Diffusion

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The Diffusion Application

- Heat transfer is a diffusion process, so the generic diffusion equation has the same structure as the heat equation.

- Heat equation

$$\delta_{ts}\rho C_p \frac{\partial T}{\partial t} - \nabla \cdot (\underline{k} \nabla T) = Q$$

- Diffusion equation

$$\delta_{ts} \frac{\partial c}{\partial t} + \nabla \cdot (-D \nabla c) = R$$

Subdomain formulation

- The diffusion equation coefficients to specify on the subdomains are as follows.
- Diffusion equation

$$\delta_{ts}\frac{\partial c}{\partial t}+\nabla\cdot(-D\nabla c)=R$$

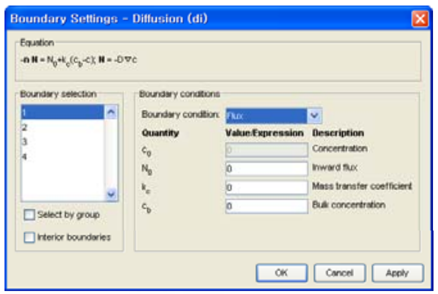
Coefficient	Description
δ_{ts}	Time scaling coefficient
D	Diffusion coefficient
D_{ij}	Diffusion coefficient tensor
R	Reaction rate

Boundary formulation

- Boundary conditions are as follows.

Coefficient	Description
$c = c_0$	Concentration
$-n\cdot(-D\nabla c) = N_0 + k_c(c_b - c)$	Flux
$n\cdot(-D\nabla c) = 0$	Insulation/Symmetry
$n\cdot(-\underline{D}\nabla c) = 0$	Axial symmetry

- You specify the boundary conditions in the boundary settings dialog box.



Model of insect population density using the diffusion equation

- This FEMLAB model examines the effects that nymph transport in multiple plant species has on the resulting population distribution.
- Nymphs equation

$$\rho \frac{\partial u}{\partial t} = \nabla \cdot \left(\frac{m}{4\tau} \nabla u\right)$$

- Distribution equation for x, y, t domain. Center of the domain is (x_0, y_0).

$$u(x, y, t_0) = u_0 \exp\left(-a\sqrt{(x - x_0)^2 + (y - y_0)^2}\right)$$

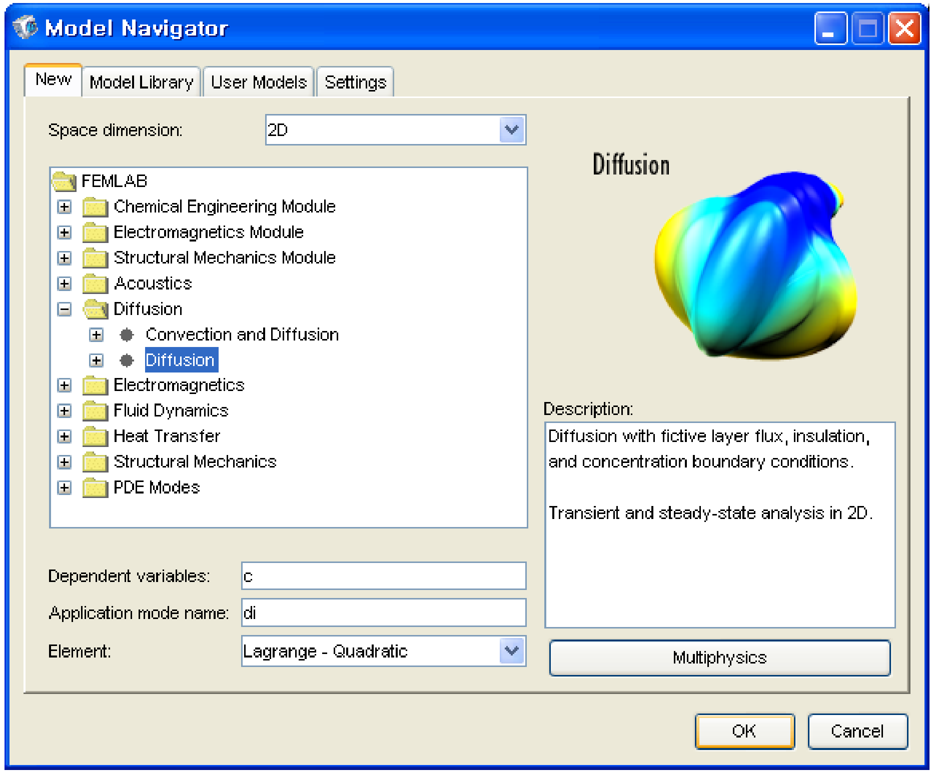
Model of insect population density using the diffusion equation

- The domain is 300 X 300 mm.

