Design of Robustness PID Controllers for MIMO Systems

Truong Nguyen Luan Vu, Moonyong Lee*
School of Chemical Engineering and Technology, Yeungnam University
(mynlee@yu.ac.kr*)

1. INTRODUCTION

Nowadays, MIMO processes are mostly used in industrial. Therefore, many researchers have been presented a lot of design method which concentrated on multi-loop PID controllers; however, it is not easy to solve all problems of MIMO control system as interaction, stability, robustness, etc. Some methods are very good performance for the system that is in case of nominal form but they are going to unstable if the system include some uncertainty models.

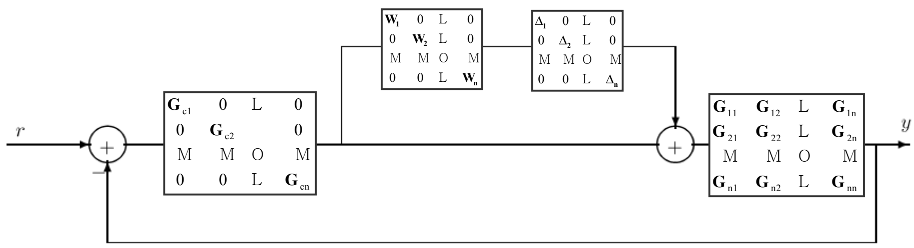

Figure 1. Multivariable feedback control system with multiplicative uncertainty.

The robust stability design can be solve this problem by synthesizing a controller which satisfies the plant uncertainty. The problem of robust performance and robust stability can be solved directly by H_∞ [1] method and μ-synthesis [2]. The proposed method is concerned to the robustness of multi-loop control system. For a given plant, the initial multi-loop PID controller parameter can be calculated by IMC theory; some famous IMC design methods are included Lee et al. [3], Jung et al. [4], etc. The weight for complex uncertainty can be found by considering a set Π of possible plant results, then H_∞ design techniques and μ-synthesis can be applied directly to select the optimal PID control parameter by detuning the closed-loop time constant λ.

2. PROPOSED DESIGN METHOD

2.1. DESIGN THE WEIGHTED FACTOR FOR COMPLEX UNCERTAINTIES

Considering the robust control theory [5], the preceding description suggests the following simple multiplicative perturbation of the uncertainty set:

$$\mathbf{\Omega}_a = \left\{ \tilde{\mathbf{G}}(j\omega) : \frac{\left|\tilde{\mathbf{G}}(j\omega) - \mathbf{G}(j\omega)\right|}{\left|\mathbf{G}(j\omega)\right|} \le \frac{\ell_a}{\mathbf{G}(j\omega)} \right\} = \left\{ \tilde{\mathbf{G}}(j\omega) : \frac{\left|\tilde{\mathbf{G}}(j\omega) - \mathbf{G}(j\omega)\right|}{\left|\mathbf{G}(j\omega)\right|} \le \ell_m \right\} \tag{1}$$

where, a nominal plant model is given by the proper transfer function $\mathbf{G}(s)$, the actual plant is represented by $\tilde{\mathbf{G}}(s)$, and the difference $\tilde{\mathbf{G}}(s)$; $\mathbf{G}(s)$ is assumed to be stable, the region to be disk shaped with radius ℓ_a, a bound on the multiplicative ℓ_m.

Thus, any member of the family $\mathbf{\Omega}$ satisfies

$$\mathbf{\Omega}_a = \left\{ \tilde{\mathbf{G}}(s) \mid \tilde{\mathbf{G}}(s) = \mathbf{G}(s)(1 + \ell_m(s)) \right\} = \left\{ \tilde{\mathbf{G}}(s) \mid \tilde{\mathbf{G}}(s) = \mathbf{G}(s)(1 + \mathrm{W}(s)\Delta(s)) \right\} \tag{2}$$

where $\mathbf{\Delta}$ is an arbitrary stable transfer function satisfying the norm condition.

$$\|\Delta\|_\infty = \sup_{\omega}|\Delta(j\omega)| \le 1 \tag{3}$$

For SISO system, each $\Delta_i(j\omega)$ is a scalar**,** so that Δ becomes a diagonal matrix with complex entries. In MIMO case, Δ is block-diagonal.

$$\Delta = \begin{bmatrix} \Delta_1 & & & \\ & \Delta_2 & & \\ & & \ddots & \\ & & & \Delta_m \end{bmatrix},\ \|\Delta_i\|_\infty \le 1. \tag{4}$$

The weight $|\boldsymbol{W}(j\omega)|$ satisfies

$$|\boldsymbol{W}(j\omega)| \ge |\boldsymbol{W}(j\omega)\Delta(j\omega)| = \left|\frac{\tilde{\mathbf{G}}(j\omega)-\mathbf{G}(j\omega)}{\mathbf{G}(j\omega)}\right| \ge \max_{\tilde{\mathbf{G}}\in\boldsymbol{\Omega}}\left|\frac{\tilde{\mathbf{G}}(j\omega)-\mathbf{G}(j\omega)}{\mathbf{G}(j\omega)}\right| \tag{5}$$

In MIMO system, it is very difficult to find exactly the multiplicative weight that can satisfy Eq. 5, in this paper, the proposed method can solve this problem easily and exactly as following procedure.

