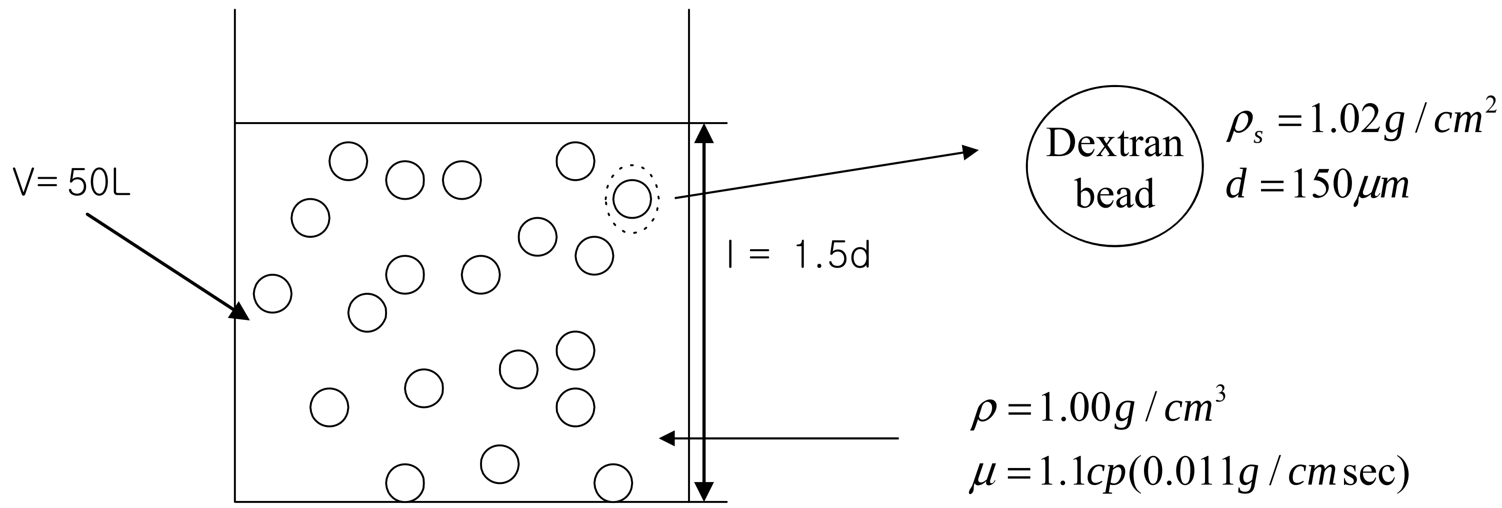
Centrifugation calculation

Solution of Examples

Example 1. Separation of Cells Growing on Support



(a) Settling time?

assumption : Beads quickly reach their maximum terminal velocity

$$t = \frac{\ell}{\upsilon_g} \left(\frac{length}{lenght / time} \right) = time$$

Force balance

$$F_G = F_B + F_D$$

$$F_G - F_B = F_D$$

$$F_G = \frac{4}{3}\pi R^3 \rho_S g$$

$$F_B = \frac{4}{3}\pi R^3 \rho \; g$$

$$F_D = 3\pi d \mu \upsilon \qquad \mathrm{Re} < 1$$

$$F_D = \kappa\left(\frac{1}{2}\rho \upsilon^2\right)\left[(\pi / 4) d^2\right] \quad \mathrm{Re} > 1$$

$$\frac{4}{3}\pi R^3 \rho_S g = \frac{4}{3}\pi R^3 \rho \; g + 3\pi d \mu \upsilon$$

$$\frac{1}{6}\pi R^3 \rho_S g = \frac{1}{6}\pi R^3 \rho g + 3\pi d \mu \upsilon$$

$$\frac{1}{6}\pi d^3 (\rho_S - \rho) g = 3\pi d \mu \upsilon$$

$$\upsilon = \frac{d^2}{18\mu}(\rho_S - \rho) g$$

$$\upsilon_g = \frac{(0.015cm)^2(1.02-1.00)g/cm^3(980cm/\sec)^2}{18(0.011g/cm\sec)}$$

