T. Advances in Advanced Process Control

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Advances in Advanced Process Control

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Outline

- Review of Model Predictive Control
- Constrained Linear Quadratic Regulation
- Constrained Model Predictive Disturbance Attenuation
- Batch Model Predictive Control
- Conclusions

What is MPC?

What we have: Finite horizon optimal control

What we want to do: Infinite horizon feedback control

⇓

Model Predictive Control

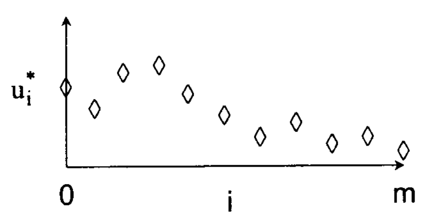
Open Loop Optimal Control

$$\min \quad \sum_{i=0}^{m} x_i^T Q x_i + \sum_{i=0}^{m-1} u_i^T R u_i$$

$$\text{subject to} \quad x_{i+1} = Ax_i + Bu_i$$

$$+ \ \alpha$$

$x_0 \quad \Rightarrow$

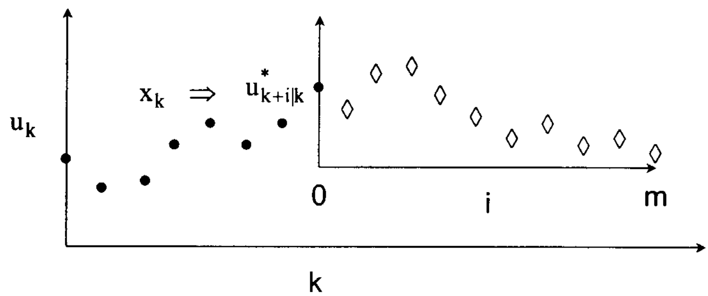


Model Predictive Control

$$\min \quad \sum_{i=0}^{m} x_{k+i|k}^T Q x_{k+i|k} + \sum_{i=0}^{m-1} u_{k+i|k}^T R u_{k+i|k}$$

$$\text{subject to} \quad x_{k+i+1|k} = A x_{k+i|k} + B u_{k+i|k}, \quad x_{k|k} = x_k$$

$+ \ \alpha$



When Is MPC Useful?

- Useless Case:

  -Unconstrained time invariant case:
  Infinite horizon linear quadratic optimal control problem admits a static feedback solution.

When Is MPC Useful? (Continued)

- Useful Case:

  -Unconstrained time varying case:
  Infinite horizon linear quadratic optimal control problem admits a static feedback solution but requires time varying characteristics of the plant over the infinite horizon

  -Constrained case:
  Both finite and infinite horizon linear quadratic optimal control problem admit open loop solution only.

Shortcomings of Classical MPC (DMC)

Truncated step response model is used

*Many model coefficients have to be stored.

*Unstable systems cannot be handled

*Truncation error is unavoidable

Advantages of State Space MPC

State space model is used

*The number of coefficient is substantially reduced.

* Unstable systems can be handled

* No truncation error

Linear Quadratic Optimal Control

$$J(x_k) = \min \sum_{i=0}^{\infty} x_{k+i|k}^T Q x_{k+i|k} + \sum_{i=0}^{\infty} u_{k+i|k}^T R u_{k+i|k}$$

subject to $x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k}, \quad x_{k|k} = x_k$

⇓

Optimal input: $u_k^* = -Fx_k$

Optimal cost: $J(x_k) = x_k^T P x_k$

where $F = (B^T PB + R)^{-1} B^T PA$

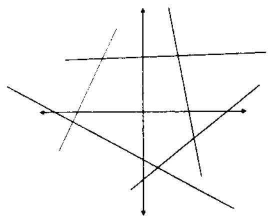
$$P = A^T (P - PB(B^T PB + R)^{-1} B^T P)A + Q$$

