CRYSTALLIZATION

1. Basic Concept

2. Crystal Size Distribution

3. Batch Crystallization

4. Recrystallization

5. Conclusion

1 BASIC CONCEPT

결정은 비생물 물질 중 가장 정렬된 3차원 배열을 하고 있다.
이러한 격자는 constituent 입자로 구성되어 있으며, 원자·이온·분자일 수 있다.

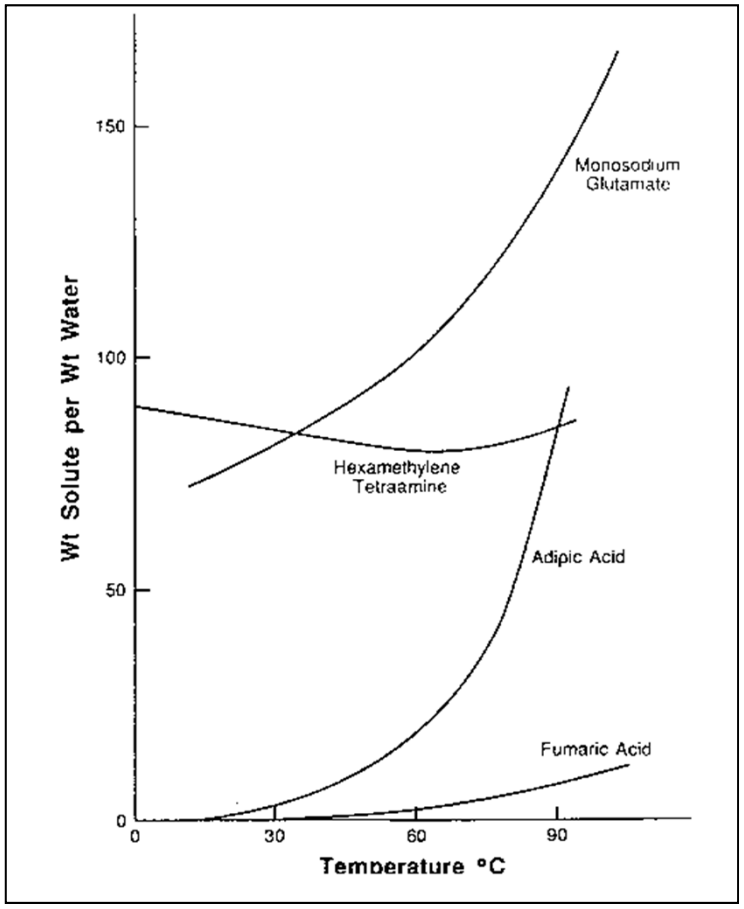
- 결정화의 중요성
  1. 결정은 순물질의 일반적인 형태이다.
  2. 일정한 형태의 결정의 생산은 마무리 작업을 용이하게 한다.
  3. 결정화는 생성물의 외양을 개선한다.

특성 { Saturation, Purity, Nucleation, Single crystal growth

1. Saturation

포화 : 용액상에서 열역학적으로 안정한 용질의 최대 농도
→ 상평형의 결과

고체 결정상과 주변의 용액(모액) 사이의 chemical potential의 결과



- 다양한 농도-온도 계수를 보여준다.
- 각각의 계수는 특정 용질과 용매의 성질
- 포화상태에서 용액은 과포화 상태로 갈 수 있다.
- 이러한 과포화 용액은 열역학적으로 불안정하다.

Fig. 1. Solubility versus temperature. Solubility almost always varies with temperature, but in widely different ways

