (Y/R)_d = \frac{1}{\tau_c s + 1}$$

$$G_c(s) = \frac{1}{G(s)}\left(\frac{1/(\tau_c s + 1)}{1 - 1/(\tau_c s + 1)}\right) = \frac{\tau s + 1}{K \tau_c s} = \frac{\tau}{\tau_c K}\left(1 + \frac{1}{\tau s}\right) \quad \text{(PI)}$$

## 3. Finite settling time for 2nd-order process

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad \text{and} \quad (Y/R)_d = \frac{1}{\tau_c s + 1}$$

$$G_c(s) = \frac{(\tau_1 + \tau_2)}{\tau_c K}\left(1 + \frac{1}{(\tau_1 + \tau_2)s} + \frac{\tau_1 \tau_2}{(\tau_1 + \tau_2)} s\right) \quad \text{(PID)}$$

- **Process with time delay**
  - **If there is a time delay, any physically realizable controller cannot overcome the time delay. (Need time lead)**
  - **Given circumstance, a reasonable choice will be**

$$(Y/R)_d = \frac{e^{-\theta_c s}}{\tau_c s + 1}$$

  - **Examples**

**1.** $G(s) = \dfrac{Ke^{-\theta s}}{(\tau s + 1)}$ and $(Y/R)_d = \dfrac{e^{-\theta s}}{\tau_c s + 1}$ $(\theta_c = \theta)$

$$G_c(s) = \frac{1}{G(s)}\left(\frac{e^{-\theta}\ /(\tau_c s + 1)}{1 - e^{-\theta s}/(\tau_c s + 1)}\right) = \frac{\tau s + 1}{K}\boxed{\frac{1}{\tau_c s + 1 - e^{-\theta s}}} \text{ (not a PID)}$$

**Physically unrealizable**

**2. With 1st-order Taylor series approx. (** $e^{-\theta s} \approx 1 - \theta s$ **)**

$$G_c(s) = \frac{\tau s + 1}{K}\frac{1}{(\tau_c + \theta)s} = \frac{\tau}{K(\tau_c + \theta)}\left(1 + \frac{1}{\tau s}\right) \text{ (PI)}$$

**3.** $G(s) = \dfrac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$ and $(Y/R)_d = \dfrac{e^{-\theta s}}{\tau_c s + 1}$ $(\theta_c = \theta)$

$$G_c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K}\frac{1}{(\tau_c + \theta)s} = \frac{(\tau_1 + \tau_2)}{K(\tau_c + \theta)}\left(1 + \frac{1}{(\tau_1 + \tau_2)s} + \frac{\tau_1\tau_2}{(\tau_1 + \tau_2)}s\right) \text{ (PID)}$$

- **Observations on Direct Synthesis Method**
  - **Resulting controllers could be quite complex and may not even be physically realizable.**
  - **PID parameters will be decided by a user-specified parameter: The desired closed-loop time constant ($\tau_c$)**
  - **The shorter $\tau_c$ makes the action more aggressive. (larger $K_c$)**
  - **The longer $\tau_c$ makes the action more conservative. (smaller $K_c$)**
  - **For a limited cases, it results PID form.**
    - **1st-order model without time delay: PI**
    - **FOPDT with 1st-order Taylor series approx.: PI**
    - **2nd-order model without time delay: PID**
    - **SOPDT with 1st-order Taylor series approx.: PID**
    - **Delay modifies the $K_c$.**

$$\frac{\tau}{K\tau_c} \rightarrow \frac{\tau}{K(\tau_c+\theta)} \text{ (1st order)} \qquad \frac{(\tau_1+\tau_2)}{K\tau_c} \rightarrow \frac{(\tau_1+\tau_2)}{K(\tau_c+\theta)} \text{ (2nd order)}$$

    - **With time delay, the $K_c$ will not become infinite even for the perfect control ($Y/R$=1).**