Plant species	$m\ (mm^2)$	$\tau\ (s)$
Wheat	5443	600
Mayweed	235	600

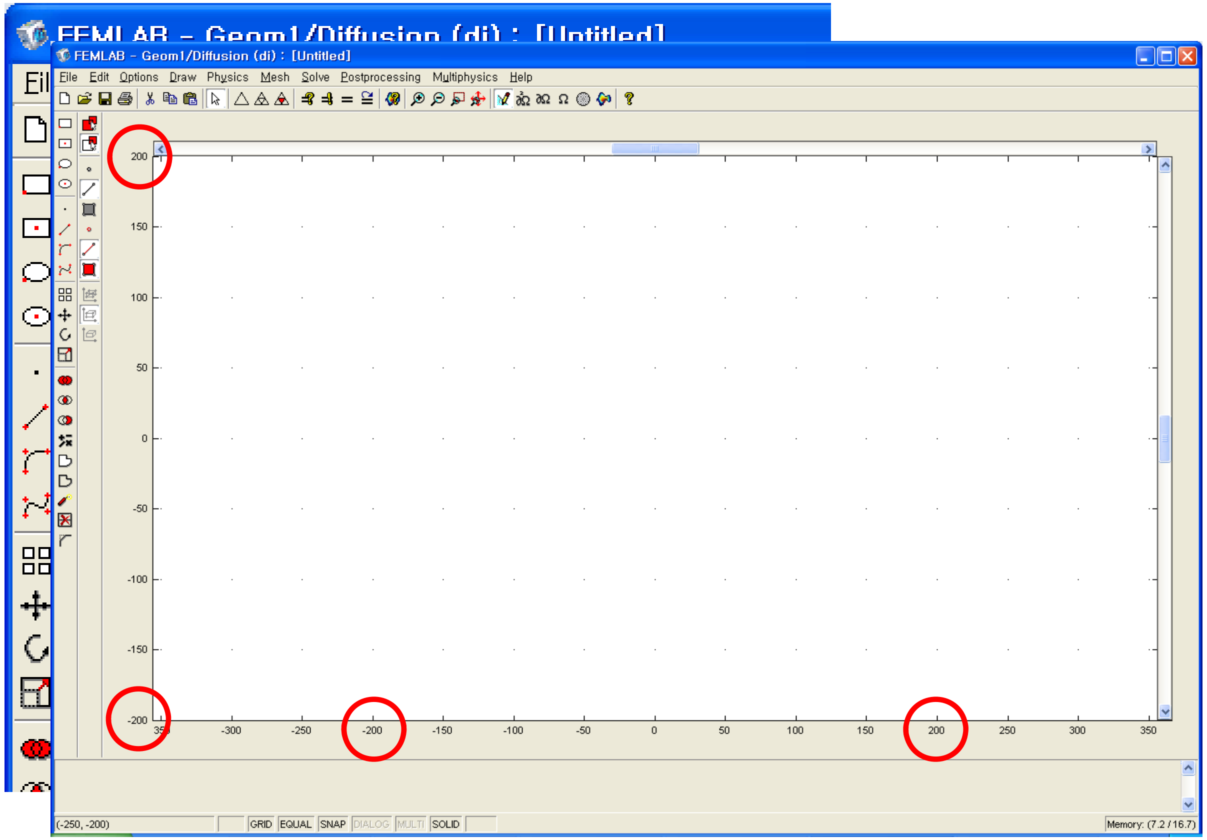
- The model simulates this process for a period of 1 hour.

Modeling using the FEMLAB Model navigator



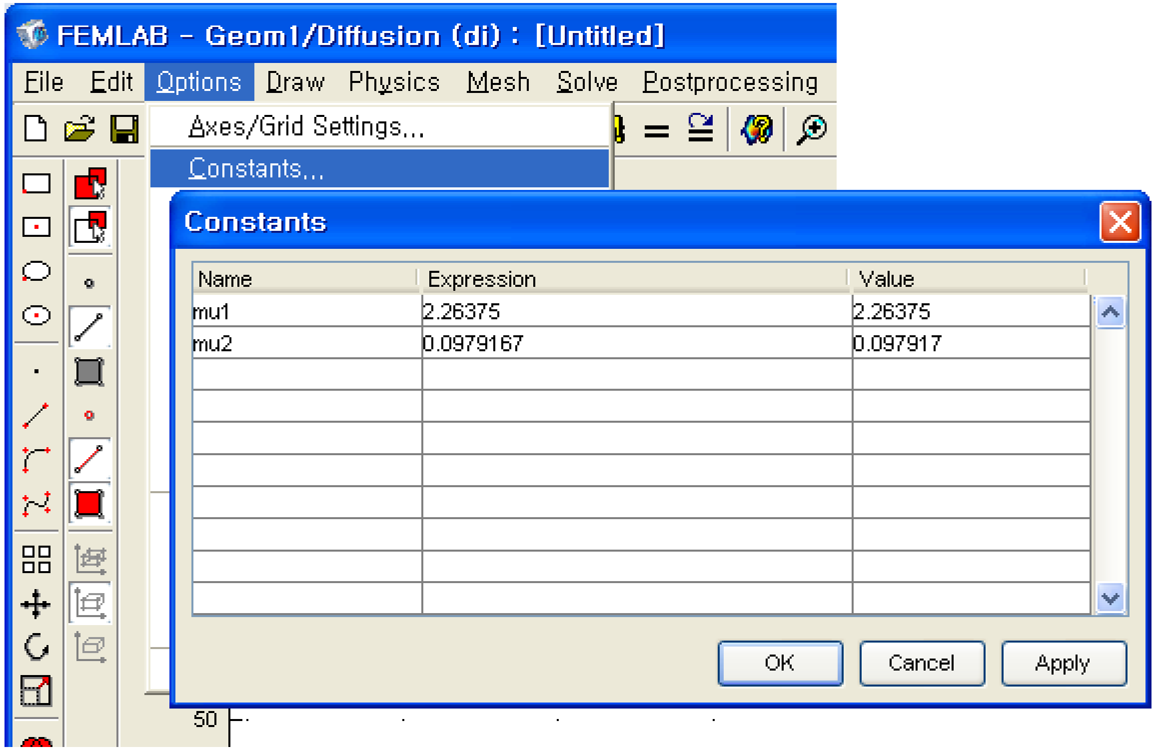
1. Open the FEMLAB
2. Select **Diffusion** from the **Diffusion** folder in the **Model Navigator**.
3. Click **OK**