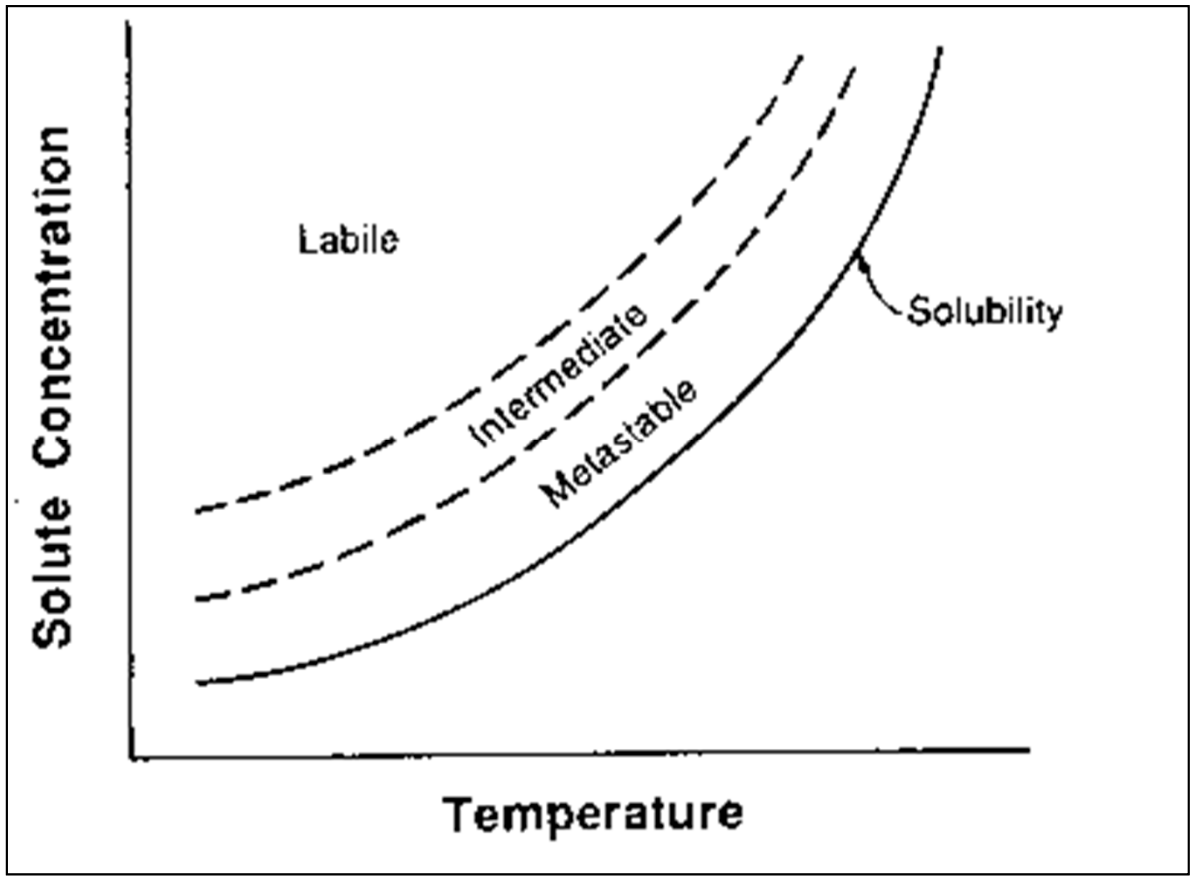


Fig. 2. Regions of supersaturation. The nature of crystal nucleation and growth is often different in the three regions shown above the solubility.

Metastability:

작은 결정의 표면 에너지의 결과

Metastable region :

용질의 평형 농도 이상 함유, 결정이 존재하지만
새로운 결정핵은 생성되지 않는다.

Intermediate region:

결정의 성장과 생성이 이루어진다.

Lable region:

핵이 자발적으로 clear solution으로 부터 생성

2. Purity

Bioseparation에서 가장 중요

quantify purity by use of a factor E,

$$E_A = \frac{\text{weight of solute A in crystal cake}}{\text{weight of solute A in filtrate}} \qquad (1\text{-}1)$$

a similar equation for impurity B

$$E_B = \frac{\text{weight of impurity B in crystals}}{\text{weight of impurity B in filtrate}} \qquad (1\text{-}2)$$

The separation factor β

$$\beta = E_A / E_B \qquad (1\text{-}3)$$

→ β가 클 경우 분리는 효과적이다.

3. Nucleation

* 결정핵 생성 mechanism에 의한 분류

(1) homogeneous nucleation : 단지 과포화에 의해 용액으로부터 결정이 생성

(2) heterogeneous nucleation : 액체내 결정의 성장에 큰 영향을 미치는 비용해성 입자로 부터 결정이 생성

(3) secondary nucleation : 다른 결정간의 접촉에 의해 핵이 생성

(4) attrition nucleation : 기존의 결정이 파쇄되어 새로운 결정으로 성장
→ secondary nucleation과 유사

모든 결정핵 생성의 mechanism은 새로운 crystal의 형성에 기여하지만 자세한 각 반응의 mechanism은 밝혀지지 않았다. 대신 nucleation rate에 대한 실험식이 사용된다.

Nucleation rate

$$\frac{dN}{dt} = k_n (c - c^*)^{'}$$

$$= B \qquad (1-4)$$

Where c : 용액의 농도
c^* : 포화 농도
k_n and I : empirical parameter
B : nucleation rate

4. Single crystal growth

"difussion controlled" is expressed as

$$\frac{dM}{dt} = kA(c - c^*) \qquad (1-5)$$

Where, k : 물질전달계수 A : 결정면적
c : 용액의 농도 c^* : 포화농도

교반에 의해 결정과 주변 용액간의 상대속도가 증가하면서 성장속도는 종종 최대치에 이른다. 이러한 경향은 작은 상대속도로 제어되는 확산이나 높은 상대속도에서 제어되는 표면반응에서 나타난다.

반응속도가 과포화에 선형적으로 의존한다고 가정하면 성장속도는

the growth rate is given by

$$\frac{dM}{dt} = \frac{A}{1/k + 1/\kappa}(c - c^*) \qquad (1-6)$$

$$l = \frac{6M}{\rho A} \qquad (1-7)$$

For a cubic of side s
$M=\rho s^3$, $A=6s^2$, $l=s$

$$M = \rho \Phi_v l^3 \qquad (1-8)$$

$$A = 6\Phi_A l^2 \qquad (1-9)$$

$$\frac{d}{dt}\left(\rho \Phi l^3\right) = \left(\frac{6\Phi_A l^2}{1/k + 1/\kappa}\right)(c - c^*)$$

$$\frac{dl}{dt} = \left(\frac{2\Phi_A / \Phi_v \rho}{1/k + 1/\kappa}\right)(c - c^*)$$

$$= k_g (c - c^*) = G \qquad (1-10)$$

Example 1. Crystallization of Adipic Acid. 10kg of adipic acid is slurried in 13.1 kg of water and heated to 90°C to solubilize the acid. The solution is then filtered to remove insoluble imputities. During the heating and filtration, 10% of the water is evaporated. The clarified solution is cooled to 35°C and filtered. The solubility at 35°C is 0.05 kg acid per kg water.

Determine the weight of crystals recovered in this operation.

Solution. We need two mass balances to solve this example. The first is a water balance:

water in = water in liquor + water evaporated

13.1 kg = water in liquor + 0.1 (13.1 kg)

water in liquor = 11.79 kg

The second balance, for the adipic acid, is

weight crystals in = weight of crystals + weight remaining in mother liquid

10 kg = weight of crystals + 0.05(11.79 kg)

weight of crystals = 9.41 kg

The recovery can be increased by lowering the final temperature. Note that if the crystals deposited were hydrated, their water of hydration must be included in the mass balances.

Example 2. Separation of Soy Sterols. A mixture of stigmasterol and sitosterol weighing 2040 kg is divided into two fractions by crystallization. The original mixture contains 86.5 % stigmasterol. The recovered crystals are 96.6% stigmasterol and weigh 1137 kg. The solids in the liquor contain 74.6% stigmasterol, found by evaporation to dryness.

Determine the β value for this separation.