Input and State Constraints

$$u^{min} \le u_{k+i|k} \le u^{max}$$

$$Gx_{k+i|k} \le g$$

Assumption: the feasible set contains a neighborhood of the origin



Mixed Constrained Linear Quadratic Optimal Control

$$J(x_k) = \min \sum_{i=0}^{\infty} x_{k+i|k}^T Q x_{k+i|k} + \sum_{i=0}^{\infty} u_{k+i|k}^T R u_{k+i|k} + \varepsilon_k^T S \varepsilon_k$$

subject to $x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k}, \quad x_{k|k} = x_k$

$$u^{min} \le u_{k+i|k} \le u^{max}$$

$$Gx_{k+i|k} \le g + \varepsilon_k$$

$$\varepsilon_k \ge 0$$

Requirement: constrained asymptotic stabilizability

Why Model Predictive Control Instead of Mixed Constrained Linear Quadratic Optimal Control

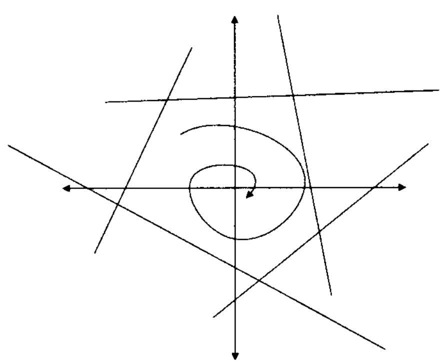
Compared to MPC, MCLQOC exhibits

- better infinite horizon performance
- better robustness property

Why MPC instead of MCLQOC

- finite dimensional formulation of MCLQOC was unknown

Key Idea in Mixed Constraints Linear Quadratic Optimal Control



For initial conditions sufficiently close to the origin, constraints are not active throughout the entire trajectory.

Maximal Output Admissible Set

(Gilbert and Tan, 1991)

Given a static state feedback control $u=-Fx$,
the maximal output admissible set is the set of all initial conditions for which constraints are not active throughout the entire trajectory.

$$O_\infty=\{x:(A-BF)^k x\in Y, k\geq 0\}$$

where

$$Y=\{x: Gx\leq g, u^{\min}\leq -Fx\leq u^{\max}\}$$

Finite Dimensional Formulation of Mixed Constrained LQOC

(Sznaier and Damborg, 1987)

Given x_k, there exists smallest N^* such that $x_{k+N^*+1|k}\in O_\infty$

$$\Downarrow$$

$$J(x_k)=\min \sum_{i=0}^{N^*} x_{k+i|k}^T Q x_{k+i|k} + x_{k+N^*+1|k}^T P x_{k+N^*+1|k} + \sum_{i=0}^{N^*-1} u_{k+i|k}^T R u_{k+i|k} + \varepsilon_k^T S \varepsilon_k$$

subject to $x_{k+i+1|k}=Ax_{k+i|k}+Bu_{k+i|k}$, $x_{k|k}=x_k$

$$u^{\min}\leq u_{k+i|k}\leq u^{\max}$$

$$Gx_{k+i|k}\leq g+\varepsilon_k$$

$$\varepsilon_k\geq 0$$

$^*x_0^T P x_0$ is the optimal cost of unconstrained LQR

Stability of Mixed Constrained LQOC

(Choi and Lee, submitted)

Lyapunov function: $J(x_k)$

Stable Plants: Globally Exponentially Stable

Marginal Plants: Exponentially Stable on Any Bounded Set B

Unstable Plants: Exponentially Stable on Any Compact Subset C of π_∞

Algorithms for N*

Exponential stability for stable (marginal) [unstable] plants:

$$a|x_k|^2 \le J(x_k) \le b|x_k|^2 \quad \text{for} \quad x_k \in R^n(B)[C]$$

$$\Delta J(x_k) \le -c|x_k|^2$$

$$\Downarrow$$

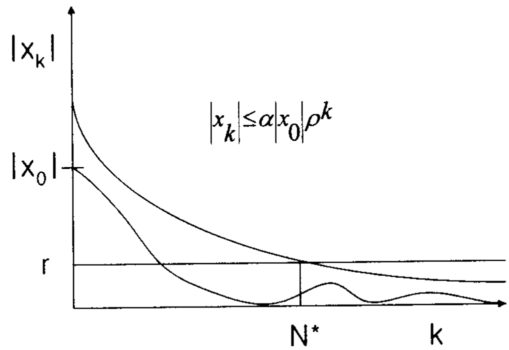
$$|x_k| \le \frac{b}{a}|x_0| \left(1-\frac{c}{b}\right)^k$$

