Tuning Name : Haalman method

Reference: Astrom , K. J. and Hagglund , T. PID controller 2nd edtion 'ISA, N. C., 1995

Method: Haalman은 desired open -loop transfer function으로 $G(s)C(s) = \frac{2}{3\theta s} e^{-s\theta}$ 을 선택하여 제어기를 찾는다. 2/3 는 setpoint의 step change에대한 mean square error를 최소화하는 것이다

Tuning rules

공정	K_C	τ_I	τ_D
$\frac{1}{1+s\tau} e^{-s\theta}$	$2\tau/3\theta$	τ	-
$\frac{1}{(1+s\tau_1)(1+s\tau_2)} e^{-s\theta}$	$2(\tau_1+\tau_2)/3\theta$	$\tau_1+\tau_2$	$\tau_1\tau_2/(\tau_1+\tau_2)$

Name: modulus optimum

Reference: Astrom , K. J. and Hagglund , T. PID controller 2nd edtion 'ISA, N. C., 1995

Method: $G(0)=1$, $\frac{d^n|G(i\omega)|}{d\omega^n}$ at $\omega=0$ 가 되도록 제어기를 정한다. set-point응답이 좋게된다. 다음과 같은 특수한 루프 전달함수 가진다. $G_{BO}=\frac{G(s)}{1-G(s)}=\frac{\omega^2}{s(s+\sqrt{2}\omega)}$

Tuning Rules :

공정		K_cK	τ_i	τ_d
$\frac{K}{1+s\tau}$	I	0.5	τ	
$\frac{K}{(1+s\tau_1)(1+s\tau_2)}$, $\tau_1>\tau_2$	P	$\frac{\tau_1}{2\tau_2}$		
	PI	$\frac{\tau_1}{2\tau_2}$	τ_1	
$\frac{K}{(1+s\tau_1)(1+s\tau_2)(1+s\tau_3)}$, $\tau_1>\tau_2>\tau_3$	PD	$\frac{\tau_1}{2\tau_3}$		τ_3
	PID	$\frac{\tau_1+\tau_2}{2\tau_3}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$
$\frac{K}{s(1+s\tau)}$	P	$\frac{1}{2\tau}$		
$\frac{K}{s(1+s\tau_1)(1+s\tau_2)}$, $\tau_1>\tau_2$	PD	$\frac{1}{2\tau_2}$		τ_1

Name : Symmetric optimum

Reference : Astrom , K. J. and Hagglund , T. PID controller 2nd edtion 'ISA, N. C., 1995

Method : modulus optimum 방법과 같이 $G(0)=1$, $\frac{d^n|G(i\omega)|}{d\omega^n}$ at $\omega=0$ 의 성질을 이용하는데 전달함수의 Bode 선도는 frequency $\omega=\omega_0$ 로 주위에 대칭적으로 나타난다. 다음과 같은 특수한 루프 전달함수를 가진다.

$$G_{SO}=\frac{\omega^2(2s+\omega)}{s^2(s+2\omega)}$$

Tuning Rules:

공정		K_cK_p	τ_i	τ_d
$\frac{K}{(1+s\tau_1)(1+s\tau_2)}$, $\tau_1>\tau_2$	PD	$\frac{\tau_1}{2\tau_2}$		τ_2
$\frac{K}{(1+s\tau_1)(1+s\tau_2)(1+s\tau_3)}$, $\tau_1>\tau_2>\tau_3$	PID	$\frac{\tau_1(\tau_2+4\tau_3)}{8\tau_3^2}$	$\tau_2+4\tau_3$	$\frac{4\tau_2\tau_3}{\tau_2+4\tau_3}$
$\frac{K}{s(1+s\tau)}$	PI	$\frac{1}{2\tau}$	4τ	
$\frac{K}{s(1+s\tau_1)(1+s\tau_2)}$, $\tau_1>\tau_2$	PD	$\frac{\tau_1}{8\tau_2^2}$		$4\tau_2$
	PID	$\frac{\tau_1+4\tau_2}{8\tau_2^2}$	$\tau_1+4\tau_2$	$\frac{4\tau_1\tau_2}{\tau_1+4\tau_2}$

Name : Magnitude optimum multiple integrations (MOMI)

Reference : Damir vrancic , Youbin Peng , Stanko Strmcnik 'A new PID controller tuning method based on multiple integrations ', Control Engineering Practice, Volume 7, Issue 5, May 1999, Pages 623 -633

Method : Magnitude optimum 방법에 파데 근사를 무한히 한 multiple integrations 을 사용해서 제어기 변수들을 구하는 방법이다.