$$\upsilon_g = 0.022\ cm/\sec$$

- check $\mathrm{Re} = \dfrac{d\upsilon\rho}{\mu}$

$$= \frac{(0.015cm)(0.022cm/\sec)1g/cm^3}{(0.011g/cm\sec)} = 0.03 < 1$$

$$V = \pi r^2 \ell$$

$$\frac{\pi}{4} d^2 \ell = V$$

$$\frac{\pi}{4}(\ell/1.5)^2 \ell = 50\times10^3\ cm^3$$

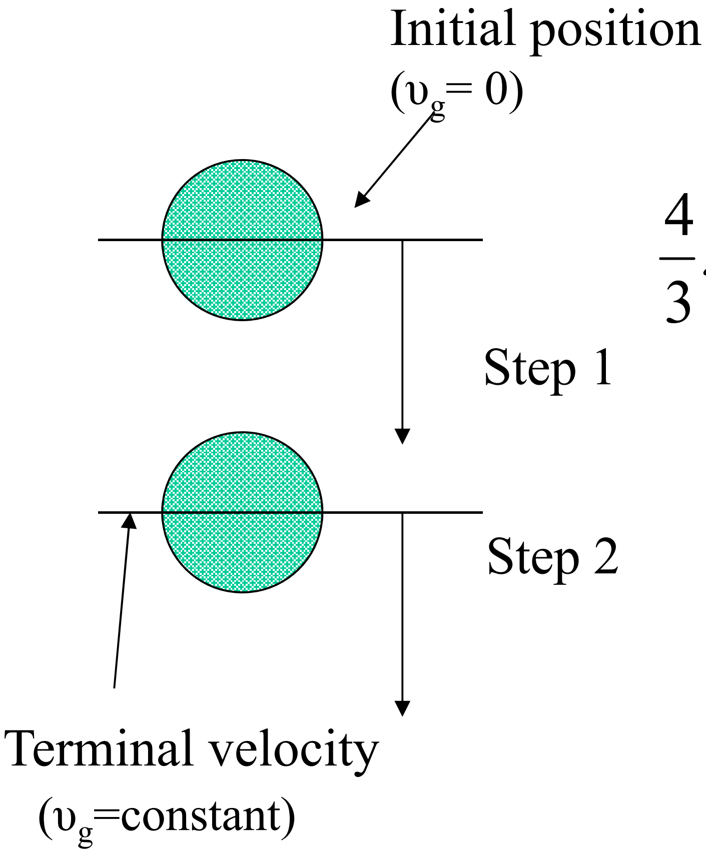
$$\ell = 52.3cm$$

$$t = \frac{\ell}{\upsilon_g}$$

$$t = \frac{52.3cm(\min/\ 60\sec)}{0.022\ cm/\sec}$$

$$= 39.6\min$$

(b) Estimate the time to reach this velocity



Force balance

$$F_G' - F_B' - F_D' = F_G - F_B - F_D$$

$$F_G' - F_B' = F_G - F_B - F_D$$

$$\frac{4}{3}\pi R^3 \rho_S g - \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi R^3 \rho_S g - \frac{4}{3}\pi R^3 \rho g + 3\pi d \mu \upsilon$$

$$\frac{1}{6}\pi d^3(\rho_S - \rho)\frac{dv_g}{dt} = \frac{1}{6}\pi d^3(\rho_S - \rho) g - 3\pi d \mu v$$

$$\frac{dv_g}{dt} = g - \frac{3\pi d \mu v}{\pi d^3/6(\rho_S - \rho)}$$

$$\frac{1}{g - \alpha \upsilon_g} dv_g = dt$$

- $\alpha = \frac{18\mu}{d/(\rho_s - \rho)}$

(Initial condition : t=0, v=0)

