불확실한 수요예측 시나리오 상에서의 화학공정의 시설확장을 위한 강건한 장기 투자 모델

이 희만, 박 선원
한국과학기술원 화학공학과

Long range robust investment model for capacity expansion of chemical processing networks under uncertain demand forecast scenarios

HEEMAN LEE and SUNWON PARK
Department of Chemical Engineering, KAIST

1. Introduction

Capacity expansion policy making about timing and sizing of processing units is of importance since it requires substantial amount of investment cost with long payout times. Optimization model for the processing network capacity expansion problem has been extensively studied by Sahinidis et al. (1989) in the form of multiperiod MILP model, but a serious defect of this model is its deterministic nature. Real world situations are characterized by a high degree of uncertainty, and this exerts an important influence on the investment, production, and pricing decisions (Paraskevopoulos, 1991). Uncertainty involved in demand is generally focused because, in general, this parameter is the hardest to estimate accurately (Berman et al., 1994). While stochastic optimization models for capacity expansion problems that incorporates uncertainty in future demand is being widely studied (Ierapetritou et al., 1994), robust optimization technique which generates processing network capacity expansion plans under various uncertain demand scenarios has rarely been considered. For the power systems, Malcolm and Zenios (1994) developed a robust optimization model for the problem of planning capacity expansion of power systems under uncertain load forecasts, and it generated capacity expansion plans that are both solution and model robust. Solution robustness means that optimal solution from the model is almost optimal for any realization of the demand scenarios, while model robustness refers to the optimal solution which has almost no excess capacity and unmet demand. In this paper the optimization problem involves capacity expansion timing and sizing of chemical processing units, and it is the purpose of this paper to propose a robust multiperiod MILP model which is solution and model robust under several uncertain product demand scenarios. This paper is organized as follows: Theoretical background for the robust optimization technique is first reviewed followed by the general mathematical formulation. Formulated optimization model is then numerically tested through a test example to illustrate the robustness of this optimization model.

2. Theoretical Background

The capacity expansion model in this study adopts robust optimization modeling framework by Malcolm and Zenios (1994). First, two sets of variables are defined: design variables and control variables. Design variables depend only on the fixed and structural constraints which are independent of uncertain demand parameters. For the capacity expansion model of processing network, individual process capacity at each time period belongs to design variable. Control variables could be adjusted once the uncertain parameters are observed, and their optimal value depends both on the realization of uncertain parameters, and on the value of the design variables. Variables denoting purchase/sales/production amount of each chemical at each time period are treated as these control variables. Next, a set of scenarios and the probability of each scenario is introduced. In this study only demand scenarios which represent the forecasted market demand changes during the given planning horizon are considered. The robust optimization model is then developed to maximize the expected net present value subtracted by the expected deviations from optimality and penalty term for model error which is expected excess capacity in this problem. Constraints for the model consists of two types: scenario dependent and scenario independent constraints. For scenario dependent constraints optimal solution set for one scenario may not satisfy the constraint of the other scenario, and error terms as model infeasibility measure are defined. Following is the compact formulation of the robust optimization model for capacity expansion problem.

Maximize $E(\xi) - \lambda E(dev(\xi)) - \omega E(|z_s|)$ (1)

subject to: $Ax = b$: scenario independent constraints (2)

$B_s x + C_s y_s + z_s = 0 \quad \forall s \in \delta$: scenario dependent constraints (3)

where

Φ	:	objective function of the robust optimization model
δ	:	set of scenarios
λ, ω	:	penalty parameters
ξ	:	net present value
$E(\xi)$	:	expected net present value
$dev(\xi)$	:	deviation of net present value
x	:	design variable set
y_S	:	control variable set for scenario s
z_S	:	infeasibility error for scenario s

3. Mathematical Formulation

The MILP model for the planning problem is as follows.