- Step 1: Defining the multiplicative (relative) uncertainty by using Eq. 5
- Step 2: Estimating the real data from Step 1 and transferring it into the mathematical model by using the Curve Fitting (CF) Method [7].

Finally, the multiplicative weight can be determined in case of the first order, second order or third order, etc., which depend on the process system.

2.2. DESIGN OF THE MULTI-LOOP IMC PID CONTROLLER

We can design the initial PID controller by using any design method which follows IMC control theory. In this research, the design method which published by Lee et al. is selected.

By following the multi-loop feedback control system, the ideal controller of the *i*th loop can be designed by

$$G_{ci}(s) = \frac{(G_{ii-}(s))^{-1}}{(\lambda_i s+1)^{n_i} - G_{ii+}(s)} \tag{6}$$

where G_{ii+} is the non-minimum part of $G_{ii.}$; G_{ii-} is minimum phase of G_{ii}; n is chosen for the IMC controller to be realizable; λ_i is adjustable constant for system robust stability and robust performance. Since $G_{ii+}(0) = 1$, Eq. 6 can be rewritten in a Maclaurin series with an integral term as

$$G_{ci}(s) = \frac{1}{s}\left(f_i(0) + f_i'(0)s + \frac{f_i''(0)}{2}s^2 + 0(s^3)\right) \tag{7}$$

where $f_i(\mathrm{s}) = G_{ci}(\mathrm{s})\mathrm{s}$

The standard PID control algorithm is given by

$$G_{ci}(s) = K_{ci}\left(1 + \frac{1}{\tau_{Ii}}s + \tau_{Di}s\right) \tag{8}$$

Comparing (7) with (8) gives the analytical tuning rules for the proportional gain and the derivative time constant of the multi-loop PID controller as follows

$$K_{ci} = f_i'(0) \quad ; \quad \tau_{Di} = \frac{f_i''(0)}{2K_{ci}} \tag{9}$$

At low frequencies, the interaction effect between control loops can not be neglected, so that we can express the closed-loop transfer function in the Maclaurin series and compare with the desired closed-loop response of the *i*th that design for IMC controller [6]. Finally, one can get the integral time constant of the multi-loop PID controller as follows

$$\tau_{Ii} = -\frac{(G_{ii+}'(0) - n_i\lambda_i)K_{ci}}{(\mathbf{G}^{-1}(0))_{ii}} \tag{10}$$

2.3. TUNING METHOD FOR MULTI-LOOP PID CONTROLLERS

According to the multivariable feedback control system that contains multiplicative uncertainty, it is shown in Fig. 1; the closed-loop transfer function can be represented as

$$\boldsymbol{H}(s) = (\boldsymbol{I} + \boldsymbol{G}_p \boldsymbol{G}_c)^{-1} \boldsymbol{G}_p \boldsymbol{G}_c \boldsymbol{W}_I \boldsymbol{\Delta}_I \tag{11}$$

The robust stability condition becomes

$$\left\| T\bar{l}_m \right\|_\infty = \left\| (\boldsymbol{I} + \boldsymbol{G}_p \boldsymbol{G}_c)^{-1} \boldsymbol{G}_p \boldsymbol{G}_c \boldsymbol{W}_I \boldsymbol{\Delta}_I \right\|_\infty = \sup_{\omega} \left| (\boldsymbol{I} + \boldsymbol{G}_p \boldsymbol{G}_c)^{-1} \boldsymbol{G}_p \boldsymbol{G}_c \boldsymbol{W}_I \boldsymbol{\Delta}_I \right| < 1 \tag{12}$$

The LHS of the Eq. 12 is the infinity norm. This means that robust stability imposes a bound on ∞ norm of complementary sensitivity function *T (jω)* weighted by $\bar{l}_m(\omega)$. The Eq. 12 contains the adjustment parameter λ, the MIMO control system will satisfy the robust stability if the optimal value of λ can be found by Eq. 12.

3. CASE STUDY

Example 1: Consider the Wood and Berry (WB) distillation column model [7].