Modeling using the FEMLAB
Options and settings



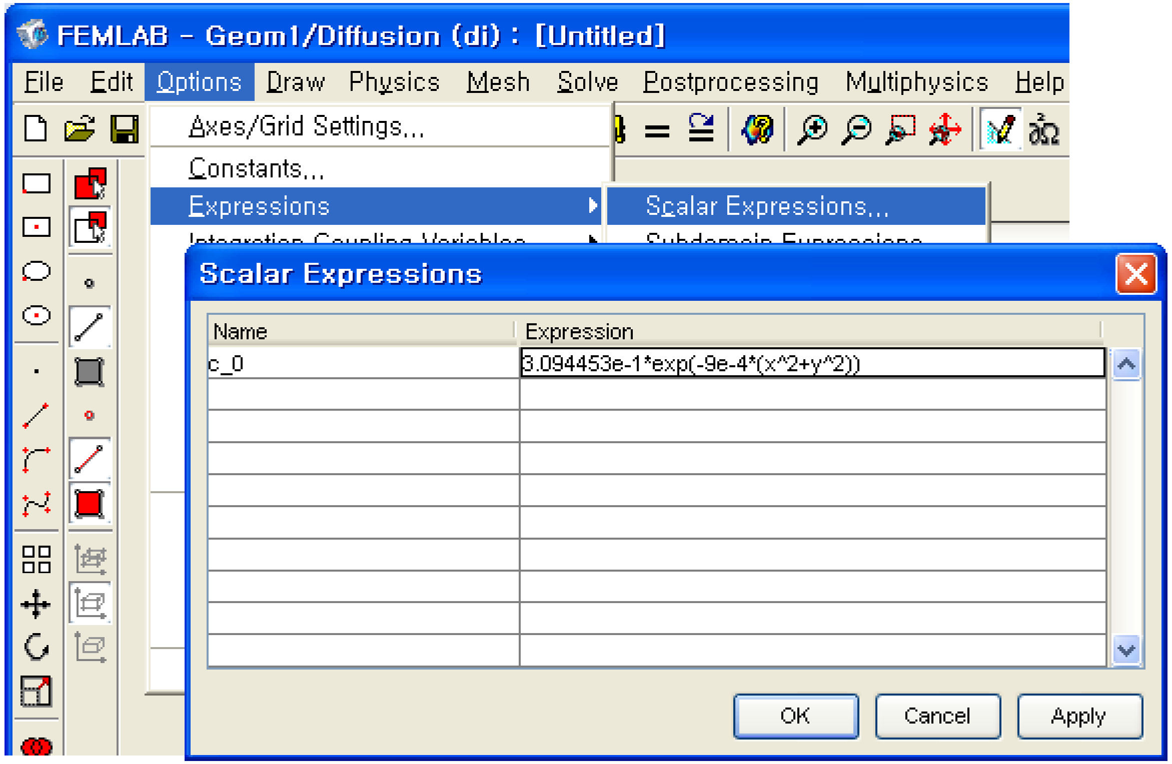
1. From the **Options** menu, choose **Axes/Grid Settings**.
2. Set **xmin** and **ymin** to -200 and **xmax** and **ymax** to 200