Let r be the radius of the largest ball contained in O_∞

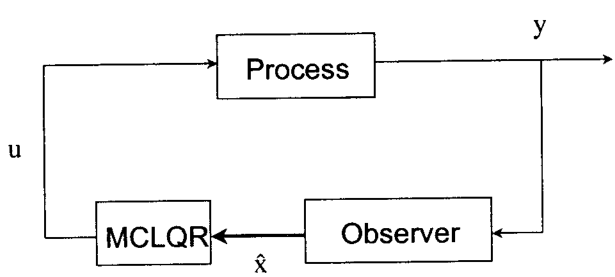
Then since $1-\frac{c}{b} \in (0,1)$, $\exists N^* \ni$

$$|x_{N^*}| \le \frac{b}{a}|x_0| \left(1-\frac{c}{b}\right)^{N^*} < r$$

Algorithms for N*



Output Feedback MCLQR



Observer: $\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L[C(A\hat{x}_k + Bu_k) - y_{k+1}]$

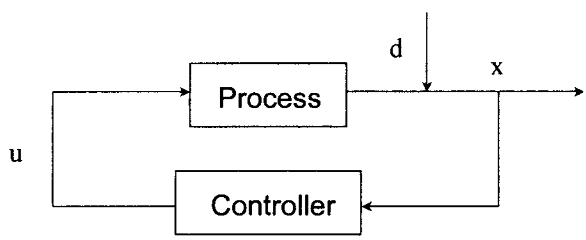
Comparison of MCLQR and MPC

* Near the steady state, MCLQR is computationally more efficient than MPC. However, away from the steady state, MPC is computationally more efficient than MCLQR.

* Performance of MCLQR is better than that of MPC.

* MCLQR is more robust than MPC.

Disturbance Attenuation



minimize the effects of disturbance to output

Why Disturbance Attenuation

Disturbances are always present in any practical control problems. However, they are not explicitly considered in regulation problem.

l_2 Gain

Closed Loop Map: $T_{zd} : d \to z = (x,u)$

l_2 Gain:

$$\|T_{zd}\|_{2e} = \sup_{K>0} \sup_{d \in l_2} \frac{\|(T_{zd} d)_K\|_2}{\|d_K\|_2}$$

where $S_K(j) = \begin{cases} s(j) & if \quad j \le K \\ 0 & if \quad j > K \end{cases}$

H_∞ Norm

For linear T_{zd} , l_2 gain reduces to H_∞ norm:

$$\|T_{zd}\|_\infty = \sup_{d \in l_2} \frac{\|T_{zd} d\|_2}{\|d\|_2}$$

Worst possible amplification of disturbance effects

H_∞ Optimal Control

Find control such that the worst possible amplification of disturbance effects is less than the desired attenuation level γ

or

Find control such that $\|T_{zd}\|_\infty \le \gamma$

H_∞ Optimal Control (Continued)

Let

$$\|T_{zd}d\|_2^2 = \frac{1}{2}\left\{\sum_{k=0}^{\infty} x_k^T Q x_k + \sum_{k=0}^{\infty} u_k^T u_k\right\}$$

Then H_∞ optimal control problem becomes

$$J_u = \min_u \max_d \frac{1}{2}\left\{\sum_{k=0}^{\infty} x_k^T Q x_k + \sum_{k=0}^{\infty} u_k^T u_k - \gamma^2 \sum_{k=0}^{\infty} d_k^T d_k\right\} \le 0$$

subject to $x_{k+1} = Ax_k + Bu_k + Dd_k$

We can assume $\gamma = 1$ using $\frac{D}{\gamma}$ instead of D

Solution of H_∞ Optimal Control

Suppose $I - D^T M D > 0$

where

$$M = Q + A^T M[I + (BB^T - DD^T)M]^{-1} A$$

Then

$$u_k^* = -B^T M \Lambda^{-1} A x_k^* \qquad d_k^* = D^T M \Lambda^{-1} A x_k^*$$

