점탄성 유체의 전기영동 흐름에 대한 Helmholtz-Smoluchowski 속도

임재영, 이원민, 박흥목*
서강대학교 화공생명공학과
(hmpark@sogang.ac.kr*)

Helmholtz-Smoluchowski velocity for viscoelastic electroosmotic flows

J.Y. Lim, W.M. Lee, H. M. Park*
Department of Chemical and Biomolecular Engineering, Sogang University
(hmpark@sogang.ac.kr*)

1. Introduction

Recent advent of various microfluidic devices gives rise to renewed interest in the electroosmotic flows Liquids at the solid-liquid interface ionizes because of the zeta potential at the wall. The electric double layer is a very thin region close to the charged surface in which there is an excess of counterions over coions to neutralize the surface charge.

Therefore, a net fluid flow in the desired direction is induced when imposing external electric field. Since the velocity at the solid surface can be assumed to be non-slip and the driving force for fluid flow is acting only within the thin electric double layer, the fluid in a capillary moves as in plug flow under the action of electroosmotic force, and at the limit of very thin double layer the velocity slips at the wall ; it goes from a uniform finite velocity to zero discontinuously at the wall. For an actual finite-thickness electric double layer the velocity drops continuously across the layer to zero at the wall. To resolve the velocity field within the electric double layer it is necessary to place many grid points massively clustered near the wall, which cause difficulties in the numerical solution of electroosmotic flows. A well-known practical alternative that alleviates the above mentioned difficulties is the adoption of the Helmholtz-Smoluchowski velocity, which is an artificial slip velocity imposed at the solid surface to take care of the electroosmotic force in capillaries. The Helmholtz-Smoluchowski slip velocity for Newtonian fluids is well documented and adopted widely by researchers in this field.

2. Governing equations

We consider a three-dimensional, incompressible and isothermal flows of viscoelastic fluids. The governing equations may be written in dimensionless variables as :

$$\nabla^{*2}\psi^* = \beta^e \sinh(\alpha\psi^*) \tag{1}$$

$$\nabla^* \cdot \mathbf{v}^* = 0 \tag{2}$$

$$\frac{\partial \mathbf{v}^*}{\partial t} + \mathbf{v}^* \cdot \nabla^* \mathbf{v}^* = -\nabla^* P^* + \frac{2(1-\beta)}{Re}\nabla^{*2}\mathbf{v} + \nabla^* \cdot \boldsymbol{\tau}^* + 2\delta \sinh(\alpha\psi^*)\nabla^*\phi^* \tag{3}$$

$$\lambda^*\left(\frac{\partial \boldsymbol{\tau}^*}{\partial t^*} + \mathbf{v}^* \cdot \nabla^* \boldsymbol{\tau}^*\right) = \frac{2\mu\beta}{Re}\mathbf{D}^* + \lambda^*(\boldsymbol{\mathcal{L}}^*\boldsymbol{\tau}^* + \boldsymbol{\tau}^*\boldsymbol{\mathcal{L}}^{T*}) - g\boldsymbol{\tau}^* \tag{4}$$

3. Results

We consider an electroosmotic flow through a square straight microchannel shown in Fig. 1. The following typical parameter values for the electroosmotic flows are assumed.

For a long microchannel where the end effects are negligible we may assume $v^x = v^y = 0$, $\frac{\partial}{\partial z} = 0$ and $\frac{\partial}{\partial t} = 0$ at the steady state, and governing equations (1)-(4) are reduced to a balance between viscous, viscoelastic and electric forces.

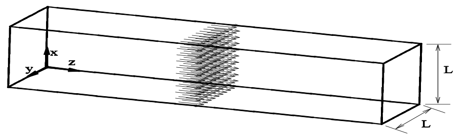

Fig. 1.

First, a set of grid points is set up in the normal direction in the electric double

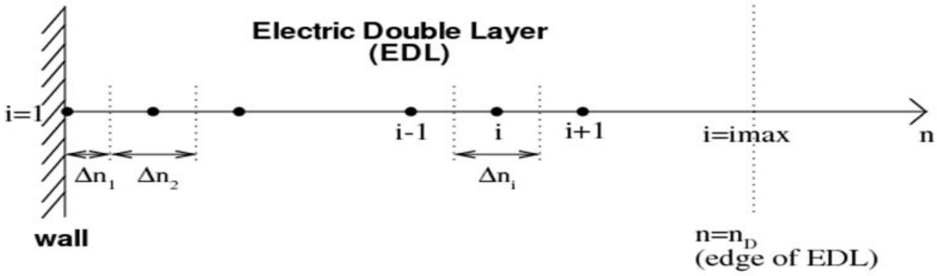

Fig. 2.

layer as shown in Fig.2, where we will seek the value of $\frac{\partial v_z}{\partial n}$.
the Helmholtz-Smoluchowski velocity U_{HS} is found by integration$(\frac{\partial v_z}{\partial n})$ over the electric double layer.

$$U_{HS} = \int_{n=0}^{n_D} (\frac{\partial v_z}{\partial n}) dn = \sum_{i=1}^{imax} (\frac{\partial v_z}{\partial n})_i \Delta n_i$$

Fig.3a. shows the comparison of the Helmholtz-Smoluchowski velocity UHS obtained from the cubic algebraic

$$a(\frac{\partial v_z}{\partial n})^3 + b(\frac{\partial v_z}{\partial n})^2 + c(\frac{\partial v_z}{\partial n}) + d = 0$$

equation :

Fig.3b is the same comparison within the electric double layer whose dimensionless thickness is shown to be about 0.005.

