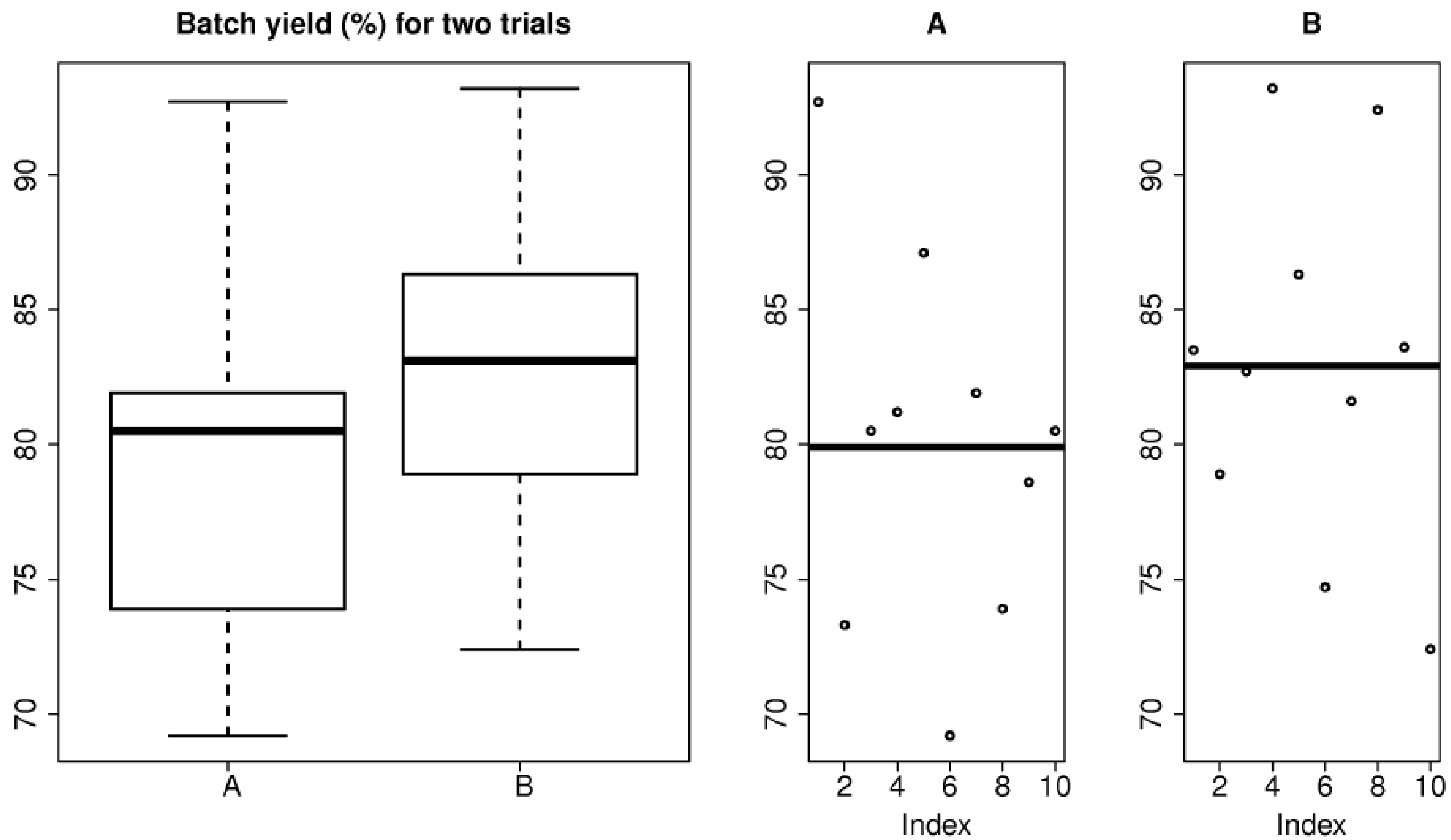
C.I: a basic tool for statistical inference (cont.)