Solution. We first calculate the E values for stigmasterol and sitosterol using Eqs. (10.1-1) and (10.1-2):

$$E_{\text{stigmasterol}} = \frac{1137\text{kg} \times 0.966\text{kg stigmasterol/kg}}{(2040 - 1137)\text{kg} \times 0.746\text{kg stigmasterol/kg}}$$

$$= \frac{1098}{673.6} = 1.63$$

$$E_{\text{sitosterol}} = \frac{1137\text{kg} \times (1.000 - 0.966)\text{kg sitosterol/kg}}{(2040 - 1137)\text{kg} \times (1.000 - 0.746)\text{kg sitosterol/kg}}$$

$$= \frac{38.66}{229.4} = 0.168$$

Thus we find $$\beta = \frac{E_{\text{stigmasterol}}}{E_{\text{sitosterol}}} = \frac{1.63}{0.17} = 9.6$$

Example 3. Streptomycin Growth.

As part of a fundamental study of crystallization, you have mounted a single crystal of this antibiotic in a clean, supersaturated solution which flows past the crystal at an adjustable rate. By examining it microscopically, you find that the crystal grows at a rate almost independent of crystal size.

At 10% supersaturation, this rate is 0.02 cm/hr at a flow of 5.0cm/sec and is 0.04cm/hr. When the flow is six times larger. From mass transfer correlations, you expect that the mass transfer coefficient varies with the square root of the fluid velocity.

Estimate k and κ from these observations.

Solution. We see that the growth rate is expected to be independent of crystal size l, so that our observation is consistent with the approximate analysis given in this section. We are also told that

$k = av^{1/2}$ where a is a constant. Thus insert the values into Eq.(1-10)

we find $$0.02\frac{cm}{hr} = \frac{2(\Phi_A/\Phi_v\rho)(c - c^*)}{[1/(a\sqrt{5.0 cm/sec}) + 1/\kappa]}$$

$$0.04\frac{cm}{hr} = \frac{2(\Phi_A/\Phi_v\rho)(c - c^*)}{[1/(a\sqrt{30 cm/sec}) + 1/\kappa]}$$

Solving, we find

$$\frac{a\Phi_A(c - c^*)}{\Phi_v\ \rho} = 0.0055\left(\frac{cm\,sec}{hr}\right)^{1/2}$$

and $$\frac{\kappa\Phi_A(c - c^*)}{\Phi_v\ \rho} = 0.065\left(\frac{cm}{hr}\right)$$

2 CRYSTAL SIZE DISTRIBUTIONS

1. Population density

the population density n :

$$n = \lim_{\Delta l \to 0} \frac{\text{change in number}}{\text{change in size}} \qquad (2-1)$$

$$\mu_j = \frac{\int_0^l l^j\, n(l)\, dl}{\int_0^\infty l^j\, n(l)\, dl} \qquad (2-2)$$

First, if $j = 0$, $\mu_j = \dfrac{\int_0^l n(l)\, dl}{\int_0^\infty n(l)\, dl}$ (2-3)

If $j = 3$, $\mu_j = \dfrac{\rho\Phi_v \int_0^l l^3\, n(l)\, dl}{\rho\Phi_v \int_0^\infty l^3\, n(l)\, dl}$ (2-4)

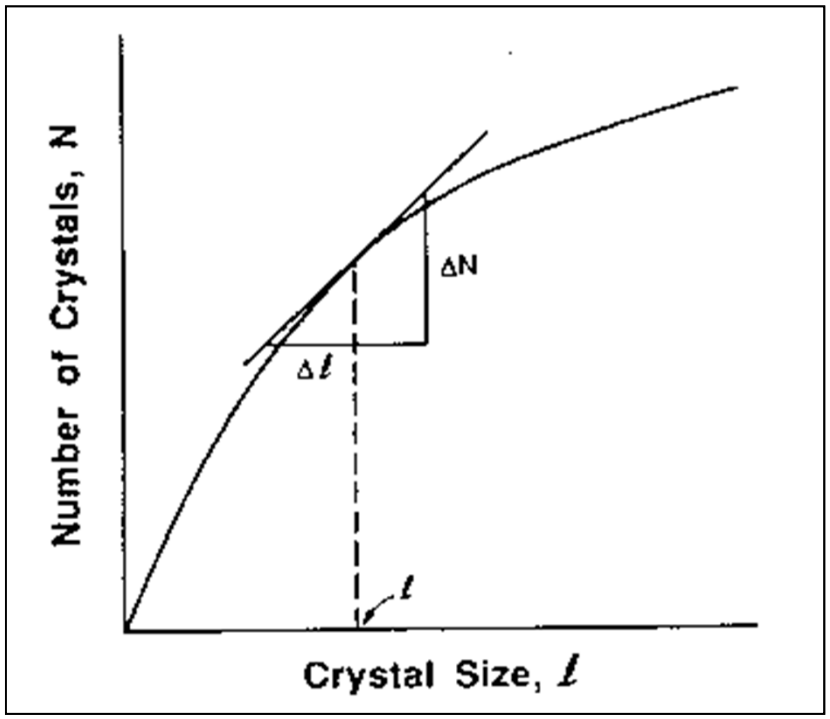


Fig. 3. The cumulative number of crystals versus size. The derivative of this curve, which has dimensions of (length)$^{-1}$, is the population density of the crystals $n\,(l,t)$.

Table 1. Fractional moment of distribution. The quantity χ (= lQ / GV), a dimensionless length.

Moment	Meaning	Basic Definition	Total for Continuous Crystallizer	Fraction for Continuous Crystallizer
0	Number of Crystals	$\frac{\int_0^l \mathbf{n(l)dl}}{\int_0^\infty \mathbf{n(l)dl}}$	$\frac{\mathbf{n_0 GV}}{\mathbf{Q}}$	$1- e^{-\chi}$
1	Size of Crystals	$\frac{\int_0^l \mathbf{l\, n(l)dl}}{\int_0^\infty \mathbf{l\, n(l)dl}}$	$n_0\left(\frac{GV}{Q}\right)^2$	$1- (1+\chi)e^{-\chi}$
2	Area of Crystals	$\frac{6\Phi_A \int_0^l \mathbf{l^2 n(l)dl}}{6\Phi_A \int_0^\infty \mathbf{l^2 n(l)dl}}$	$12\Phi_A n_0\left(\frac{GV}{Q}\right)^3$	$1-(1+\chi+\frac{1}{2}\chi^2)\,e^{-\chi}$
3	Mass of Crystals	$\frac{\rho\Phi_v \int_0^l \mathbf{l^3 n(l)dl}}{\rho\Phi_v \int_0^\infty \mathbf{l^3 n(l)dl}}$	$6\Phi_V n_0\left(\frac{GV}{Q}\right)^4$	$1-(1+\chi+\frac{1}{2}\chi^2+\frac{1}{6}\chi^3)\,e^{-\chi}$