Modeling using the FEMLAB
Options and settings



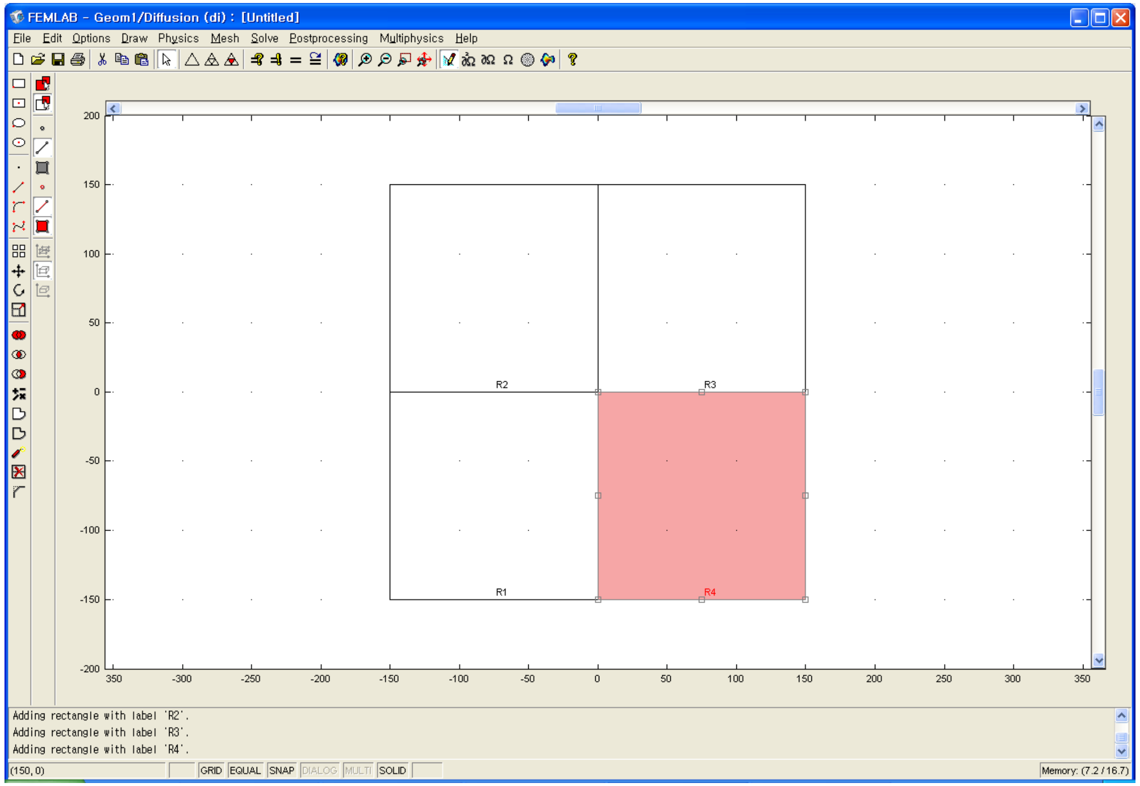
1. Open the **Constants** dialog box from the **Options** menu.
2. Enter the following constants.

Modeling using the FEMLAB
Options and settings



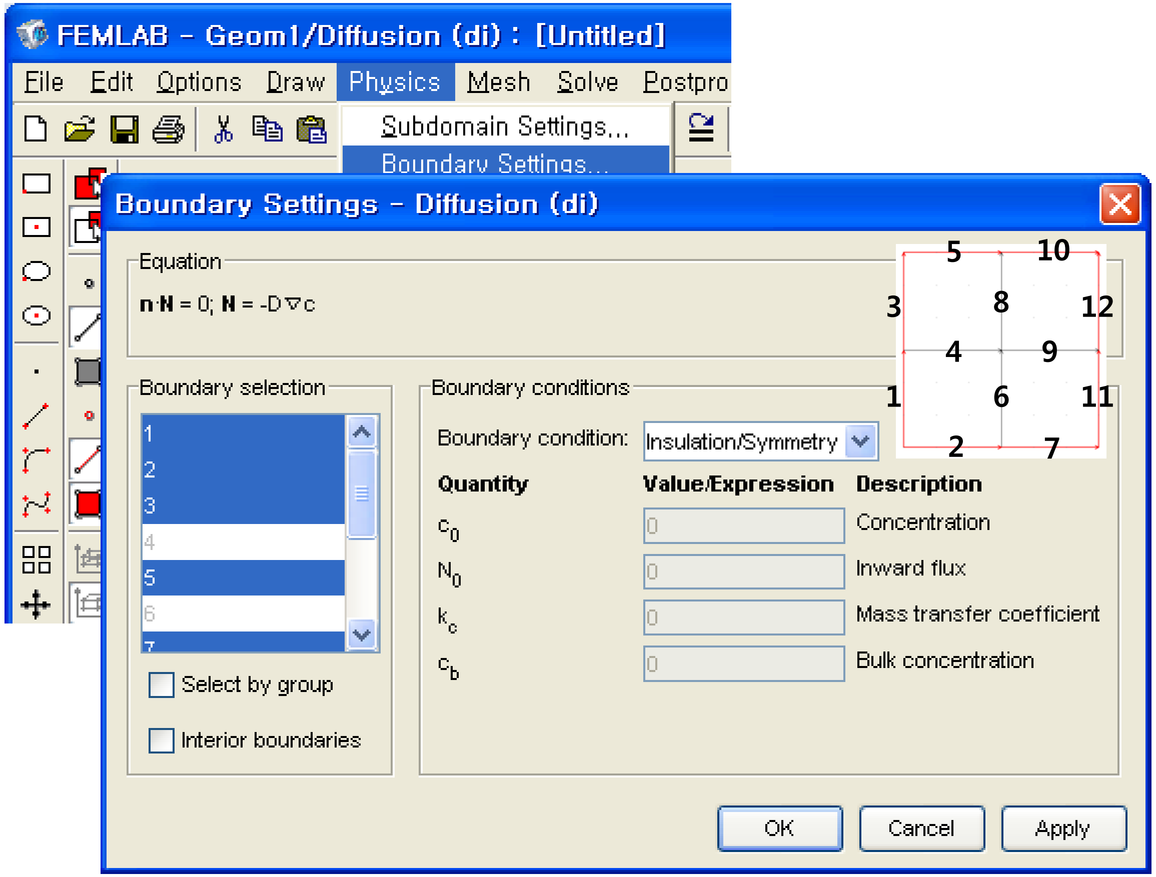
1. On the Options menu, point to **Expressions** and then click **Scalar Expressions**.
2. Enter the following expression.