# INTERNAL MODEL CONTROL (IMC)

- **Motivation**
  - The resulting controller from direct synthesis method may not be physically unrealizable.
  - If there is RHP zero in the process, the resulting controller from direct synthesis method will be unstable.
  - Unmeasured disturbance and modeling error are not considered in direct synthesis method.
- **Source of trouble**
  - From direct synthesis method

$$G_c = \boxed{\frac{1}{G}\left(\frac{(Y/R)_d}{1-(Y/R)_d}\right)}$$

Resulting controller may have higher-order numerator than denominator

Direct inversion of process causes many problems

Process is unknown

- **IMC**
  - **Feedback the error between the process output and model output.**
  - **Equivalent conventional controller:** $$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}}$$
  - **Using block diagram algebra**

$$C = GP + L \quad P = G_c^* E \quad E = R - (C - \tilde{C}) = R - C + \tilde{G}P$$

$$P = G_c^*(R - C + \tilde{G}P)$$

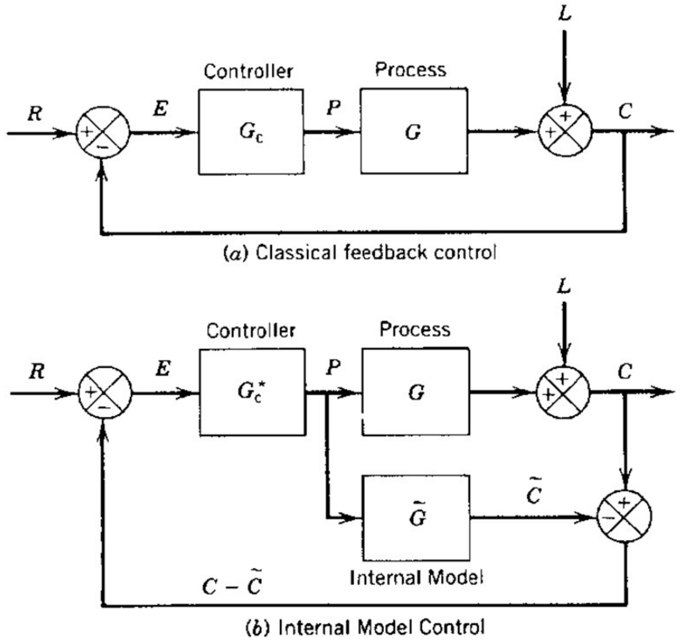
$$\Rightarrow \quad P = G_c^*(R - C)/(1 - G_c^* \tilde{G})$$

$$C = GG_c^*(R - C)/(1 - G_c^* \tilde{G}) + L$$

$$(1 + GG_c^* - G_c^* \tilde{G})C = GG_c^* R + (1 - G_c^* \tilde{G})L$$

$$C = \frac{G_c^* G}{1 + G_c^*(G - \tilde{G})} R + \frac{(1 - G_c^* \tilde{G})}{1 + G_c^*(G - \tilde{G})} L$$

If $\tilde{G} = G$, $C = G_c^* G R + (1 - G_c^* G)L$



- **IMC design strategy**
  - **Factor the process model as**

$$\tilde{G} = \tilde{G}_+ \tilde{G}_-$$

**Uninvertibles**

  - $\tilde{G}_+$ **contains any time delays and RHP zeros and is specified so that the steady-state gain is one**
  - $\tilde{G}_-$ **is the rest of $G$.**
- **The controller is specified as**

$$G_c^* = \frac{1}{\tilde{G}_-} f$$

  - **IMC filter $f$ is a low-pass filter with steady-state gain of one**
  - **Typical IMC filter:**

$$f = \frac{1}{(\tau_c s + 1)^r}$$

  - **The $\tau_c$ is the desired closed-loop time constant and parameter $r$ is a positive integer that is selected so that the order of numerator of $G_c^*$ is same as the order of denominator or exceeds the order of denominator by one.**