Crystal from continuous process

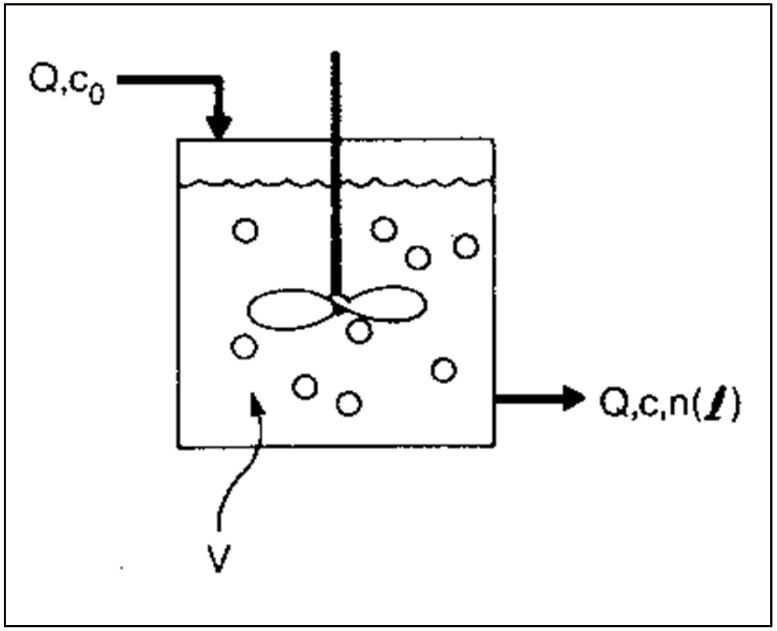


Fig. 4.Continuous crystallization. In this case, a supersaturated feed is always entering the well stirred crystallizer. A product stream containing the same crystal concentration as in the bulk is withdrawn at the feed rate.

We can now make a balance on the number of crystals within a given size Δl

[accumulation of crystals] = [crystals growing into range]
- [crystals growing out of range]
- [crystals of given range flowing out] (2-5)

At steady state,

$$0=[VnG]|_{l} - [VnG]|_{l+\Delta l} - Qn\Delta l \qquad (2\text{-}6)$$

In the limit as $\Delta l \to 0$, this balance becomes

$$\frac{Vd(Gn)}{dl} + Qn = 0 \qquad (2 - 7)$$

If we assume that growth rate G is not a function of the size l, we find

$$\frac{dn}{dl} + \frac{nQ}{GV} = 0 \qquad (2 - 8)$$

When l is small, the population density n_0 will be dominated by the nucleation of new crystals. In other words, as $l \to 0$,

$$n_0 = \frac{dN/dt}{dl/dt} = \frac{B}{G} \qquad (2 - 9)$$

Using this as a boundary condition for Eq.(10.2 - 8)

$$n = n_0 e^{-lQ/GV} \qquad (2 - 10)$$

Dominant crystal size

By definition, the mass for a given size range is

$$dM = \rho \Phi_v l^3 n dl \qquad (2-11)$$

From table 10.2-1, the total mass of crystals is

$$M_T = 6\rho\Phi_v n_0 \left(\frac{GV}{Q}\right)^4 \qquad (2-12)$$

Therefore, the change in the fraction of crystal mass $w(= M / M_T)$ is given by

$$\frac{dw}{dl} = \frac{l^3}{6}\left(\frac{Q}{GV}\right)^4 \frac{n}{n_0} = \frac{l^3}{6}\left(\frac{Q}{GV}\right)^4 e^{-lQ/GV} \qquad (2-13)$$

If we set the derivative of Eq.(10.2-13) with respect to l equal to zero and solve for l, we obtain the maximum or dominant size l_D :

$$l_D = \frac{3GV}{Q} \qquad (2-14)$$

Example 4. Characterizing an Ammonium sulfate crystallization. A continuous crystallization vessel containing 100liters of ammonium sulfate slurry is fed with 50 liters/hr of supersaturated solution. The withdrawal rate of product slurry is also 50 liters/hr. A nucleation rate B of 7.18* 10^7nuclei/liters hr, and growth rate G of 0.056 mm/hr are expected. Determine the following:

(a) the dominant crystal size,
(b) the number of crystals equal to or smaller than this size,
(c) the fraction of crystals in this range, and
(d) the product slurry concentration.

In these calculations, assume cubic crystals with a density of 1.769 g/cm^3.

Solution.

(a) From Eq.(2-14), the dominant crystal size is

$$l_D = \frac{3GV}{Q} = 3\left[0.056\,\frac{mm}{hr}\right]\,\frac{100liters}{50liters/hr}$$

$$= 0.336mm$$

(b) The number of crystals N equal to or smaller than this size I found from Table 10.2-1 :

$$N = \int_0^{l_D} n\, dl = n_0 \left(\frac{GV}{Q}\right)\left(1 - e^{-l_D Q/GV}\right)$$

$$= \left(\frac{BV}{Q}\right)\left(1 - e^{-l_D Q/GV}\right)$$

But $l_D Q / GV = 3$ as in part (a). Therefore, the number of crystals per liter is

$$N = 7.18 \times 10^7 \left(\frac{1}{\text{liter hr}}\right)\left(\frac{100 \text{ liters}}{50 \text{ liters/hr}}\right)(1 - e^{-3})$$

$$= 1.36 \times 10^8 \frac{1}{\text{liter}}$$

(c) The fraction of the crystals in this size range is

$$(1-e^{-3}) = 0.95$$

(d) The product concentration is found also from Table 10.2 -1:

$$M_T = 6\Phi_v \rho n_0 \left(\frac{GV}{Q}\right)^4$$

$$= 6 \times 1 \frac{1769 \text{g}}{(1000 \text{mm}^3/\text{cm}^3)}\left(\frac{7.18 \times 10^7 / \text{liter hr}}{0.56 \text{ mm/hr}}\right) \times \left(0.56 \frac{\text{mm}}{\text{hr}} \frac{100 \text{liter}}{50 \text{ liter/hr}}\right)^4$$

$$= 210 \text{ g/liter}$$

Example 5. Crystal size distribution for Ammonium sulfate. Using the growth rate and other conditions given in Example 4, calculate the anticipated crystal size distribution of the slurry. In particular, estimate the mass fraction of crystals retained on different standard screens.