Modeling using the FEMLAB Geometry modeling



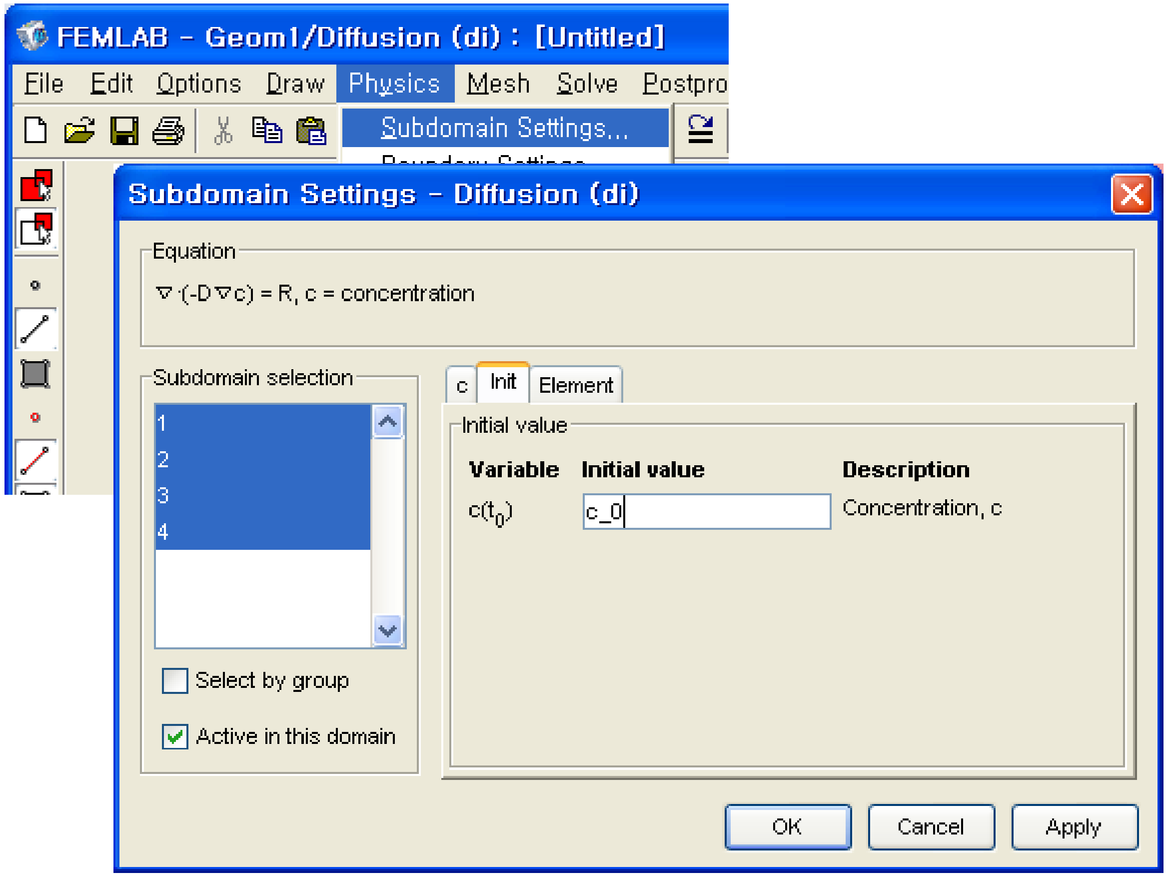
1. Click the **Rectangle/Square** button and draw a square with corners in (-150,-150) and (0,0).
2. Draw three more squares in the following locations.

Modeling using the FEMLAB
Physics settings (boundary)



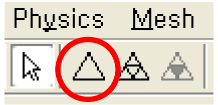
1. Open the **Boundary Settings** from the **physics** menu.
2. Make sure the **Boundary condition** is **Insulation/Symmetry** for <u>all outer boundaries</u>.

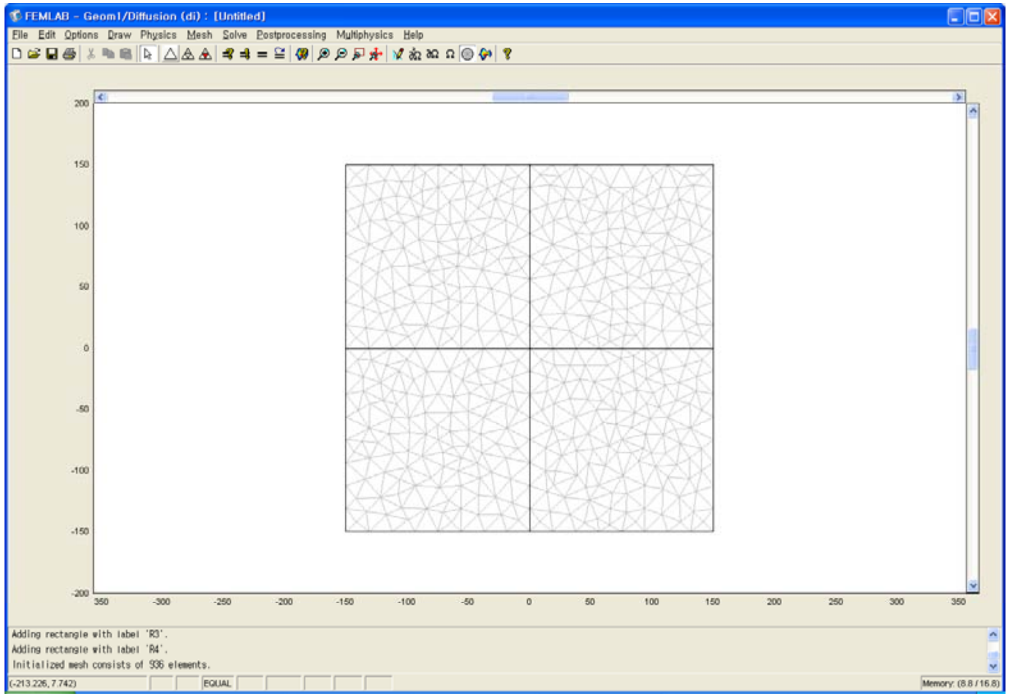
Modeling using the FEMLAB
Physics settings (subdomain)