- **Example**
  - **FOPDT model with 1/1 Pade approximation**

$$\tilde{G} = \frac{K(1 - \theta s/2)}{(1 + \theta s/2)(\tau s + 1)}$$

$$\tilde{G}_+ = 1 - \theta s/2 \qquad \tilde{G}_- = \frac{K}{(1 + \theta s/2)(\tau s + 1)}$$

$$G_c^* = \frac{1}{\tilde{G}_-} f = \frac{(1 + \theta s/2)(\tau s + 1)}{K} \frac{1}{(\tau_c s + 1)}$$

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} = \frac{(1 + \theta s/2)(\tau s + 1)}{K(\tau_c + \theta/2)s} \quad \text{(PID)}$$

$$K_c = \frac{1}{K} \frac{(\tau + \theta/2)}{(\tau_c + \theta/2)} \qquad \tau_I = \tau + \theta/2 \qquad \tau_D = \frac{\tau\theta/2}{\tau + \theta/2}$$

# IMC based PID controller settings

**Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ [4][a]**

| *Case* | *Model* | $K_c K$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|---|
| A | $\dfrac{K}{\tau s + 1}$ | $\dfrac{\tau}{\tau_c}$ | $\tau$ | — |
| B | $\dfrac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$ | $\dfrac{\tau_1 + \tau_2}{\tau_c}$ | $\tau_1 + \tau_2$ | $\dfrac{\tau_1 \tau_2}{\tau_1 + \tau_2}$ |
| C | $\dfrac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$ | $\dfrac{2\zeta\tau}{\tau_c}$ | $2\zeta\tau$ | $\dfrac{\tau}{2\zeta}$ |
| D | $\dfrac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}, \beta > 0$ | $\dfrac{2\zeta\tau}{\tau_c + \beta}$ | $2\zeta\tau$ | $\dfrac{\tau}{2\zeta}$ |
| E | $\dfrac{K}{s}$ | $\dfrac{1}{\tau_c}$ | — | — |
| F | $\dfrac{K}{s(\tau s + 1)}$ | $\dfrac{1}{\tau_c}$ | — | $\tau$ |

[a]Based on Eq. 12-30 with $r = 1$.