$$-\frac{1}{\alpha}\ln(g - \alpha \upsilon_g) = t + c$$

$$c = -\frac{1}{2}\ln g$$

$$-\frac{1}{\alpha}\ln(g - \alpha v_g) = t + \ln g$$

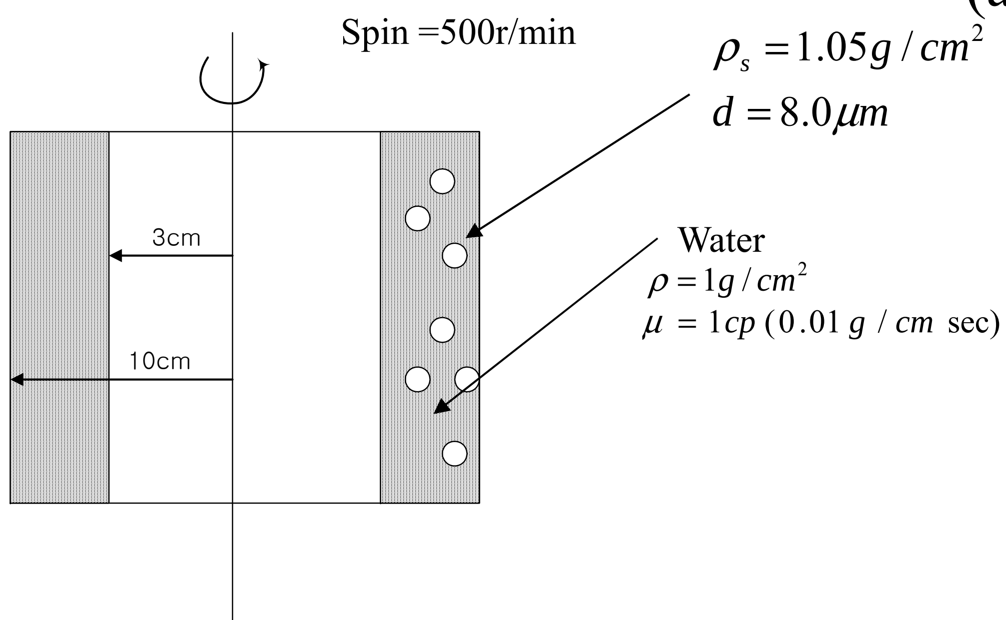
$$v_g = \left[\frac{g d^2(\rho_s - \rho)}{18\mu}\right]\left[1 - \exp(-18\mu t / d^2(\rho_s - \rho))\right]$$

$$\frac{18\mu}{d^2((\rho_s - \rho))} t >> 1$$

$$t >> \frac{d^2(\rho_s - \rho)}{18\mu} = \frac{(0.015cm)^2(0.02g/cm^3)}{18(0.01g/cm\sec)}$$

$$>> 2\times10^{-5}\sec$$

Example 2 Centrifugation of Yeast Cells.



(a) How long does it take to have a complete separation ?

$$\upsilon = \frac{d^2}{18\mu}(\rho_S - \rho) g$$

$$\frac{dr}{dt} = v_\omega = \frac{d^2}{18\mu}(\rho_s - \rho)\omega^2 r$$

$$\frac{1}{r} dr = \frac{d^2}{18\mu}(\rho_s - \rho) dt$$

$$[\ln r]_3^{10} = \frac{d^2}{18\mu}(\rho_s - \rho) t$$

$$\ln \frac{10cm}{3cm} = \frac{d^2}{18\mu}(\rho_s - \rho)\omega^2 t$$

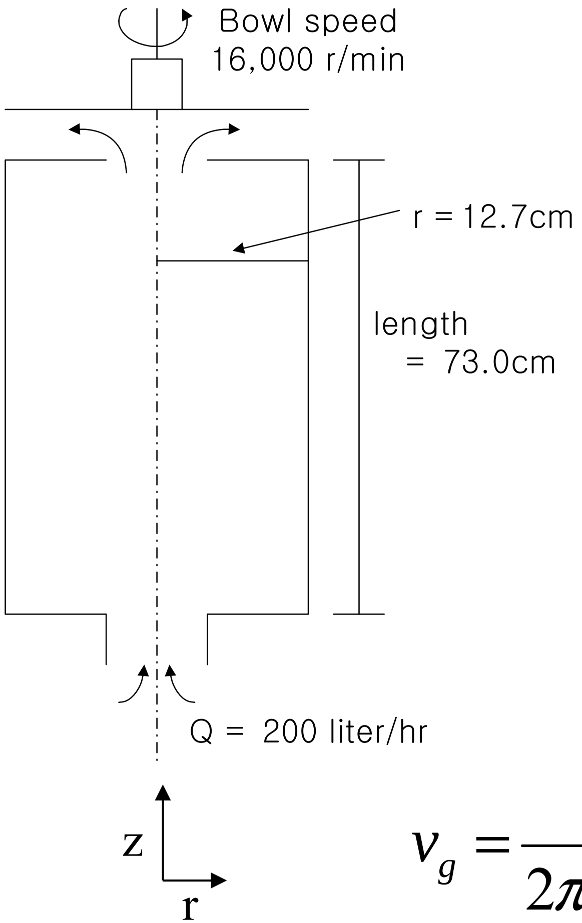
$$\ln \frac{10cm}{3cm} = \frac{(8 \times 10^{-4} cm)^2}{18(0.01 g / cm \sec)} \cdot$$