1. Open the **Subdomain Settings** dialog box from the **Physics** menu.
2. On the **c** page set **D isotropic** to mu1 in subdomain 1 and 4, and to mu2 in subdomain 2 and 3.
3. On the **Init** page set **$c(t_0)$** to c_0 in all subdomains.

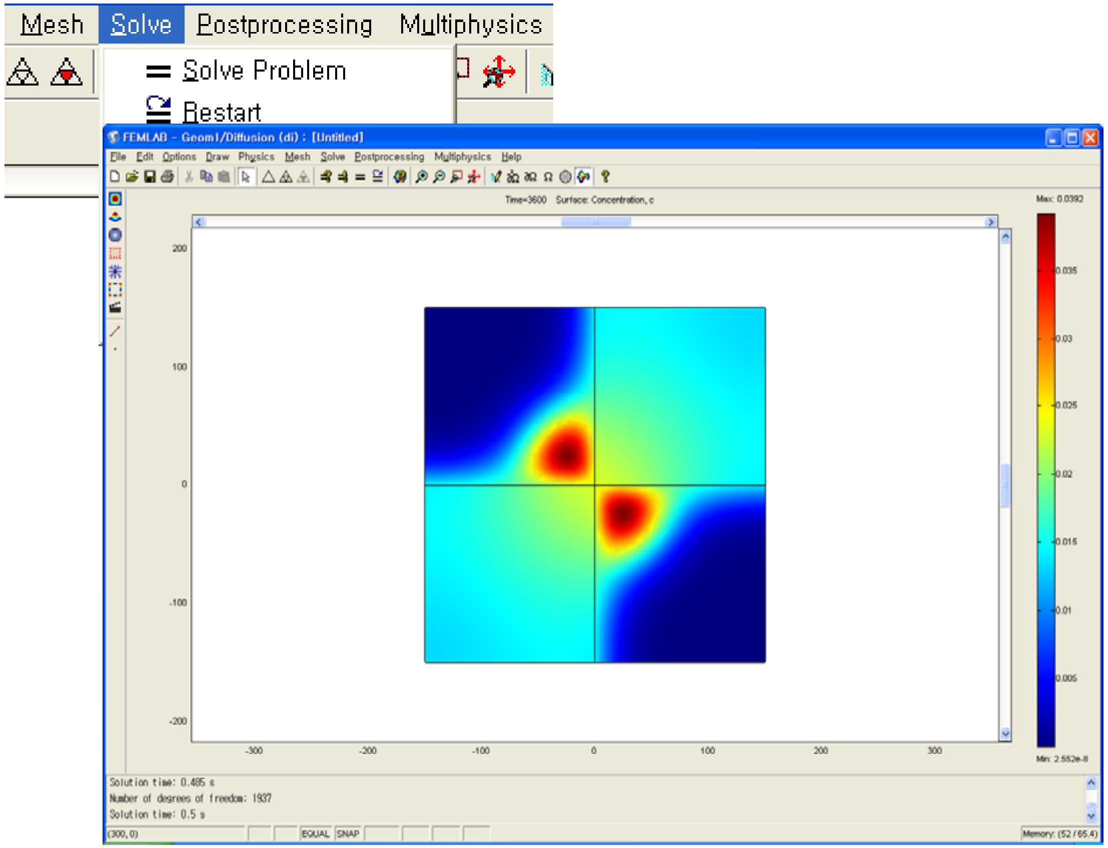
Modeling using the FEMLAB
Mesh generation