$$x_{k+1}^* = \Lambda^{-1} A x_k^* \qquad J_u(x_0) = x_0^T M x_0$$

where

$$\Lambda = I + (BB^T - DD^T)M$$

H_∞ MPC

$$J_N(x_k) = \min_u \max_d \frac{1}{2}\left\{\sum_{j=0}^{\infty} x_{k+j|k}^T Q x_{k+j|k} + \sum_{j=0}^{\infty} u_{k+j|k}^T u_{k+j|k} - \sum_{j=0}^{\infty} d_{k+j|k}^T d_{k+j|k}\right\}$$

subject to $x_{k+j+1|k} = Ax_{k+j|k} + Bu_{k+j|k} + Dd_{k+j|k}$ $x_{k|k} = x_k$

$+ \ \alpha$

Infinite horizon dynamic game problem

Unconstrained Problem is solved but is not interesting.

Constrained H_∞ MPC for Stable Plants

$$J_N(x_k) = \min_u \max_d \frac{1}{2}\left\{\sum_{j=0}^{\infty} x_{k+j|k}^T Q x_{k+j|k} + \sum_{j=0}^{\infty} u_{k+j|k}^T u_{k+j|k} - \gamma^2 \sum_{j=0}^{\infty} d_{k+j|k}^T d_{k+j|k}\right\}$$

subject to $x_{k+j+1|k} = Ax_{k+j|k} + Bu_{k+j|k} + Dd_{k+j|k}$ $x_{k|k} = x_k$

$u^{\min} \le u_{k+j|k} \le u^{\max}$ $0 \le j \le N-1$

$u_{k+j|k} = 0$ $j > N$

Problem is not defined for marginal or unstable plants

Reduction of H_∞ MPC to QP

* Reduce to a finite horizon dynamic game problem by substituting the analytic solution of the inner maximization over $[N,\infty)$
* Reduce to a QP by substituting the analytic solution of the resulting finite horizon dynamic game problem.

QP Formulation of Constrained H_∞ MPC

$$J_N(x_k) = \min_u \left\{ \frac{1}{2}\sum_{j=0}^{N-1} u_{k+j|k}^T u_{k+j|k} + \left[\frac{1}{2} x_{k|k}^T S_0 x_{k|k} - x_{k|k}^T v_0 - q_0 \right] \right\}$$

subject to $v_j = A^T[I + DP_j S_{j+1}]^T [v_{j+1} - S_{j+1} B u_{k+j|k}],\ \ v_N = 0$

$u^{\min} \le u_{k+j|k} \le u^{\max} \qquad 0 \le j \le N-1$

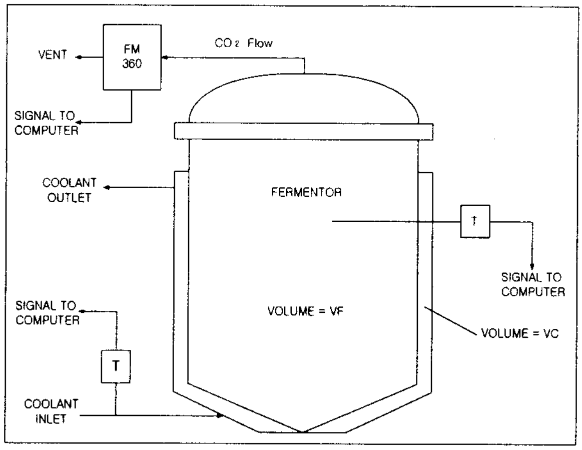
where

$$S_j = Q + A^T S_{j+1}[I + DP_j S_{j+1}]A, \quad S_N = Q_N$$

$$P_j = [I - D^T S_{j+1} D]^{-1} D^T$$

Complexity of this QP is compatible with that of standard MPC

Example



Example (Continued)

$$J(x_k) = \min_u \max_d \frac{1}{2}\left\{\sum_{j=1}^{\infty} x_{k+j|k}^T Q x_{k+j|k} + \sum_{j=1}^{N-1} u_{k+j|k}^T u_{k+j|k} - \sum_{j=1}^{\infty} d_{k+j|k}^T d_{k+j|k}\right\}$$