Maximize

A) Objective function

= Expected net present value - Expected deviation - Expected excess capacity

$$= \sum_{s=1}^{NS} p_s \xi_s - \lambda \sum_{s=1}^{NS} p_s \left| \xi_s - \sum_{s=1}^{NS} p_s \xi_s \right| - \omega \sum_{s=1}^{NS} \sum_{t=1}^{NT} \sum_{i=1}^{NP} p_s Zp_{sit} \quad (4)$$

subject to:

B) Deterministic objective function

Net present value = Profit by chemical sales - Cost of chemical purchase - Process unit operating cost - Capacity expansion cost

$$= \sum_{j=1}^{NC} \sum_{l=1}^{NM} \sum_{t=1}^{NT} \sum_{s=1}^{NS} (\gamma_{jlt} Sale_{sjlt} - \Gamma_{jlt} Pur_{sjlt})$$

$$- \sum_{i=1}^{NP} \sum_{t=1}^{NT} \sum_{s=1}^{NS} \delta_{it} W_{sit} - \sum_{i=1}^{NP} \sum_{t=1}^{NT} (\alpha_{it} QE_{it} + \beta_{it} Y_{it}) \quad (5)$$

C) Production/Inventory capacity limitations

$$Q_{it} = Q_{i,t-1} + QE_{it} \qquad i = 1, \ldots, NP \quad t = 1, \ldots, NT \quad (6)$$

$$Y_{it} QE^{L}{}_{it} \le QE_{it} \le Y_{it} QE^{U}{}_{it} \qquad i = 1, \ldots, NP \quad t = 1, \ldots, NT \quad (7)$$

D) Availability/Demand limitation of each chemical

$$Pur_{sjlt} \le a^{l}_{jlt} \qquad s=1,\dots,NS \quad j=1,\dots,NC \quad l=1,\dots NM \quad t=1,\dots,NT \qquad (8)$$

$$Sale_{sjlt} \le d_{sjlt} \qquad s=1,\dots,NS \quad j=1,\dots,NC \quad l=1,\dots NM \quad t=1,\dots,NT \qquad (9)$$

E) Material balance equations for each chemical in process network

$$WI_{sijt} = \mu_{ij} W_{sit} \qquad s=1,\dots,NS \quad i=1,\dots,NP \quad j=1,\dots,NC \quad t=1,\dots,NT \qquad (10)$$

$$WO_{sijt} = \eta_{ij} W_{sit} \qquad s=1,\dots,NS \quad i=1,\dots,NP \quad j=1,\dots,NC \quad t=1,\dots,NT \qquad (11)$$

$$\sum_{l=1}^{NM} Sale_{sjlt} + \sum_{i=1}^{NP} WI_{sijt} - \sum_{l=1}^{NM} Pur_{sjlt} - \sum_{i=1}^{NP} WO_{sijt} = 0 \quad s=1,\dots,NS \quad j=1,\dots,NC \quad t=1,\dots,NT \qquad (12)$$

F) Production limitations

$$W_{sit} - Q_{it} + Zp_{sit} = 0 \qquad s=1,\dots,NS \quad i=1,\dots,NP \quad t=1,\dots,NT \qquad (13)$$

4. Test Example

The robust optimization formulation developed in section 2 is applied to the following small size problem. Fig. 1 describes the processing network for the test problem. Chemical 1 is used as a resource material for process 1. Process 1 produces chemical 2, and it is fed into process 2 to produce chemical 3. Chemical 2 can also be purchased from the market. Chemical 3 is then sold to satisfy the market demand.

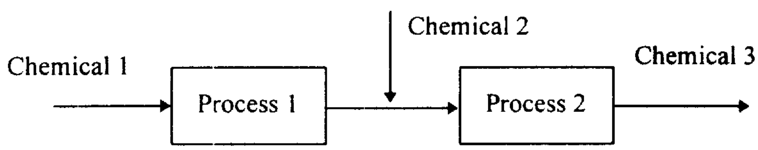

Fig. 1. Chemical processing network for the test example.

Graphs in Fig. 2 show the forecasted price data, availabilty, and demand trend of three chemicals for the next 12 time periods. It is assumed that all the price parameters have been already discounted at the specified interest rate and include the effect of taxes in the net present value. Demand of chemical 3 is uncertain, and three demand scenarios are considered. All three demand scenarios assumes constant market growth until time period 6. According to demand scenario 1 market for chemical 3 is forecasted to be reduced, while scenario 2 forecasted it to be saturated and to be constant after time period 6. Demand scenario 3 assumes constant growth of market demand for chemical 3.