Solution. For these conditions, the generalized dimensionless length χ is

$$\chi = \frac{\ell Q}{GV} = \left(\frac{\ell}{0.056\text{mm/hr}}\right)\left(\frac{50\text{liters/hr}}{100\text{liters}}\right) = \frac{\ell}{0.112\text{mm}}$$

For a screen of 20 mesh, the mesh size is 0.841 mm, so

$$\chi = \frac{0.841\text{mm}}{0.112\text{mm}} = 7.51$$

The weight fraction of crystals from 0 to this size is found from Table 10.2 - 1:

$$\mu_3 = 1 - e^{-\chi}\left(1 + \chi + \frac{\chi^2}{2} + \frac{\chi^3}{6}\right)$$

$$= 1 - 3^{-7.51}\left(1 + 7.51 + \frac{7.51^2}{2} + \frac{7.51^3}{6}\right) = 0.972$$

The fraction retained is

$$1-\mu_3 = 0.054$$

The results for this and other screen sizes are given in Table 10.2-2

Table 2. Crystal size distribution.

Ammonium Sulfate Crystallization					Sucrose Crystallization		
Screen Size Mesh	Screen Size (mm)	Dimensionless Size χ	Fraction Passed μ_3	Fraction Retained $1-\mu_3$	Mesh Size	Average Size l (mm)	Population Density as ln n
20	0.841	7.51	0.946	0.054	20[b]	1.02[b]	9.8
28	0.595	5.31	0.777	0.223	28	0.72	12.45
35	0.420	3.75	0.517	0.483	35	0.51	14.61
48	0.297	2.65	0.274	0.726	48	0.36	16.46
100	0.149	1.33	0.046	0.954	65	0.25	16.98

Example 6. The analysis of Sucrose crystallization. The following screen analysis is for a product prepared during a study of sucrose crystallization :

Product Size, mesh	Cumulative Percent
+ 20	3
+ 28	14
+ 35	38
+ 48	76
+ 65	92

Sucrose has density of 1.588 g/ cm^3. The slurry density and retention time were given as 355 g/ liter and 2.5 hr, respectively. From those data, determine :

(a) the crystal growth rate
(b) the nucleation rate
(c) the dominant crystal size, and
(d) the slurry concentration.

Solution.

The sieve opening for a 20 mesh screen is 0.841 mm and for a 28 mesh screen is 0.595mm. Therefore, the size range Δl of crystals on 28 mesh is

$$\Delta l = 0.841\text{-}0.595 = 0.246\text{mm}$$

The average size l on the 28 mesh screen is

$$l = \frac{0.841 + 0.595}{2} = 0.718\text{mm}$$

The population density of the 28 mesh crystals can be estimated from Eq.(10.2-1)

$$n_{28} = \frac{(0.14 - 0.03)\ 335\ \text{g/liter}}{1.588\ \text{g/cm}^3\left[1\text{cm}^3/1000\text{mm}^3\right](\text{crystals/\#})\ (0.718\text{mm/crystals})^3\ (0.246\text{mm})}$$

$$= 255{,}000\ \frac{\text{crystals}}{\text{mm liter}}$$

$$\ln n_{28} = 12.45$$

(a) The growth rate is

$$G = -\frac{1}{\text{slope}}\left(\frac{Q}{V}\right) = -\frac{1}{-9.9(2.5\text{hr})} = 0.040\text{mm/hr}$$

(b) The nucleation rate is given by

$$B = n_0 G = e^{19.6}\left(\frac{1}{\text{mm liter}}\right)\left[0.040\ \frac{\text{mm}}{\text{hr}}\right] = 1.3\times10^7\ \frac{1}{\text{liter hr}}$$

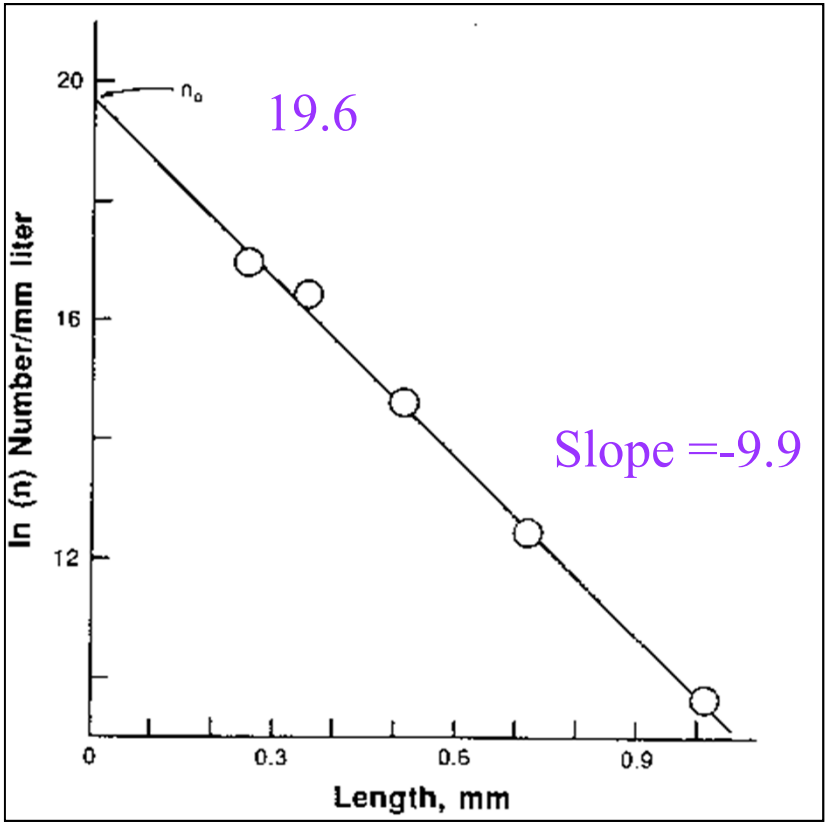


Fig. 5. The population density in sucrose crystallization. The logarithm of the population density is proportional to crystal size, as predicted by Eq. (2-10)