$$(0.05 g / cm^3)(500 \times 2\pi^2 / 60 \sec) t$$

$$t = 2500 \sec$$

$$= 41.6 \min$$

Example 3 Tubular Centrifugation of *E.coli*.



(a) Calculate the settling velocity υ_g for the cells.

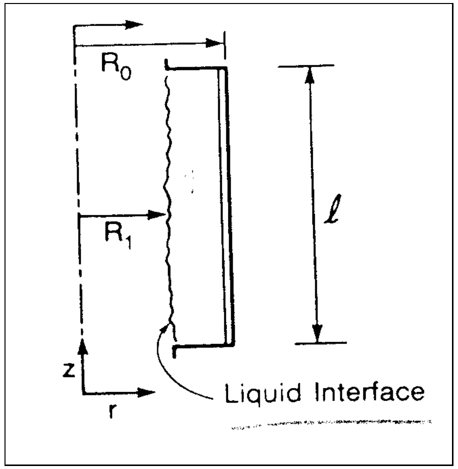
$$Q = v_g \left[\frac{2\pi \ell R^2 \omega^2}{g} \right]$$

$$v_g = \frac{Qg}{2\pi \ell R^2 \omega^2}$$

$$v_g = \frac{200 \times 10^3 cm^3/hr(980 cm/\sec^2)(hr/3600 \sec)}{2\pi(73 cm)(12.7 cm)^2([2\pi(16{,}000)]/60(\sec/\min)\min)^2}$$

$$= 2.6 \times 10^{-7} cm/\sec$$

Tubular Bowl Centrifuge Theory



$$Q = Av$$

$$\frac{dz}{dt} = \frac{Q}{\pi(R_0^2 - R_1^2)}$$

$$\frac{dr}{dt} = v_\omega = \frac{d^2}{18\mu}(\rho_s - \rho)\omega^2 r \qquad \frac{dr}{dt} = v_g\left(\frac{r\omega^2}{g}\right)$$

$$\frac{dr}{dz} = \frac{dr/dt}{dz/dt} = v_g\left(\frac{r\omega^2}{g}\right)\frac{\pi(R_0^2 - R_1^2)}{Q}$$

Logarithmic mean radius

$$\overline{R} = \frac{R_0 - R_1}{\ln(R_0/R_1)}$$

$$\frac{1}{r}dr = v_g\left(\frac{\omega^2}{g}\right)\frac{\pi(R_0^2 - R_1^2)}{Q}dz$$

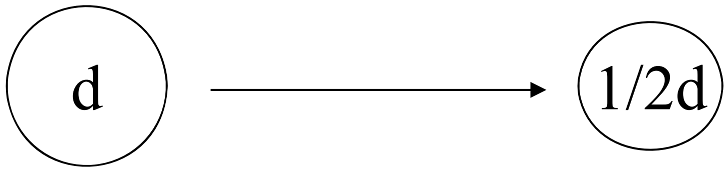
$$\frac{R_0^2 - R_1^2}{\ln(R_0/R_1)} = \frac{(R_0 + R_1)(R_0 - R_1)}{\ln(R_0/R_1)} = 2R^2$$

$$Q = \frac{v_g \ell\left(\frac{\omega^2}{g}\right)\pi(R_0^2 - R_1^2)}{\ln(R_0/R_1)}$$

$$Q = v_g\left[\frac{2\pi\ell R^2\omega^2}{g}\right]$$

$$Q = v_g[\Sigma]$$

(b) After disruption the diameter of debris is about one-half of the original cell diameter and the viscosity is increased four times Estimate the volumetric capacity of this same centrifuge operating under these new conditions.



