Guided Tutorial for Pole-zero effects on step response

Lesson 1: *The poles of a transfer function indicate the dynamic behavior of the system. A system is stable unless any of the poles lies on right-half plane (RHP).*

1. The red dot represents location of poles of the system and the blue dot for zeros. For a system described by a transfer function

$$G(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-a_1)(s-a_2)\cdots(s-a_n)}$$

roots of the denominator, a_1, a_2, …, a_n, are known as poles of the transfer function.

Open the JavaScript module. Initially, the system has two poles and one zero showing inverse response. Try to make the transfer function as $G(s)=1/(s+1)$ by deleting one pole and zero after checking the "Delete points" checkbox and click a point. (To delete one pole of complex pair, complex poles must be moved to real axis so that the double real roots can be obtained.). The response is stable.

Lesson 2: *Closer the pole to origin, more sluggish is the response of the system*

2. Click on the "Linear Move" checkbox. Then green line appears and the move will be restricted along the green line. Now move the pole to the left to value −2 by dragging it. The transfer function of the system is $G(s) = 2/(s+2) = 1/(0.5s+1)$. Thus, the time constant of the system is τ=0.5. The system is still stable. The output goes to final value of 1 faster, as the time constant is decreased.

3. Now, move the pole to −0.5. Again, it's a first order system with time constant τ=2. The step response gets sluggish. The system is stable because the pole is negative. As you move the pole closer to zero, the response gets sluggish. You can move the pole left to right and observe the speed of response.

Lesson 3: *Step response of system with poles at origin is a ramp function. Systems with a positive pole are "unstable*."

4. Move the pole location to origin. The transfer function is $G(s)=1/s$. Step response of the system is a ramp function.

5. Check the "Auto scale Y-axis" in the "Setup" tab, then move the pole location to 1. We now have a positive pole, which results in exponential growth with time. Positive pole results in an unstable system. Why does the response go to −∞?

Lesson 4: *Complex roots for a system always exist as a* ***complex conjugate pair****. The real part of the complex root determines exponential growth or damping of the system, while the* *complex part results in sinusoidal response.*

6. Add a pair of complex poles and delete the real pole. Then move the poles to about $-0.5 \pm j0.75$. (The locations of the poles are shown in the right space.) We now have two poles that are complex conjugates. The system shows slight oscillations. Negative real part of the root results in exponential decay in the amplitude of the sinusoid. Check the checkbox "Circular move". A green circle appears, and the poles will move on the circle with fixed radius which implies constant τ. By moving around the circle, observe the oscillation of the response.

7. Move the pole to the imaginary axis (pole location would be $0 \pm j0.95$ or $0 \pm j0.9$). The poles are now purely imaginary, which are marginally stable. The response of the system is oscillatory with constant amplitude. As the real part of the poles is 0, there is neither attenuation nor increase in the amplitude.

8. Now move the poles to the right-hand side of the Y-axis (usually referred to as "right half plane"). Place the poles at $0.25 \pm j0.8$. The response is sinusoidal with exponentially growing amplitude. As we move pole towards the real axis, the exponential increase becomes faster and faster, and the oscillations become less and less.

Thus, in general, a pole in right half plane results in an unstable system while a pole in left half plane represents a stable system. Pole(s) in right half plane and left half plane has positive real part and negative real part, respectively.

Lesson 5: *The response of a second order system can also be determined by its time constant and damping coefficient. Damping coefficient determines the general shape of the dynamic behavior, while time constant indicates the time scale of the response.*

We will repeat steps 6 and 7 above. First, bring the pole back to location at the negative real axis after unchecking the "Circular move". Move the poles exactly to location -1. The two poles should merge into a single pole with a concentric circle.

9. As in cases (6 and 7), click on "Circular move" and move the pole around the circle towards the imaginary axis (Y-axis). You will see the response of the system becomes increasingly oscillatory which implies the damping coefficient decreases.

10. For poles located at -1, the system is **critically damped**. The damping factor ζ=1, and the system does not show oscillatory behavior. Thus, *real repeated roots correspond to a critically damped system*.

11. The transfer function of the system in case (6), *i.e.*, poles at $-0.5 \pm j0.75$ is

$$G(s) = \frac{1}{(s + 0.5 + j0.75)(s + 0.5 - j0.75)} = \frac{1}{(1.23s^2 + 1.23s + 1)}$$

The time constant is τ=1.109 and damping factor is ζ=0.555. The system is underdamped. Thus, *complex roots correspond to an underdamped system.*

Lesson 6: *Distinct real roots correspond to an overdamped system.*

12. Uncheck all move restriction and move both poles back to −1 showing red concentric circle. Now drag a pole in the X axis direction to −2. (You can move one pole only when both poles are on real axis.) This will introduce a pole at −2. This corresponds to an overdamped system, given by

$$G(s) = \frac{2}{(s + 1)(s + 2)} = \frac{1}{(0.5s^2 + 1.5s + 1)}$$

Thus, we have a second order system with τ=0.707 and ζ=1.06. This is an overdamped system.

Lesson 7: *Right half plane zero results in an inverse response. An overshoot is observed if we have a left half plane zero that is closer to the origin than all the poles of the transfer function.*

13. At this point, we should have two poles, one at −1 and the other at −2. Let us now introduce a zero and see its effect on the dynamics of the system. Check the "Add zero" checkbox and the click exactly at (1, 0) to introduce a single zero at 1. The zero will be shown by a blue dot. We have introduced a zero in right half plane. The system shows inverse response, *i.e.*, on introducing a positive step change, the output first decreases, and then increases to settle down at the steady state value of 1. **Inverse response will be observed whenever we have a zero in right half plane.**

14. Now check the "Linear move" checkbox Move the zero to location −0.5. As the zero is now in the left half plane, we do not see inverse response as before. However, the system shows an overshoot. This is because the zero is closer to the origin than any of the poles. This is usually true in simple cases.

15. Move the zero to location −2.5. There is neither inverse response nor an overshoot observed.

You can now play with this module by introducing more poles and zeros (number of zeros should not exceed the number of poles), moving them around and seeing the response of higher order systems with various pole-zero locations.