subject to

$$x_{k+j+1|k} = Ax_{k+j|k} + Bu_{k+j|k} + Dd_{k+j|k}$$

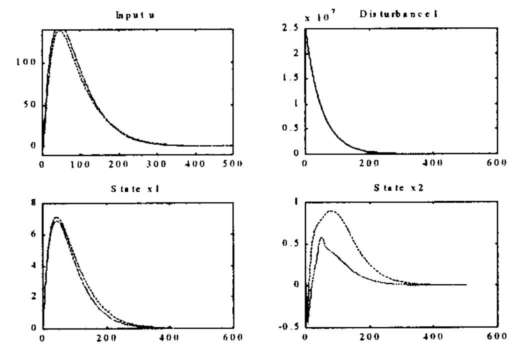
$$-80 \le u_{k+j|k} \le 80 \ (0 \le j \le 19), \quad u_{k+j|k} = 0 \quad (j \ge 20),$$

where

$$A = \begin{bmatrix} 0.9725 & 0.0261 \\ 0.2002 & 0.7158 \end{bmatrix} \qquad B = \begin{bmatrix} -0.0001 \\ -0.0080 \end{bmatrix}$$

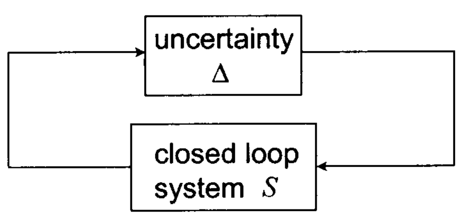
$$D = 1.0\times10^{-6}\begin{bmatrix} 0.0201 & 0.1652 \\ 0.0022 & 0.9427 \end{bmatrix} \quad Q = 500\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Example (Continued)



H_∞ MPC outperforms over MPC

Small Gain Theorem



The overall system is stable if $\|\Delta\|_{2e}\|S\|_{2e} < 1$

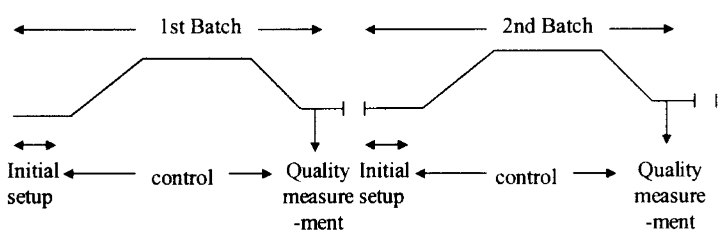
Robust Control under Unstructured Uncertainty

$$\|S\|_{2e} < \gamma \qquad \text{if} \qquad J_N(0) \le 0$$

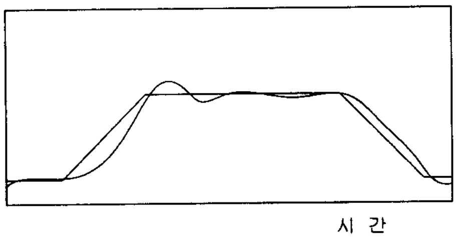
Hence if $J_N(0) \le 0$, small gain theorem dictates that constrained H_∞ MPC robustly stabilizes the plant with unstructured uncertainties satisfying

$$\|\Delta\|_{2e} < \frac{1}{\gamma}$$

Operation of Batch Process



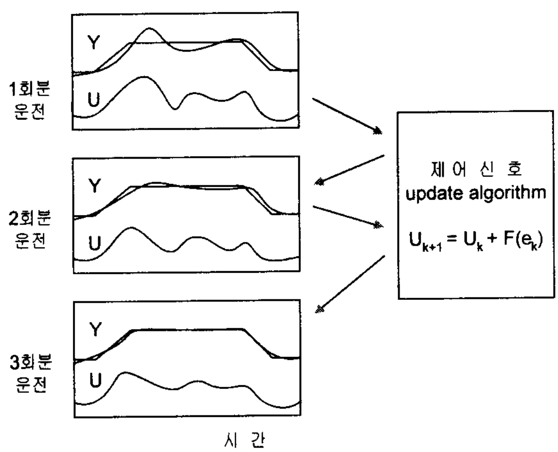
Feedback Control of Batch Process



Servo problem over a finite horizon

* removal of error by integrator is impossible

Iterative Learning Control of Batch Process



Quadratic Criterion Based Iterative Learning Control (Q-ILC)