Original cell (μ, Q_o) — Disrupted cell (4μ, Q_d)

$$Q = v_g\left[\frac{2\pi \ell R^2 \omega^2}{g}\right]$$

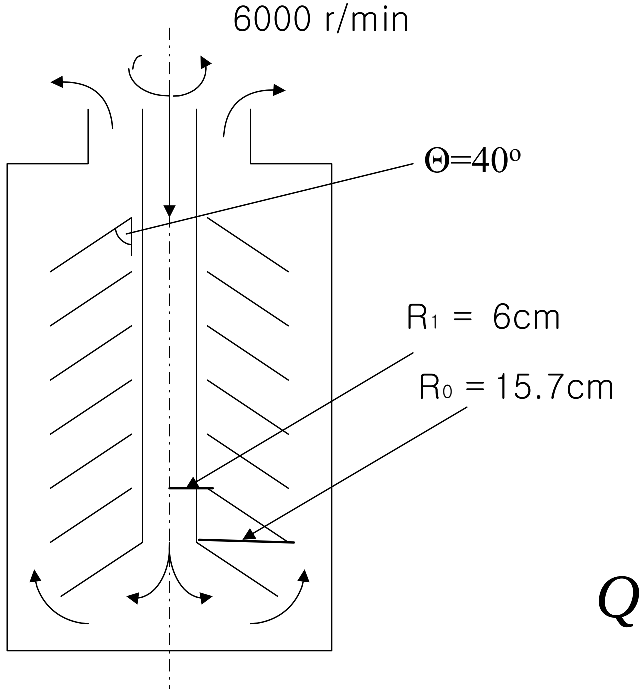
$$Q = \frac{d^2}{18\mu}(\rho_s - \rho) g \left(\frac{2\pi \ell R^2 \omega^2}{g}\right)$$

$$\frac{Q_d}{Q_o} = \frac{{d_d}^2 / \mu_d}{{d_o}^2 / \mu_o}$$

$$\frac{Q_d}{Q_o} = \frac{(1/2d)^2 / 4\mu_d}{d^2 / \mu}$$

$$\frac{Q_d}{Q_o} = \frac{1}{16}$$

Example 4 Disc Centrifugation of Chlorella.



$v_g = 1.07\times 10^{-4}$

n= 80 (disc number)

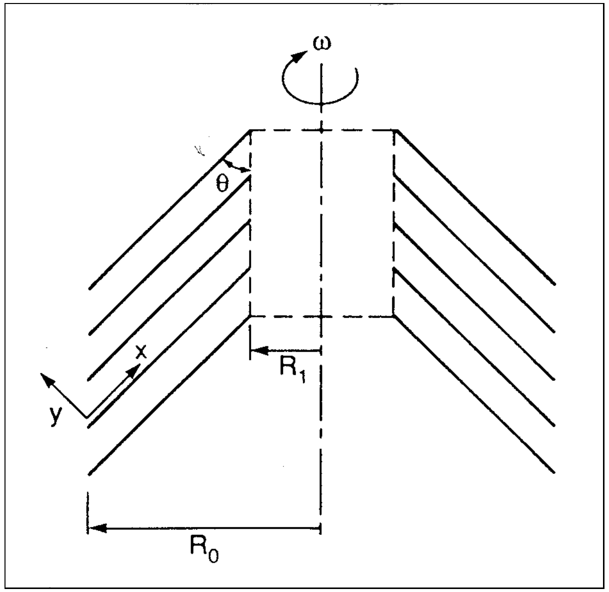
(a) estimate the volumetric capacity for this centrifuge.

$$Q = v_g\left[\frac{2\pi n\omega^2}{3g}\left(R_0^3 - R_1^3\right)\cot\Theta\right]$$

$$Q = 1.07\times 10^{-4}\, cm/\sec\left\{\frac{2\pi(80)}{3(980\, cm/\sec^2)}\left[\frac{2\pi(6000)}{60(\sec/\min)\min}\right]^2\times\left[(15.7cm)^3 - (6cm)^3\right]\cot(40)\right\}$$