(c) The dominant crystal size is

$$l_D = 3G\left(\frac{V}{Q}\right) = 3[0.040\text{mm/ hr}]2.5\text{hr} = 0.30\text{mm}$$

(d) The slurry concentration is given by

$$M_T = 6\Phi_v \rho n_0 \left(\frac{GV}{Q}\right)^4$$

$$= 6\left[1.588\,\frac{\text{g}}{\text{cm}^3}\right]\left[\frac{1\text{cm}^3}{1000\text{mm}^3}\right]\ e^{19.6}\left(\frac{\text{crystals}}{\text{liter mm}}\right)\left(0.040\,\frac{\text{mm}}{\text{hr}}2.5\text{hr}\right)^4$$

$$= 320\text{ g/liter}$$

3. BATCH CRYSTALLIZATION

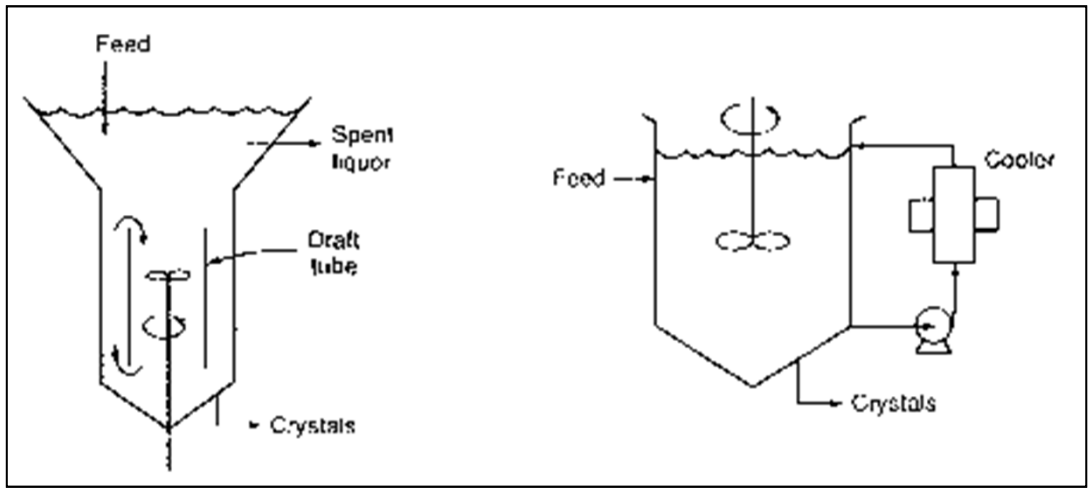


Fig. 6. Batch crystallizers. These are nothing more than stirred tanks which are slowly cooled. They are often baffled for better mixing.

1. The cooling curve

For the unsteady operation in a batch system

$$V\frac{\partial n}{\partial t} = -V\frac{\partial}{\partial l}(nG) \qquad (3-1)$$

In which the growth rate is given by Eq.(10.1-10)

$$G = \frac{\partial l}{\partial t} = k_g(c - c^*) \qquad (3-2)$$

For the nucleation rate B,

$$B = \frac{\partial N}{\partial t} = G n_0 = k_n (c - c^*)^i \qquad (3\text{-}3)$$

The balance on the amount of supersaturation in the batch unit :

$$\begin{bmatrix} \text{change in} \\ \text{supersaturation} \end{bmatrix} = \begin{bmatrix} \text{change from} \\ \text{altered temperature} \end{bmatrix} + \begin{bmatrix} \text{change from} \\ \text{crystal growth} \end{bmatrix} + \begin{bmatrix} \text{change from} \\ \text{nucleation} \end{bmatrix} \qquad (3\text{-}4)$$

많은 batch crystallization은 metastable zone에서 일어나므로, Eq.(.3-4)를 Eq. (1-6)와 결합시켜서

$$0 = V \frac{dc^*}{dt} + \frac{A}{(1/k + 1/\kappa)} (c - c^*) \qquad (3\text{-}5)$$

Where A : the total crystal area

c^* : the saturation concentration (온도의 함수)

$$\frac{dc^*}{dt} = \left(\frac{dc^*}{dT} \right) \left(\frac{dT}{dt} \right) \qquad (3\text{-}6)$$

The total crystal area depends on the mass of seeds M_s and on the linear growth rate G

$$A = [\text{number of crystals}]\left[\frac{\text{area}}{\text{crystal}}\right] = \left[\frac{M_s}{\rho \Phi_v l_s^3}\right]\left\{6\Phi_A (l_s + Gt)^2\right\} \qquad (3-7)$$

Where M_s : the total mass of seed

ρ : density , l_s : initial size of seed crystal

G is the linear growth rate, given by Eq. (10.1 -10)

$$G = \frac{dl}{dt} = \left[\frac{2\Phi_A / \Phi_v \rho}{1/k + 1/\kappa}\right](c - c^*) \qquad (3-8)$$

Combine (10.3 - 5) – (10.3 - 8)

$$\frac{dT}{dt} = -\left[\frac{M_s / V}{dc^* / dt}\right]\frac{3G}{l_s^3}(l_s + Gt)^2 \qquad (3-9)$$

온도의 미소변화에 대해 온도에 대한 용해도를 상수라 하면

$$T = T_0 - \left[\frac{M_s / V}{dc^* / dt}\right]\frac{3Gt}{l_s}\left(1 + \frac{Gt}{l_s} + \frac{1}{3}\left(\frac{Gt}{l_s}\right)^2\right) \qquad (3-10)$$

This result is sometimes written in terms of different variables :

$$\frac{T - T_0}{T_P - T_0} = \frac{M_s}{M_P}(3\eta\tau)\left[1 + \eta\tau + \frac{1}{3}(\eta\tau)^2\right] \qquad (3\text{-}11)$$

Batch scale-up

Example 7. Modified Tetracycline crystallization. You have been successfully running crystallizations of a substituted form of this antibiotic in mixed chlorinated solvents. Because some of these may be carcinogenic, you have been asked to explore the use of mixed alcohols.