Process Model: $\mathbf{y}_k = \mathbf{G}\ \mathbf{u}_k$

where $\mathbf{y} = [y(1)\ \ y(2)\ \ \cdots\ \ y(N)]^T, \mathbf{u} = [u(0)\ \ u(1)\ \ \cdots\ \ u(N-1)]^T$

$$\mathbf{G} = \begin{bmatrix} h_{1,1} & 0 & \cdots & 0 \\ h_{1,2} & h_{2,1} & \cdots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ h_{1,N} & h_{2,N-1} & \cdots & h_{N,1} \end{bmatrix}$$

$$\Rightarrow \mathbf{y}_{k+1} = \mathbf{y}_k + \mathbf{G}\ (\mathbf{u}_{k+1} - \mathbf{u}_k)$$

Let $\mathbf{e}_k = \mathbf{r}_k - \mathbf{y}_k,\ \Delta\mathbf{u}_{k+1} = \mathbf{u}_{k+1} - \mathbf{u}_k$

Output Error Transition Model : $\mathbf{e}_{k+1} = \mathbf{e}_k - \mathbf{G}\ \Delta\mathbf{u}_{k+1}$

Q-ILC (Continued)

$$\min_{\Delta\mathbf{u}_{k+1}} \frac{1}{2}\left\{\mathbf{e}_{k+1}^T\mathbf{Q}\mathbf{e}_{k+1} + \Delta\mathbf{u}_{k+1}^T\mathbf{R}\Delta\mathbf{u}_{k+1}\right\}$$

subject to $\mathbf{e}_{k+1} = \mathbf{e}_k - \mathbf{G}\ \Delta\mathbf{u}_{k+1}$

$+\ \alpha$

$\Downarrow$

Optimal input adjustment (unconstrained case)

$$\mathbf{u}_{k+1} = \mathbf{u}_k - (\mathbf{G}^T\mathbf{Q}\mathbf{G} + \mathbf{R})^{-1}\mathbf{G}^T\mathbf{Q}\,\mathbf{e}_k$$

Drawbacks of Q-ILC

Q-ILC can achieve zero tracking error in spite of the presence of batchwise repetitive disturbances.

However, Q-ILC cannot handle real time disturbances.

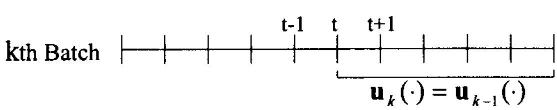
$$\Downarrow$$

Q-ILC needs to be combined with real time control

Batch Model Predictive Control (BMPC)

Let $\mathbf{G} = [G(0) \quad G(1) \quad \cdots \quad G(N-1)]$

and $\mathbf{e}_k(t) = \mathbf{e}_k$ when $\Delta u_k(t) = \Delta u_k(t+1) = \cdots = \Delta u_k(N-1) = 0$



From $\mathbf{e}_k = \mathbf{e}_{k-1} - \mathbf{G}\Delta\mathbf{u}_k$

$$\mathbf{e}_k(t) = \mathbf{e}_{k-1} - G(0)\,\Delta u_k(0) - \cdots - G(t-1)\Delta u_k(t-1)$$

Similarly

$$\mathbf{e}_k(t+1) = \mathbf{e}_{k-1} - G(0)\,\Delta u_k(0) - \cdots - G(t)\Delta u_k(t)$$

BMPC (Continued)

Hence

$$\mathbf{e}_k(t+1) = \mathbf{e}_k(t) - G(t)\Delta u_k(t), \quad \mathbf{e}_k(0) = \mathbf{e}_{k-1}(N)$$

Predictor:

$$\mathbf{e}_k(t+m \mid t) = \mathbf{e}_k(t) - \begin{bmatrix} G(t) & \cdots & G(t+m-1) \end{bmatrix} \begin{bmatrix} \Delta u_k(t) \\ \vdots \\ \Delta u_k(t+m-1) \end{bmatrix}$$

$$= \mathbf{e}_k(t) - \mathbf{G}^m(t)\Delta \mathbf{u}_k^m(t)$$

BMPC (Continued)

BMPC:

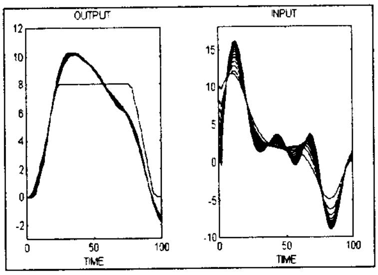
$$\min_{\Delta \mathbf{u}_k^m(t)} \frac{1}{2} \left\{ \mathbf{e}_k^T(t+m \mid t)\mathbf{Q}\mathbf{e}_k(t+m \mid t) + \Delta \mathbf{u}_k^{mT}(t)\mathbf{R}\Delta \mathbf{u}_k^m(t) \right\}$$

subject to

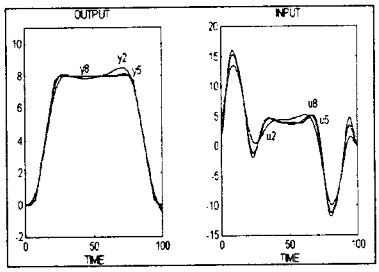
$$\mathbf{e}_k(t+m \mid t) = \mathbf{e}_k(t) - \mathbf{G}^m(t)\Delta \mathbf{u}_k^m(t)$$

$+ \ \alpha$

Comparison between MPC and QMPC

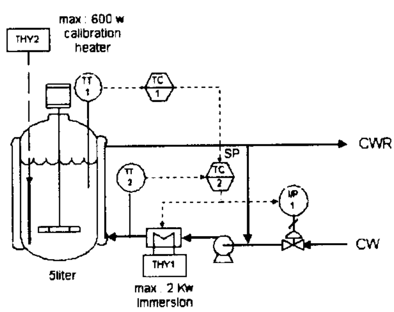


MPC

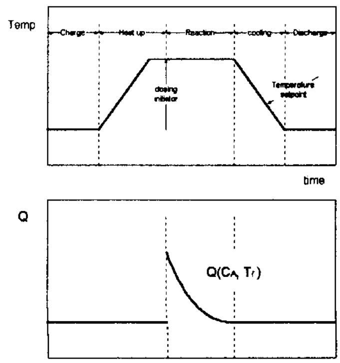


BMPC

Example

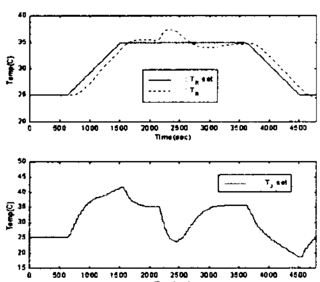


Example (Continued)

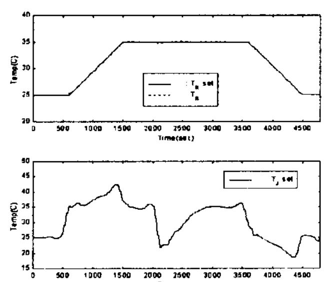


Example (Continued)

No real time disturbance



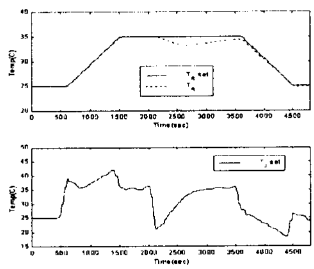
PID control



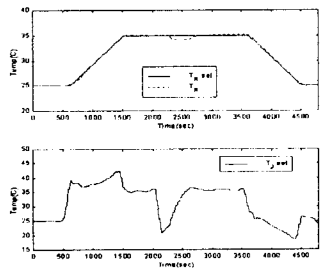
Batch MPC(6th run)

Example (Continued)

Real time disturbance: 15% change in heat of reaction



Q-ILC(7th run) (Learning only)



Batch MPC(7th run)

Conclusions

- As far as regulation concerned, constrained infinite horizon linear quadratic optimal is the ultimate alternative of MPC and its implementable algorithm is now available.
- Constrained model predictive disturbance attenuation algorithm is now available and can be used for both effective rejection of disturbances and robust control under unstructured uncertainty.
- For batch processes, conventional MPC is likely to work very poor due to batchwise disturbances and thus needs to be fortified with learning mechanism. Batch MPC that can handle both real time and batchwise disturbances is now available.