$$= 3.1\times 10^4\, cm^3/\sec = 31\, liter/\sec$$

Disc Type Centrifuge Theory



$$\frac{dx}{dt} = v_0 - v_\omega \sin\Theta$$

$$\cong v_0$$

$$= \left[\frac{Q}{n(2\pi r\ell)}\right] f(y)$$

$$v_0 = \left[\frac{Q}{n(2\pi r\ell)}\right] f(y)$$

$$\frac{dy}{dt} = v_\omega \cos\Theta$$

$$\frac{dy}{dt} = v_g \left(\frac{\omega^2 r}{g}\right) \cos\Theta$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left[\frac{2\pi r n\ell v_g \varpi^2)}{Qgf(y)}\right] r^2 \cos\Theta$$

$$= \left[\frac{2\pi r n\ell v_g \varpi^2)}{Qgf(y)}\right] (R_0 - x\sin\Theta)^2 \cos\Theta$$

$$Q = v_g \left[\frac{2\pi r n\omega^2}{3g} (R_0^3 - R_1^3) \cot\Theta\right]$$

$$Q = v_g [\Sigma]$$

Example 5 Scale-up of Centrifugation in Yeast Separation.

Recombinant protein in suspension → Centrifuging → Centrifuged protein

- Type of a centrifuge = Bottle centrifuge
- Spin = 2000 r/min
- Time = 30 min
- R_1 = 5 cm, R_0 = 15 cm

(a) Somebody want to scale-up the size and type of centrifuge for separation $10m^3$ of this suspension per day.

Solve) $Q = \upsilon_g [\Sigma]$

Step 1) About υ_g

$$\frac{dr}{dt} = v_\omega = \frac{d^2}{18\mu}(\rho_s - \rho)\omega^2 r$$

$$\frac{dr}{dt} = v_g \frac{\omega^2 r}{g}$$

$$\frac{1}{r} dr = v_g \left(\frac{\omega^2}{g}\right) dt$$

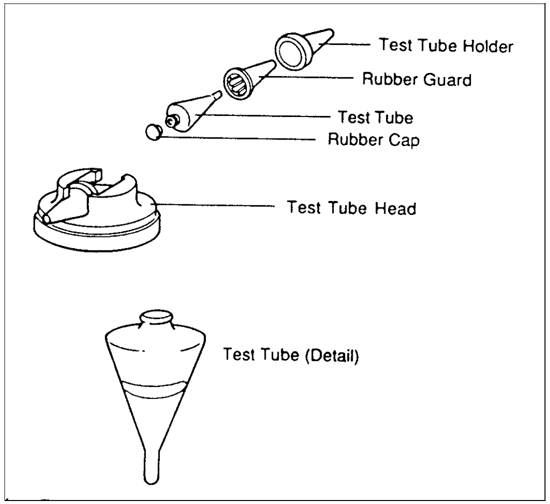


Figure. 1 Gyro tester

$$\int_{R_1}^{R_0} \frac{1}{r} dr = v_g (\frac{\omega^2}{g}) dt$$

$$v_g = \frac{g \ln(R_0 / R_1)}{t\omega^2}$$

$$v_g = \frac{980 cm/\sec^2 \ln(15/5)}{[2\pi(2000)/60\sec]^2 30\min(60\sec/\min)}$$

$$= 1.53 \times 10^{-5} cm/\sec$$

- To use Gyro tester : accurate and reliable data

Operating condition: sample 10 ㎖

spin = 7500 min

interval time = 10 sec

Spin Time (sec)	Solid Position	Supernatant Clarity	Solids
0	31	Cloudy	Soft
10		Turbid	Soft
20		Turbid	Slimy
30		Hazy	Plastic
40	47	Clear	Tightly packed

$$v_g = \frac{980 cm/\sec^2 \ln(47/31)}{[2\pi(7500)/60\sec]^2 40\sec}$$

$$= 1.65 \times 10^{-5} cm/\sec$$

Step 2) About [Σ]

- Disc bowl of the Gyro tester

Operating condition

n = 18 (disc number)