# IMC based PID controller settings

**Table 12.1** IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

| Case | Model | $K_cK$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|---|
| A | $\dfrac{K}{\tau s+1}$ | $\dfrac{\tau}{\tau_c}$ | $\tau$ | — |
| B | $\dfrac{K}{(\tau_1 s+1)(\tau_2 s+1)}$ | $\dfrac{\tau_1+\tau_2}{\tau_c}$ | $\tau_1+\tau_2$ | $\dfrac{\tau_1\tau_2}{\tau_1+\tau_2}$ |
| C | $\dfrac{K}{\tau^2s^2+2\zeta\tau s+1}$ | $\dfrac{2\zeta\tau}{\tau_c}$ | $2\zeta\tau$ | $\dfrac{\tau}{2\zeta}$ |
| D | $\dfrac{K(-\beta s+1)}{\tau^2s^2+2\zeta\tau s+1},\ \beta>0$ | $\dfrac{2\zeta\tau}{\tau_c+\beta}$ | $2\zeta\tau$ | $\dfrac{\tau}{2\zeta}$ |
| E | $\dfrac{K}{s}$ | $\dfrac{2}{\tau_c}$ | $2\tau_c$ | — |
| F | $\dfrac{K}{s(\tau s+1)}$ | $\dfrac{2\tau_c+\tau}{\tau_c^2}$ | $2\tau_c+\tau$ | $\dfrac{2\tau_c\tau}{2\tau_c+\tau}$ |
| G | $\dfrac{Ke^{-\theta s}}{\tau s+1}$ | $\dfrac{\tau}{\tau_c+\theta}$ | $\tau$ | — |
| H | $\dfrac{Ke^{-\theta s}}{\tau s+1}$ | $\dfrac{\tau+\dfrac{\theta}{2}}{\tau_c+\dfrac{\theta}{2}}$ | $\tau+\dfrac{\theta}{2}$ | $\dfrac{\tau\theta}{2\tau+\theta}$ |
| I | $\dfrac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$ | $\dfrac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$ | $\tau_1+\tau_2-\tau_3$ | $\dfrac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$ |
| J | $\dfrac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2s^2+2\zeta\tau s+1}$ | $\dfrac{2\zeta\tau-\tau_3}{\tau_c+\theta}$ | $2\zeta\tau-\tau_3$ | $\dfrac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$ |
| K | $\dfrac{K(-\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$ | $\dfrac{\tau_1+\tau_2+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$ | $\tau_1+\tau_2+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}$ | $\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\dfrac{\tau_1\tau_2}{\tau_1+\tau_2+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$ |
| L | $\dfrac{K(-\tau_3 s+1)e^{-\theta s}}{\tau^2s^2+2\zeta\tau s+1}$ | $\dfrac{2\zeta\tau+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_e+\theta}$ | $2\zeta\tau+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}$ | $\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\dfrac{\tau^2}{2\zeta\tau+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$ |
| M | $\dfrac{Ke^{-\theta s}}{s}$ | $\dfrac{2\tau_c+\theta}{(\tau_c+\theta)^2}$ | $2\tau_c+\theta$ | — |
| N | $\dfrac{Ke^{-\theta s}}{s}$ | $\dfrac{2\tau_c+\theta}{\left(\tau_c+\dfrac{\theta}{2}\right)^2}$ | $2\tau_c+\theta$ | $\dfrac{\tau_c\theta+\dfrac{\theta^2}{4}}{2\tau_c+\theta}$ |
| O | $\dfrac{Ke^{-\theta s}}{s(\tau s+1)}$ | $\dfrac{2\tau_c+\tau+\theta}{(\tau_c+\theta)^2}$ | $2\tau_c+\tau+\theta$ | $\dfrac{(2\tau_c+\theta)\tau}{2\tau_c+\tau+\theta}$ |

- **Modification of IMC and DS methods**
  - **For lag dominant models ($\theta/\tau$<<1), IMC and DS methods provide satisfactory set-point response, but very slow disturbance responses because the value $\tau_I$ is very large.**
  - **Approximate the FOPDT with IPDT model and use IMC tuning relation for IPDT model**

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \Rightarrow G(s) = \frac{K^* e^{-\theta s}}{s} \quad \text{where } K^* \triangleq K/\tau$$

  - **Limit the value of $\tau_I$**

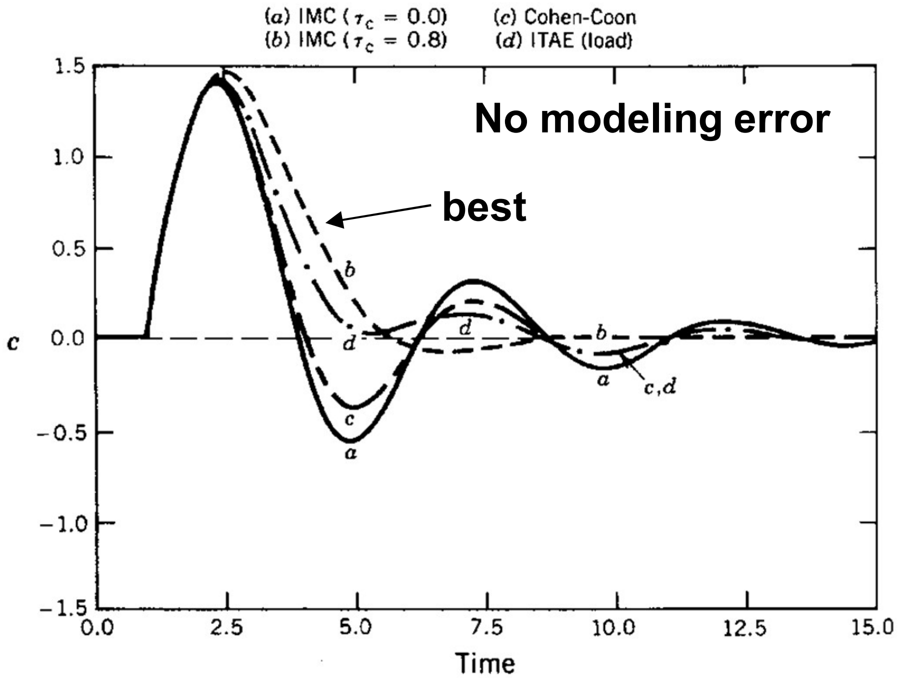
$$\tau_I = \min\{\tau_I, 4(\tau_c + \theta)\}$$