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{13}$$

In case of gain, delay and time constant uncertainty in each element of this model, the multiplicative uncertainty can be defined by Eq. 1 and the multiplicative weighted factor will be found by Eq.5, the upper boundary of the multiplicative weighted factor can be calculated by CF method and then the third order of the multiplicative weighted factor will be found by converting into zero/pole/gain form. For two main loops, the multiplicative weighted factor will be found as

$$W_{Iii} = \{\frac{3.683(s+0.579)(s^2+0.099s+0.013)}{(s+0.077)(s^2+1.401s+1.176)}, \frac{4.129(s+0.227)(s^2+0.106s+0.009)}{(s+0.123)(s^2+0.598s+0.225)}\} \tag{14}$$

In the simulation study, step changes in set-point were sequentially made in the individual loops. n_i in (6) was chosen as 1 for all loops according to the process model order. By using Eq. 12, the optimum λ values were found as 4.31 and 7.6 for loop 1 and 2, respectively. All the control parameters used in the example are listed in Table 1 and Table 2.

Table 1. PID parameter and IAE values by the various methods

Process		Proposed	BLT	DLT
WB	K_c	0.25, -0.07	0.38, -0.08	0.34, -0.14
	τ_I	8.36, 7.38	3.25, 23.60	17.20, 15.90
	τ_D	0.09, 0.39	-	15.49, 1.36
	IAE_1	5.83, 6.17	5.11, 16.8	7.34, 5.64
	IAE_2	5.27, 10.7	3.38, 32.7	6.59, 10.91
	IAE_t	27.97	57.99	30.48

Table 2. Total IAE values of the closed-loop responses under the parameter changes.

WB (±10)	27.50, 29.97	51.89, 61.26	30.41, 37.19
WB (±20)	29.14, 33.46	48.54, 67.80	37.91, 42.83
WB (±30)	32.60, 39.82	45.99, 76.15	43.14, 47.66

Fig.2 shows the closed-loop response by several methods. The superior performance of the proposed method is readily apparent from the fig. 1 and the IAE values in Table 1 and Table 2.

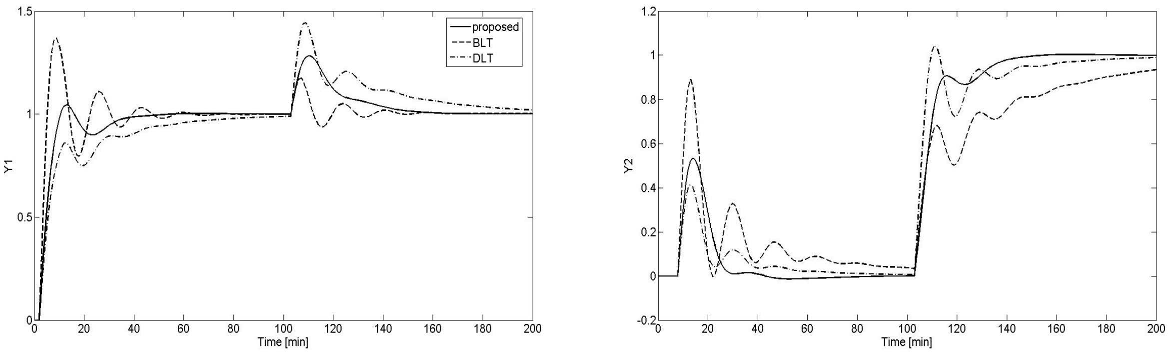

Fig. 2. Closed-loop responses to sequential step changes in set-point for WB column

4. CONCLUSION

The simulation study shows that the proposed method has more desirable feature in robust stability and performance. The IAE values imply that the multi-loop PID controller is designed by proposed method will make the MIMO control system more robust stability and performance over those by the Biggest Log Modulus (BLT) and the Decentralized Lambda Tuning (DLT) method.

ACKNOWLEDGEMENT

This research was supported by the Second Phase of BK (Brain Korea) 21.

Reference

1. K. Zhou, J. C. Doyle, and K. Glover, “Robust and optimal control,” Upper Saddle River, NJ: Prentice-Hall, 1996.
2. G. J. Balas, J. C. Doyle, K. Glover, A. Packard, and R. Smith, “μ-synthesis and Synthesis Toolbox,” Natick, MA: The Mathworks, Inc., 1995
3. M. Lee, K. Lee, C. Kim, and J. Lee, “Analytical design of multi-loop PID controllers for desired closed-loop responses,” *AIChE Journal*, vol. 50, no. 7, pp. 1631-1635, 2004.
4. J. Jung, J. Y. Choi, and J. Lee, “One-parameter method for a multi-loop control system design” *Ind. Eng. Chem. Res.*, vol. 38, pp. 1580-1588, 1999.
5. M. Morari and E. Zafiriou, “Robust process control,” Prentice Hall, 1997.
6. C.G. Economou and M. Morari, “Internal model control 6: multi-loop design,” *Ind. Eng. Chem. Proc. Process. Des. Dev.*, vol. 25, pp. 411-419, 1989.
7. H. H. Gene, S. S. Mohammed and D. M. Paul, “Analytical, numerical, and computational methods for science and engineering,” Prentice Hall, Englewood Cliffs, New Jersey 07632.