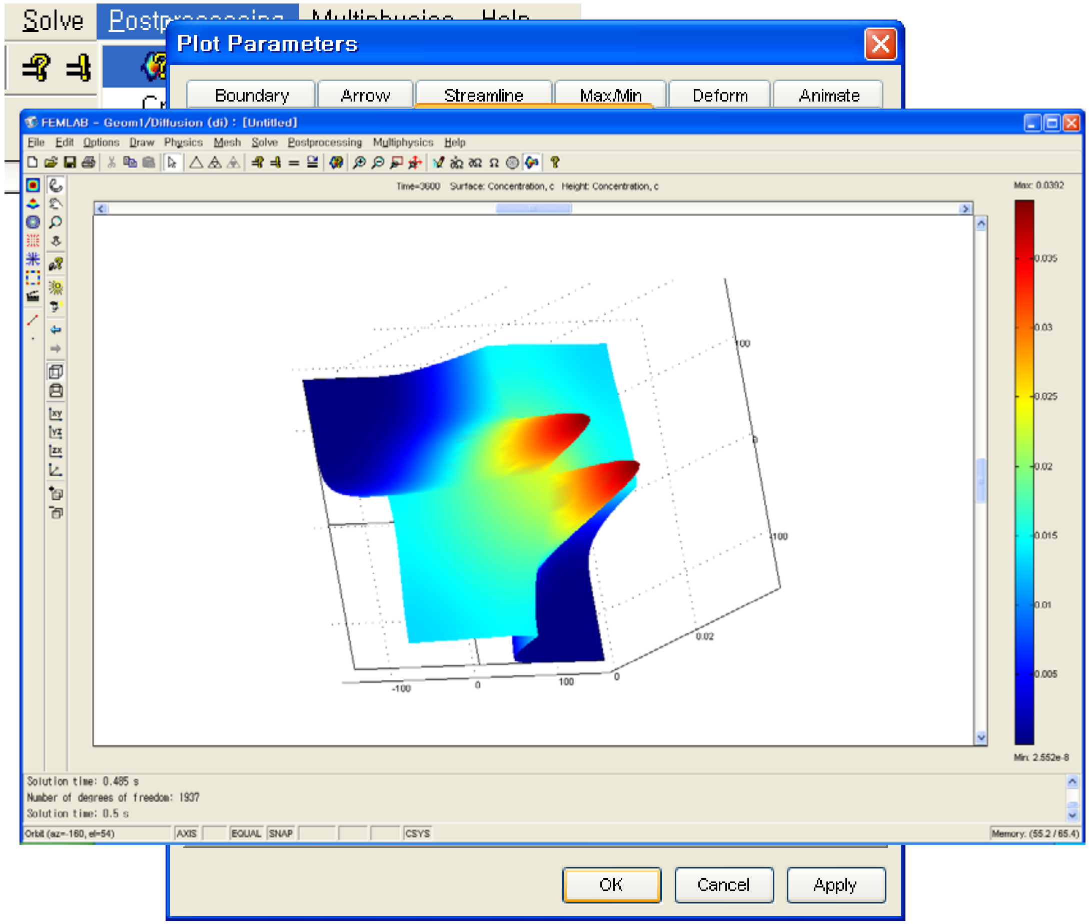
1. Initialize the mesh and refine it once.

Modeling using the FEMLAB
Solving the model



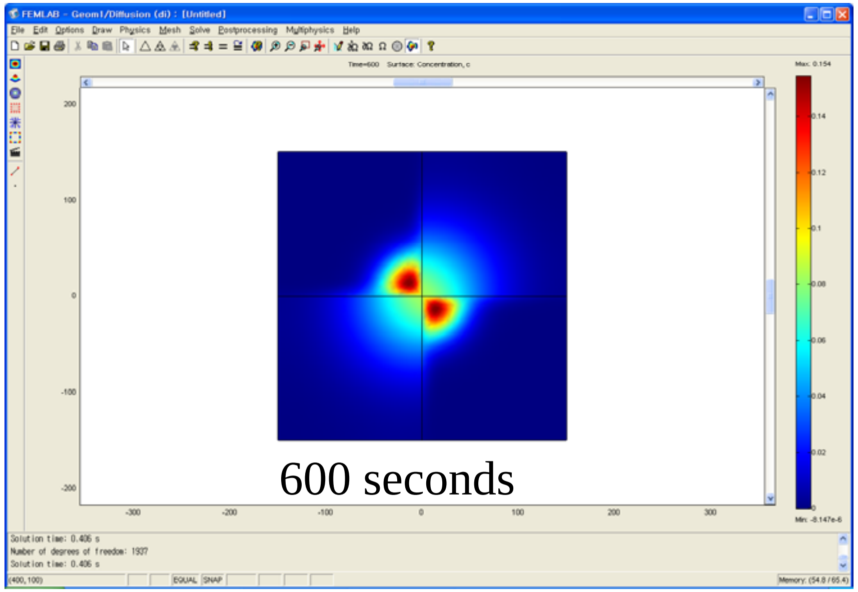
1. Open the **Solver Parameters** dialog box.
2. Select the **Time dependant** solver and change the **Times** to 0:200:3600.
3. Click the **Solve** button.