It is shown that U_{HS} and v_z are almost the same over the channel cross section as well as in the electric double layer.

Fig.4 shows comparisons of τ_{nz}, τ_{zz} and τ_{nn} obtained by solving the cubic equation and those obtained by solving governing partial differential equations within electric double layer.

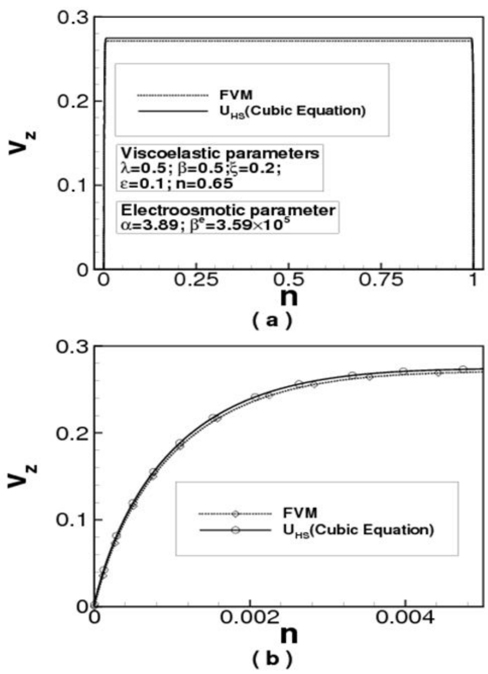

Fig. 3.

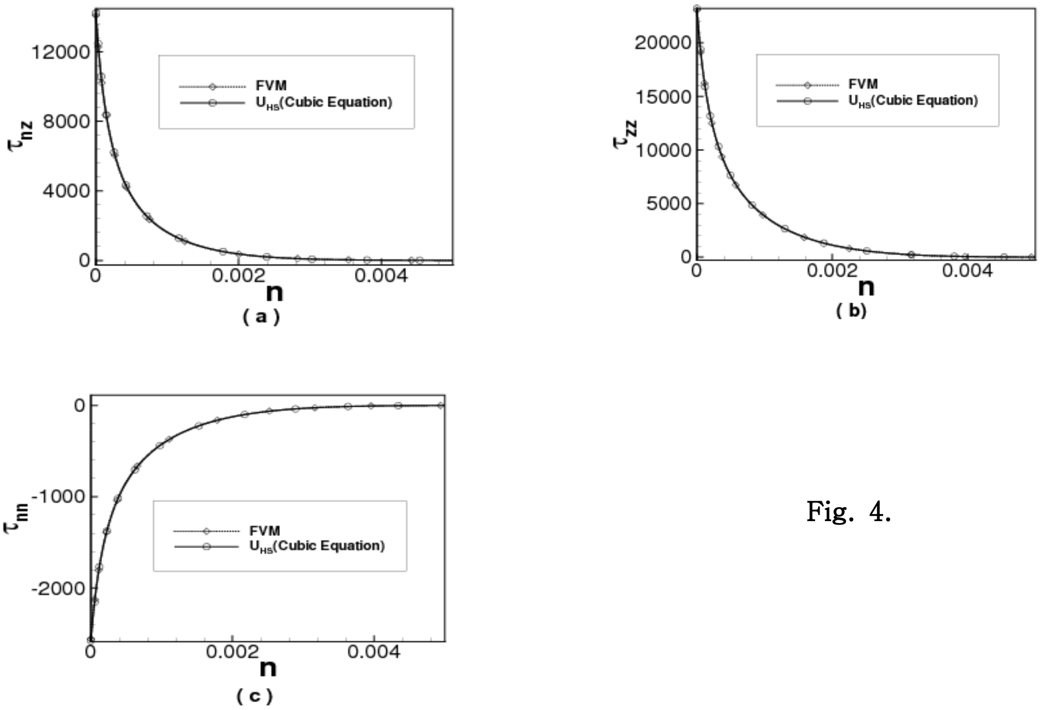

Fig. 4.

4. Conclusion

A simple method is devised to find volumetric flow rate of viscoelastic electroosmotic flows through microchannels.

It is based on the concept of the Helmholtz-Smoluchowski velocity which is widely adopted in the electrotic flows of Newtonian fluids. The Helmholtz-Smoluchowski velocity for viscoelastic fluids can be found by solving a simple cubic algebraic equation. The volumetric flow rates obtained using this Helmholtz-Smoluchowski velocity are found to be almost the same as those obtained by solving the governing partial differential equations for various viscoelastic fluids. Many biofluids such as blood and DNA solutions are viscoelastic and the volumetric flow rates of these viscoelastic electroosmotic flows through microchannels can be found easily using the method developed in the present investigation. The rheological properties of DNA suspensions are tabulated in Kolodner, where one can find out the DNA concentration above which the usual assumption of Newtonian fluid breaks down.

5. Reference

[1] R. J. Yang, L. M. Fu and Y. C. Lin 2001 Electroosmotic flow in microchannels. J. Colloid Interface Sci. **239**, 98–105.

[2] H. Keh and H. C. Tseng 2001 Transient electrokinetic flow in fine capillaries. J. Colloid Interface Sci. **242**, 450–459.

[3] R. F. Probstein, Physicochemical Hydrodynamics, Butterworth Publishers, MA, 1989.

[4] S. Das, S. Chakraborty 2006 Analytical solutions for velocity, temperature and concentration distribution in electroosmotic microchannel flows of a non-Newtonian bio-fluid. Analytica Chimica Acta **559**, 15–24.