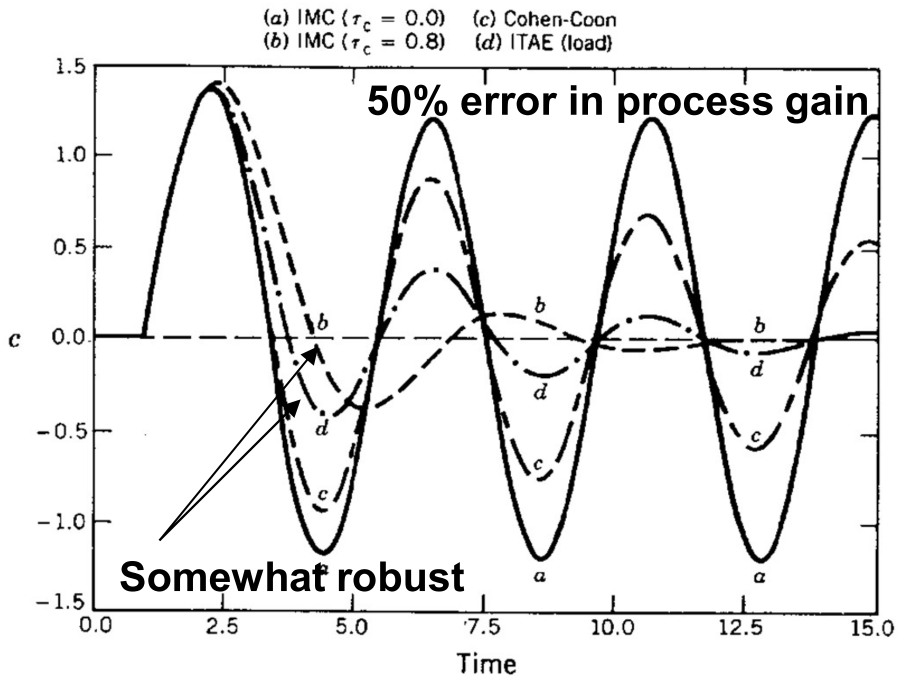
  - **Design the controller for disturbance rejection**

# COMPARISON OF CONTROLLER DESIGN RELATIONS

- **PI controller settings for different methods**

$$G(s) = \frac{2e^{-s}}{s+1}$$



# EFFECT OF MODELING ERROR

- **Actual plant**

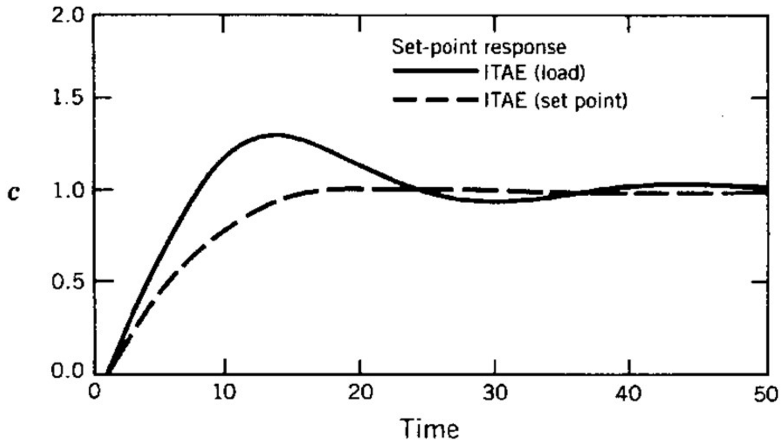
$$G(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