Modeling using the FEMLAB Postprocessing and visualization



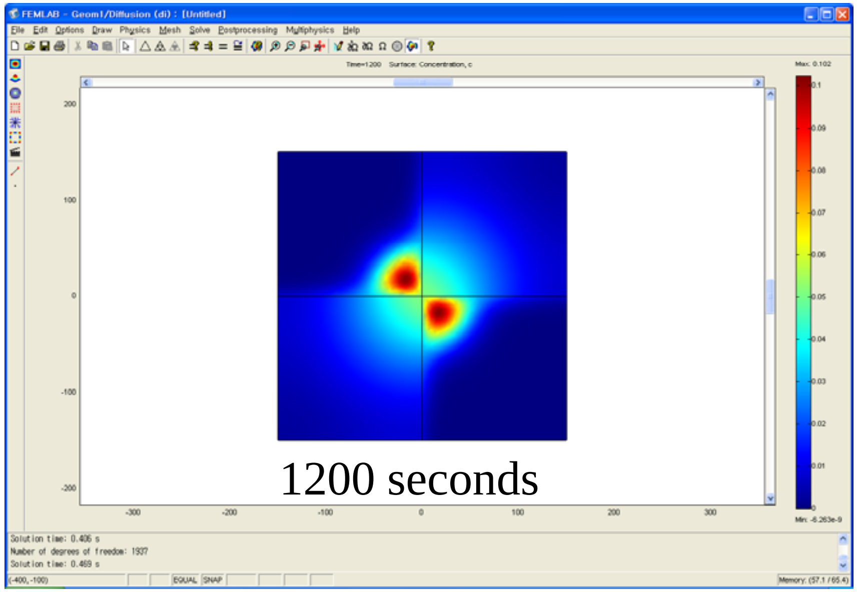
1. Open the **Plot Parameters** dialog box.
2. Click the **Surface** tab.
3. Under **Surface data**, make sure that, Concentration, c is selected in the **Predefined quantities** list
4. Select the **Height data** check box.
5. Under **Height data**, select Concentration, c in the **Predefined quantities** list.
6. Click **OK**.

Result analysis



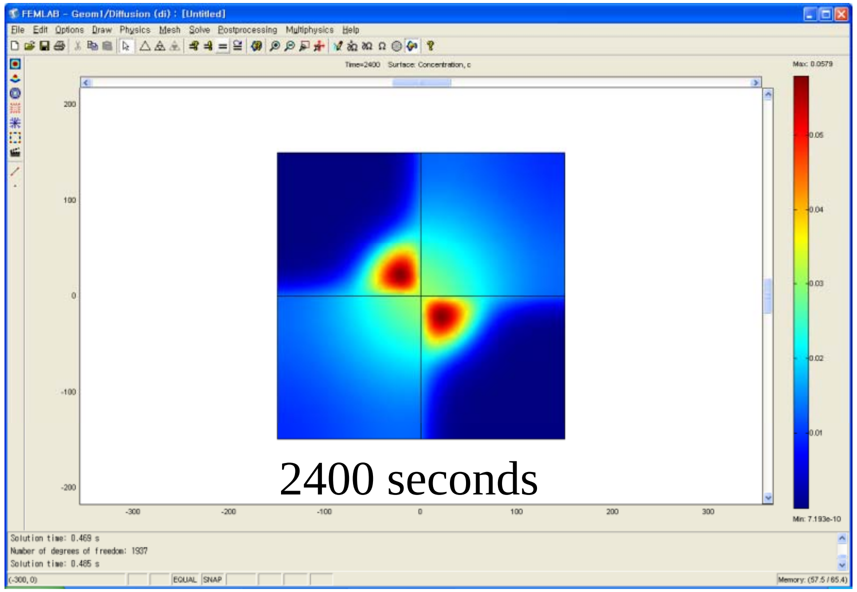
600 seconds

Mayweed	Wheat
Wheat	Mayweed

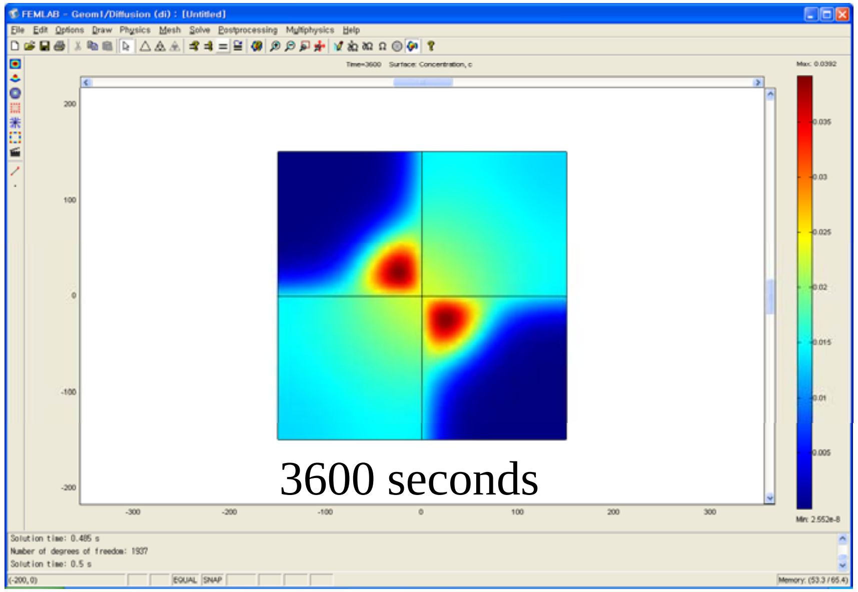


1200 seconds

Result analysis



Mayweed	Wheat
Wheat	Mayweed



Conclusions

- As time proceeds, the population distribution becomes distorted due to differences in the plant transport properties.
- Initially it appears that the nymphs are attracted to the mayweed. However, as the process moves towards a steady-state condition, the population becomes uniform in both plant species.