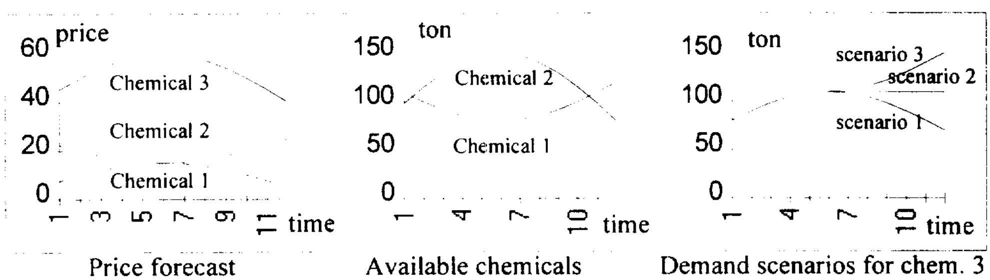

Fig. 2. Price and demand scenarios.

5. Results and Discussion

Table 1 shows the computational results with/without robust optimization technique. The upper part of the Table 1 shows the optimization result with the deterministic optimization model. As expected, the optimization model produces the best results if the actually realized demand pattern follows the forecasted demand scenario on which the deterministic model is based. A serious defect of this model occurs when the realized demand pattern doesn't follow the forecasted demand scenario. Large deviation between profits and excess capacity shown in the upper part of the Table 1 reflect that investment risk increases as the inconsistency between the realized demand pattern and the forecasted demand scenario increases. The lower part of the Table 1 shows the optimization result by robust optimization model. The model was run three times by varying λ, ω. As the penalty factors increase, expected deviation of profits as well as excess capacity is reduced. λ is focused on the effective utilization of installed capacity while ω is more on the insensitivity to the various scenarios actually realized (Malcolm and Zenios, 1994). Nevertheless it should be noted that the profit can be reduced by the emphasis on robustness of the solution.

Table 1. Computational Results. * denotes optimal plan for each scenario

Without robust optimization technique						
Plan based on	Realized Scenario 1	Realized Scenario 2	Realized Scenario 3	Expected Deviation of Profit		Excess Capacity
Scenario 1	30778.6*	32283.7	32283.7	722.4		663.8
Scenario 2	30759.4	32453.6*	32453.6	813.22		674.2
Scenario 3	30209.4	31903.6	32844.5*	1039.03		791.2
With robust optimization technique						
Plan λ, ω	Realized Scenario 1	Realized Scenario 2	Realized Scenario 3	Expected Profit	Expected Deviation	Excess Capacity
1, 0.1	30759.4	32398.1	32398.1	31742.6	786.6	578.2
1.1, 0.1	30778.6	32228.2	32228.2	31648.4	695.8	567.8
1.1, 0.2	30773.9	32223.4	32223.5	31643.6	695.8	539.5

6. Conclusions

A robust optimization model for determining capacity expansion timing and sizing of chemical processing network is presented by employing robust optimization technique in the model. Unlike the non-robust profit maximization plan which is sensitive to demand uncertainty, robust solution is shown to be insensitive to different uncertain demand scenarios by incorporating penalty terms for the expected deviation of net present values and excess capacity in the objective function. Finally, effectiveness of the model is illustrated through a test example.

Acknowledgment - Partial financial support from Korea Science and Engineering Foundation through Automation Research Center at Postech is gratefully appreciated.

Literature Cited

Berman O., Z. Ganz and J. M. Wagner, A stochastic optimization model for planning capacity expansion in service industry under uncertain demand. *Naval Research Logistics* **41**, 545-564 (1994).

Ierapetritou M. G. and E. N. Pistikopoulos, Novel optimization approach of stochastic planning models. *Ind. Eng. Chem. Res.* **33**, 1930-1942 (1994).

Malcolm S. A. and S. A. Zenios, Robust optimization for power systems capacity expansion under uncertainty. *J. Opl Res. Soc.* **45**, 1040-1049 (1994).

Paraskevopoulos D., E. Karakitsos and B. Rustem, Robust capacity planning under uncertainty. *Management Science* **37**, 787-800 (1991).

Sahinidis N. V., I. E. Grossmann, R. E. Fornari and M. Chathrathi, Optimization model for long range planning in the chemical industry. *Computers chem. Engng.* **13**, 1049-1063 (1989).