Tuning Rules :

process	K_c	τ_i	τ_d
$K\dfrac{1+b_1s+b_2s^2+\cdots+b_ms^m}{1+a_1s+a_2s^2+\cdots+a_ms^m}e^{-\theta s}$	$\dfrac{A_3}{2\left(A_1A_2-A_3K_{PR}-T_dA_1^2\right)}$	$\dfrac{A_3}{A_2-T_dA_1}$	$\dfrac{A_3A_4-A_2A_5}{A_3^2-A_1A_5}$

$$A_0=K$$

$$A_1=K(a_1-b_1+\theta)$$

$$A_2=K\left[b_2-a_2-\theta b_1+\frac{\theta^2}{2!}\right]+A_1a_1 \quad \cdots$$

$$A_k=K\left((-1)^{k+1}(a_k-b_k)+\sum_{i=1}^{k}(-1)^{k+1}\frac{\theta^i b_{k-1}}{i!}\right)+\sum_{i=1}^{k-1}(-1)^{k+1-1}A_ia_{k-i}$$

Comment :

1. multiple integrations 는 일반적으로 매우 작은 오버슛과 oscillation 이 없다.
2. 공정이 큰 set에 대해서 좋거나 비교할만한 settling time 가진다.

Name : Magnitude optimum multiple integration tuning method for filtered PID controller

Reference : Damir vrancic , Stanko Strmcnik , Dani Juricic 'A magnitude optimum multiple integration tuning method for filtered PID controller ', Automatica, Volume 37, Issue 9, September 2001, Pages 1473 -1479

Method : MOMI method 에서 derivative filter 포함하고 있는 PID 제어기에 대해서 고려한 방법이다.

Tuning rules :

process	K_c
$K\frac{1+b_1s+b_2s^2+\cdots+b_ms^m}{1+a_1s+a_2s^2+\cdots+a_ms^m}e^{-\theta s}$	$\frac{A_3^2-A_1A_5+T_f(A_2A_3-A_0A_5)+T_f^2A_1A_3+T_f^3A_0A_3}{2\Delta}$

K_c/τ_i	$K_c\tau_d$
$\frac{A_2A_3-A_1A_4+T_f\left(A_2^2-A_0A_4\right)+T_f^2A_1A_2+T_f^3A_0A_2}{2\Delta}$	$\frac{A_3A_4-A_2A_5}{2\Delta}$

$$\begin{aligned}\Delta = & A_1A_2A_3+A_0A_1A_5-A_1^2A_4-A_0A_3^2 \\ & +T_f\left(A_1A_2^2+A_0^2A_5-A_0A_1A_4-A_0A_2A_3\right) \\ & +T_f\left(A_1^2A_2-A_0A_1A_3\right)+T_f^3\left(A_0A_1A_2-A_0^2A_3\right)\end{aligned}$$

$$A_k=K\left((-1)^{k+1}(a_k-b_k)+\sum_{i=1}^{k}(-1)^{k+1}\frac{\theta^i b_{k-1}}{i!}\right)+\sum_{i=1}^{k-1}(-1)^{k+1-1}A_i a_{k-i}$$

Comment :

1. MOMI 보다 필요한 input data의 수가 줄었다. - K_P와 5개의 area (A_1~ A_5)
2. Area (A_1~ A_5) 가 numercal integration을 이용한 process open – loop step response 에서 계산할수 있다.