You plan to do this by cooling saturated solutions of this material which initially are at 20°C. Near this temperature the solubility changes $1.14 * 10^{-3}$ g tetracyline/cm^3 solvent/°K. The seed crystals are 0.01cm, roughly cubic, with a density of 1.06 g/cm^3. They are added at a concentration of only 35 ppm. The crystal size in this crop is about 0.088 cm. The supersaturaion is expected to be around 0.077 g/cm^3 ; the growth is diffusion controlled, with a mass transfer coefficient of 6.5×10^{-5} cm/sec. Estimate how the crystallizer temperature should vary with time to achieve these goals.

Solution. To solve this problem, we need only calculate the various quantities in Eq.(10.3-11). First, we note that the increase in crystal size η is given by

$$\eta = \frac{\mathbf{l_P - l_s}}{\mathbf{l_s}} = \frac{\mathbf{0.088 - 0.010}}{\mathbf{0.010}} = \mathbf{7.8}$$

Second, we find the time for crystal growth by calculating the growth rate G (=dl/dT) from Eq.(1-10) or (3-8), which for diffusion control is

$$G = 2k\left(\frac{\Phi_A}{\Phi_v \rho}\right)(c - c^*)$$

$$= (2)6.5\times10^{-5}\ \frac{cm}{sec}\left(\frac{1}{1(1.06 g/cm^3)}\right)\ \frac{0.077g}{cm^3}\ \left(\frac{3600\ sec}{hr}\right) = 0.034 cm/hr$$

This assumes that Φ_v and Φ_A are one, which is the case for cubic crystals.

The process time t_P is found from

$$G\, t_P = l_P - l_s$$

$$(0.034\ cm/hr)\ t_P = 0.088cm - 0.010cm$$

$$t_P = 2.3hr$$

The cooling curve can now be found from Eq. (10.3 -10) or (10.3 -11), which may be written as

$$T = T_0 - \left[\frac{M_S / V}{dc^* / dT}\right]\eta\tau\left[1 - \eta\tau + \frac{1}{3}(\eta\tau)^2\right]$$

$$= 20^\circ - \left[\frac{35\times10^{-6}\ \text{g/cm}^3}{1.14\times10^{-3}\ \text{g/cm}^3\ ^\circ\text{C}}\right]\frac{(3)\times 7.8t}{2.3\text{hr}}\left[1 + \frac{7.8t}{2.3\text{hr}} + \frac{1}{3}\left(\frac{7.8t}{2.3\text{hr}}\right)^2\right]$$

$$= 20^\circ - 0.312t(1 + 3.39t + 3.83t^2)$$

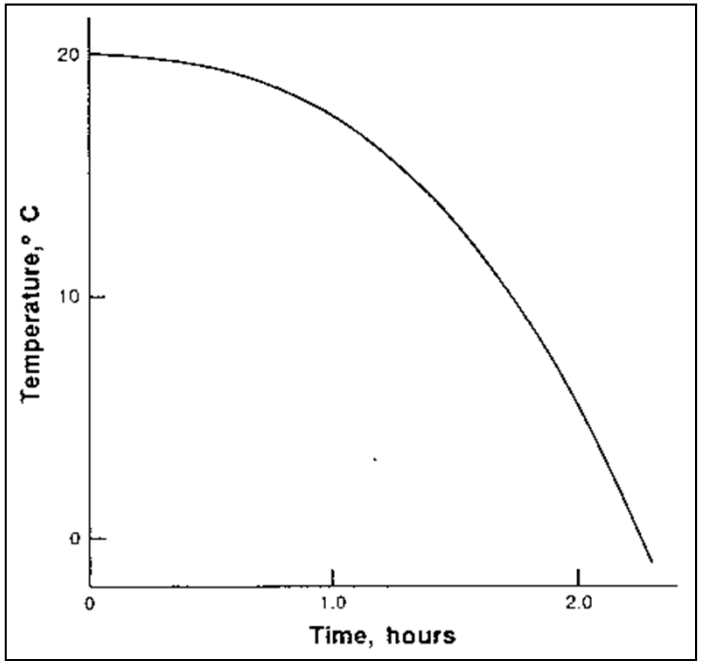


Fig. 7. The cooling curve for a modified tetracycline. The temperature is initially nearly constant, but later drops abruptly. This general shape is characteristic of the cooling curves for batch crystallization.

4 RECRYSTALLIZATION

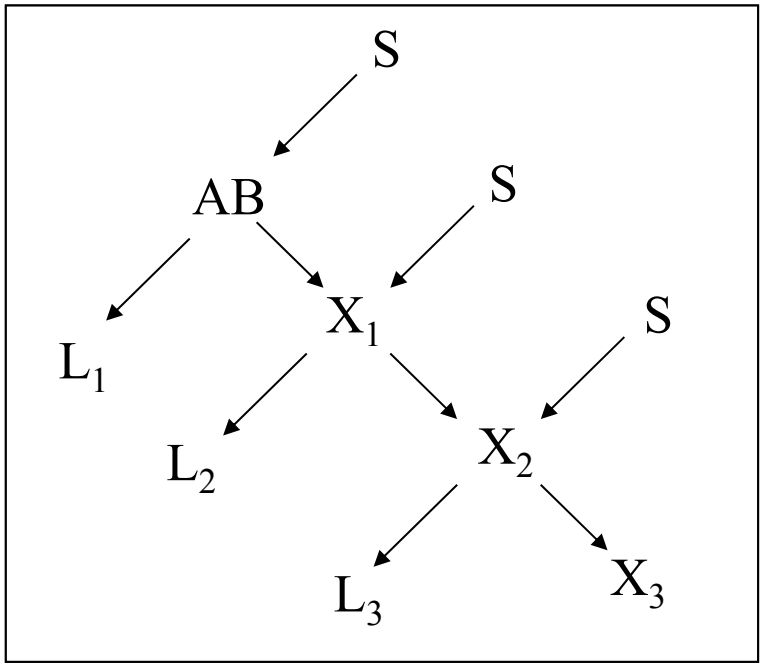


Fig. 8. Simple recrystallization.
This scheme, which uses fresh solvent for each recrystallization, gives a high purity but a low yield.