Θ = 51° (angle)

spin = 8500 r/min

R_0=4.7cm, R_1=2.1cm

$$\Sigma = \frac{2\pi n \varpi^2}{3g}(R_0^3 - R_1^3)\cot\Theta$$

$$= \frac{2\pi n 18^2}{3(980 cm/\sec^2)}\left(\frac{2\pi(8500)}{60\sec}\right) \cdot [(4.7cm)^3 - (2.1cm)^3]\cot 51^o$$

$$= 2.33 \times 10^6 cm^2$$

Maximum flow

$$Q_{Max} = \upsilon_g[\Sigma]$$

$$= (1.65\times10^{-5})cm/\sec(2.33\times10^{6})cm^2$$

$$= 38cm^3/\sec = 0.038 liter/\sec$$

(b) At this point, we have a meeting with the client who change order

Order : Q = 1000ℓ/hr

removal of solids be continuous.(=>nozzle ejecting bowl type)

$$\Sigma = (\frac{Q}{v_g})$$

$$= \frac{10\times10^6 cm^3/hr}{1.65\times10^{-5} cm/\sec}\left(\frac{1}{3600\sec/hr}\frac{m^2}{10^4 cm^2}\right)$$

$$= 17{,}000m^2 \xrightarrow[\text{twice}]{} \Sigma = 34{,}000m^2$$

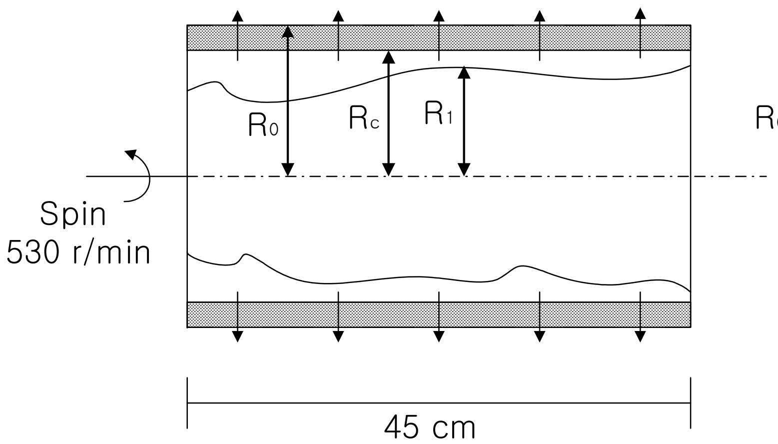
Example 6 Centrifugal Filtration of Pogesterone.

$P_0 = 0.016 g/cm^3$
$V = 250 cm^3$
(slurry)

Filtering → Product

- $A_f = 8.3\ cm^2$
- $\Delta P = 1\ atm$
- $\rho_c = 1.09$ (solid/cm^3 cake)
- $\rho = 1 g/cm$ (slurry density)
- time = 32min(1920 sec)

(a) To use this experiment to estimate the time to filter 1600ℓ of this slurry through a centrifugal filter.



R_0 = 51cm

$$t = \frac{\mu \alpha \rho_c R_c^2}{2\rho\omega^2 (R_o^2 - R_1^2)} \cdot \left[\left(\frac{R_o}{R_c}\right)^2 - 1 - 2\ln\frac{R_o}{R_c} \right]$$

i) Properties of the cake

$$t = \frac{\mu \alpha \rho_o}{2\Delta p}\left(\frac{V}{A}\right)^2$$

$$1920\sec = \frac{\mu\alpha(0.016 g/cm^3)}{2(1atm)(1.01\times 10^6 g/cm\sec^2 atm)}\left(\frac{250}{8.3}cm^3/cm^2\right)^2$$

$$\mu\alpha = 2.67\times 10^8 \sec^{-1}$$

ii) mass balance

$$\rho_c \pi (R_o^2 - R_c^2)\ell = \rho_o V$$

$$1.09 g/cm^3 \pi ((51cm)^2 - R_c^2) 45cm = (0.016 g/cm^3) 1600\times 10^3 cm^3$$

$$R_c = 49.3cm$$

iii) filtration time

$$t = \frac{\mu\alpha\rho_c R_c^2}{2\rho\omega^2(R_o^2 - R_1^2)}\cdot\left[\left(\frac{R_o}{R_c}\right)^2 - 1 - 2\ln\frac{R_o}{R_c}\right]$$

$$= \frac{2.67\times 10^8 \sec^{-1}(1.09 g/cm^3)(49.3cm)^2}{2(1g/cm)\left(\frac{530\cdot 2\pi}{60\sec}\right)^2[(51cm)^2 - (45.5cm)^2]}\cdot\left[\left(\frac{51cm}{49.3cm}\right)^2 - 1 - 2\ln(51cm/49.3cm)\right]$$

$$= 8.5\min$$