Name : Gain and Phase Margin Specifications

Reference : Weng K. H. , Chang C. H. and Lisheng S. C. 'Tuning of PID Controllers Based on Gain and Phase Margin Specifications' Automatica VOL. 31, 497 -502, 1995

Method :

$$A_m = \frac{1}{\left|G_c(j\omega_c)G_p(j\omega_c)\right|} \quad , \quad \arg\left[G_c(j\omega_c)G_p(j\omega_c)\right] = -\pi \qquad (1)$$

$$\phi_m = \arg\left[G_c(j\omega_c)G_p(j\omega_c)\right] + \pi \, , \quad G_c(j\omega_c)G_p(j\omega_c) = 1 \qquad (2)$$

Gain and Phase Margin이 (1), (2) 로 정의 될 때

$$\arctan x \approx \begin{cases} \frac{1}{4}\pi x & (|x| \le 1), \\ \frac{1}{2}\pi - \frac{\pi}{4x} & (|x| > 1) \end{cases}$$

의 성질을 이용해서 특성화된 Gain and Phase Margin 을 사용해서 PID controller 을 튜닝하는 방법.

Tuning Rule:

Process	k_c	τ_i	τ_d
$\frac{K}{1+\tau s}e^{-\theta s}$	$\frac{\omega_c \tau}{A_m K}$	$\left(2\omega_c - \frac{4\omega_c^2\theta}{\pi} + \frac{1}{\tau}\right)^{-1}$	
$\frac{K}{(1+s\tau_1)(1+s\tau_2)}e^{-\theta s}, \tau_1 \ge \tau_2$	$\frac{\omega_c \tau}{A_m K}$	$\left(2\omega_c - \frac{4\omega_c^2\theta}{\pi} + \frac{1}{\tau}\right)^{-1}$	τ_2

$$A_m = \frac{\pi\tau}{4k_c k_p \theta}\left(1 + \sqrt{1 - \frac{4\theta}{\pi\tau_i} + \frac{4\theta}{\pi\tau}}\right)$$

$$\phi_m = \frac{1}{2}\pi - \frac{k_c k_p \theta}{\tau} + \frac{\pi}{4k_c k_p}\left(1 - \frac{\tau}{\tau_i}\right)$$

$$\omega_p = \frac{A_m \phi_m + \frac{1}{2}\pi A_m (A_m - 1)}{\left(A_m^2 - 1\right)\theta}$$

Name : Under – damped response with Specifications on Gain and Phase Margins

Reference : Weng K. H. , Chang C. H. and junhong Zhou 'Self-Tuning PID control of a plant with Under – damped response with Specifications on Gain and Phase Margins' IEEE Transactions on Control Sytem Tech. , Vol. 5, NO. 4, July 1997

Method :

Gain and Phase Margin Specifications 방법을 The Second-order plus dead time(SOPTD) Under- damped plant model에 대해서 적용한 방법이다.

Tuning Rules :

공정	K_c	τ_i	τd
$\frac{K_p}{\left(s^2+2\varsigma_p\omega_n s+\omega_n^2\right)}e^{-\theta s}$, $0<\varsigma_p<1$	$\frac{2\omega_p\left(\pi\varsigma_p\omega_n+\pi\omega_p-2\omega_p^2\theta\right)}{\pi A_m K_p}$	$\frac{2\left(\pi\varsigma_p\omega_n+\pi\omega_p-2\omega_p^2\theta\right)}{\pi\omega_n^2}$	$\frac{\pi}{2\left(\pi\varsigma_p\omega_n+\pi\omega_p^2\theta\right)}$

$$\omega_p=\frac{A_m\phi_m+\frac{1}{2}\pi A_m(A_m-1)}{\left(A_m^2-1\right)\theta}$$

Name : Non -symmetrical Optimum Method

Reference : Luc Loron , 'Tuning of PID Controllers by the Non -symmetrical Optimum Method ',Automatica . Vol 33 No1 103 -107, 1997

Method : resonant peak M 이 최소화 되는 것들 중 gain을 최적화 하는 것을 이용해서 tuning 하는 방법이다.

Tuning rules :

공정	K_c	τ_i
$\frac{K}{s(\tau s+1)}$	$K\lambda$	$\alpha\omega$

$$\lambda = \sqrt{\frac{M_C^2}{M_C^2-1}} \quad , \alpha = \left(\frac{1+\sin\phi_m}{\cos\phi_m}\right)$$