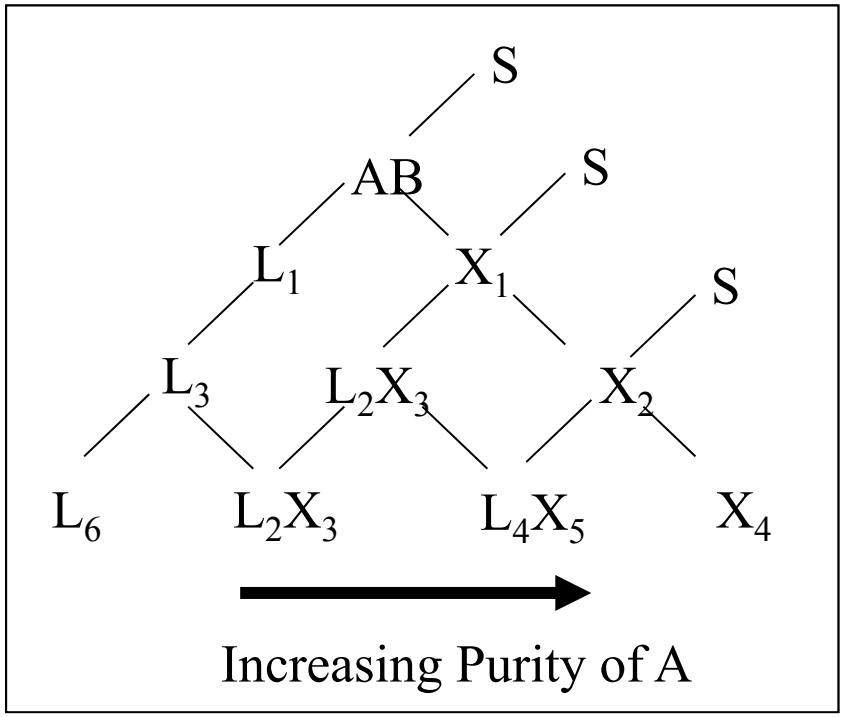


Fig. 9. Fractional recrystallization.
This scheme makes more efficient use of the various liquors, and hence gives a higher yield than that in the previous figure.

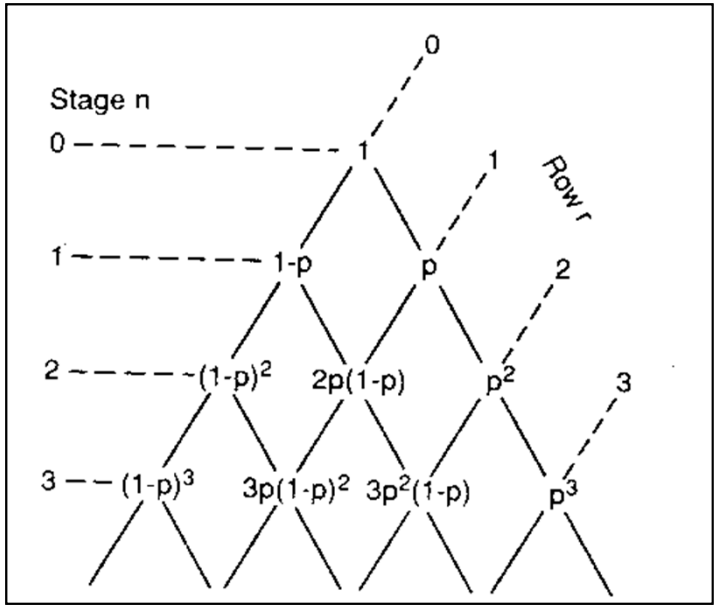


Fig. 10. Fractional crystallization as a stage operation. This scheme produces very pure crystals by a process equivalent to that of Craig extraction detailed in Section 5.5.

$$Y_A=(E_A/(1+E_A))^N \qquad (4\text{-}1)$$

$$f(r,n)=\frac{n!}{r!(n-r)!}p^r(1-p)^{n-r} \qquad (4.2)$$

Example 8. Multiple simple crystallization of Soybean Sterols. A mixture of soy sterols contains 20% stigmasterol and 80% sitosterol. This mixture and subsequent products are recrystallized six times following the scheme in Fig. 10.4-1. The crystallizations are conducted under conditions where the E values are 2.0 and 0.5 for stigmasterol and sitosterol, respectively.

(a) Calculate the yields of stigmasterol and sitosterol in the final product.
(b) Determine the purity of the final product.

Solution. (a) The recovered yields of stigmasterol and sitosterol are calculated using Eq.(10.4-1) :

$$Y(\text{stigmasterol}) = \left(\frac{E}{E+1}\right)^6 = \left(\frac{2}{3}\right)^6 = 0.88$$

$$Y(\text{sitosterol}) = \left(\frac{0.5}{0.5+1}\right)^6 = 0.0014$$

(b) The purity of the final crystals is given by

$$\text{purity} = \frac{(0.088 \times 20)\text{g stigmasterol}}{(0.088 \times 20)\text{g stigmasterol} + (0.0014 \times 80)\text{g sitosterol}}$$

$$= 0.94$$

5 CONCLUSIONS

- **C**rystallization is a powerful tool for the recovery of the final highly purified product.
- **C**rystallization enhances the performance of subsequent operations.
- **C**rystallization improves the appearance of the final product.
- **T**here is a dearth of useful information for organic materials and for batch operations.
- **A**nalyzing crystallization is complex and difficult.
- **S**cale-up is risky and often compromise both product purity and crystal characteristics.