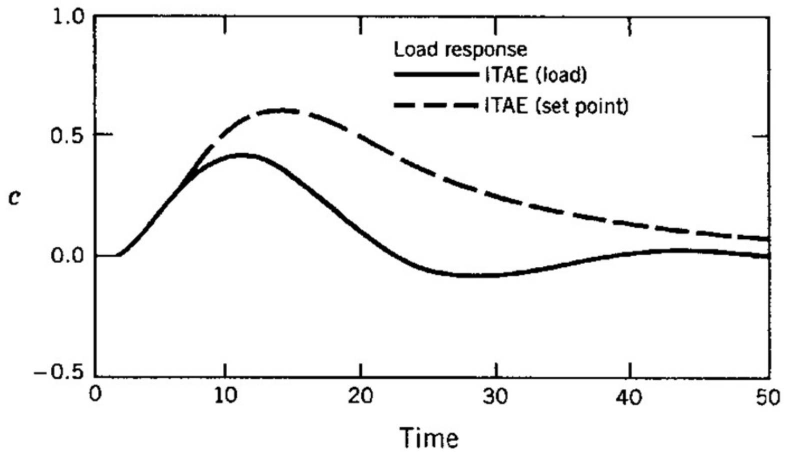
- **Approx. model**

$$\tilde{G}(s) = \frac{2e^{-4.7s}}{12s+1}$$

  - Satisfactory for this case
  - Use with care

**As the estimated time delay gets smaller, the performance degradation will be pronounced.**




- **All kinds of tuning method should be used for initial setting and fine tuning should be done!!**

# GENERAL CONCLUSION FOR PID TUNING

- The controller gain should be inversely proportional to the products of the other gains in the feedback loop.
- The controller gain should decrease as the ratio of time delay to dominant time constant increases.
- The larger the ratio of time delay to dominant time constant is, the harder the system is to control.
- The reset time and the derivative time should increase as the ratio of time delay to dominant time constant increases.
- The ratio between derivative time and reset time is typically between 0.1 to 0.3.
- The ¼ decay ratio is too oscillatory for process control. If less oscillatory response is desired, the controller gain should decrease and reset time should increase.
- Among IAE, ISE and ITAE, ITAE is the most conservative and ISE is the least conservative setting.

# TROUBLESHOOTING CONTROL LOOPS

- **Causes of performance degradation of controller**
  - Changing process conditions, usually throughput rate
  - Sticking control valve stem
  - Plugged line in a pressure or DP transmitter
  - Fouled heat exchangers, especially reboilers for distillation
  - Cavitating pumps

- **Starting points of trouble shooting**
  - What is the process being controlled?
  - What is the controlled variable?
  - What are the control objectives?
  - Are closed-loop response data available?
  - Is the controller in the M/A mode? Is it reverse or direct acting?
  - If the pressure is cycling, what is the cycling frequency?
  - What control algorithm is used? What are the controller settings?
  - Is the process open-loop stable?
  - What additional documentation is available?

- **Checking points**
  - **Components in the control loop (process, sensor, actuator, …)**
    - **Field instruments vs. instruments in central control room**
    - **Recent changes to the equipment or instrumentation (cleaning HX, catalyst replacement, transmitter span, …**
    - **Sensor lines (particles, bubbles)**
    - **Control valve sticking**
    - **Controller tuning parameters**