Data acquired



C.I: a basic tool for statistical inference (cont.)

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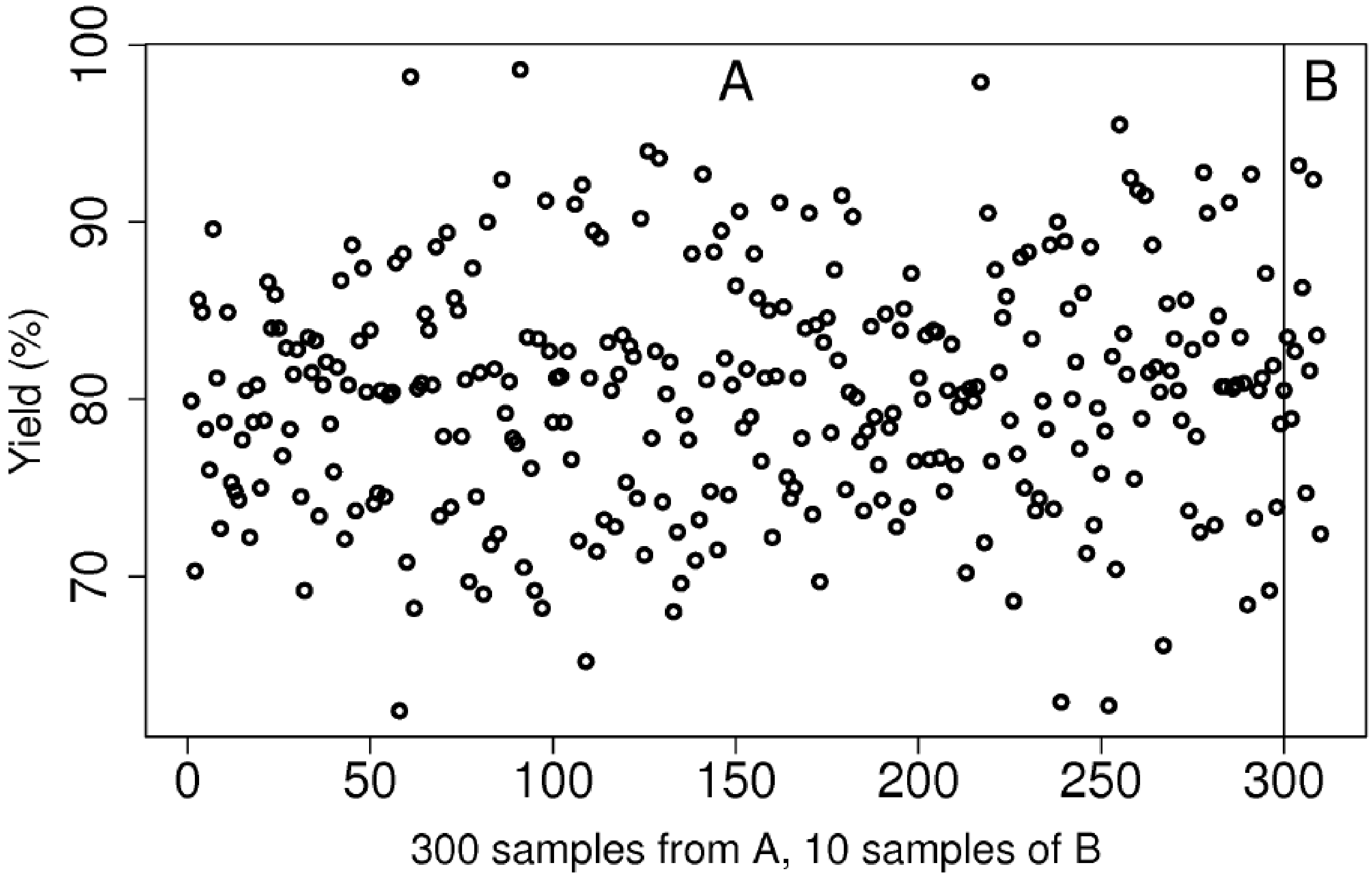
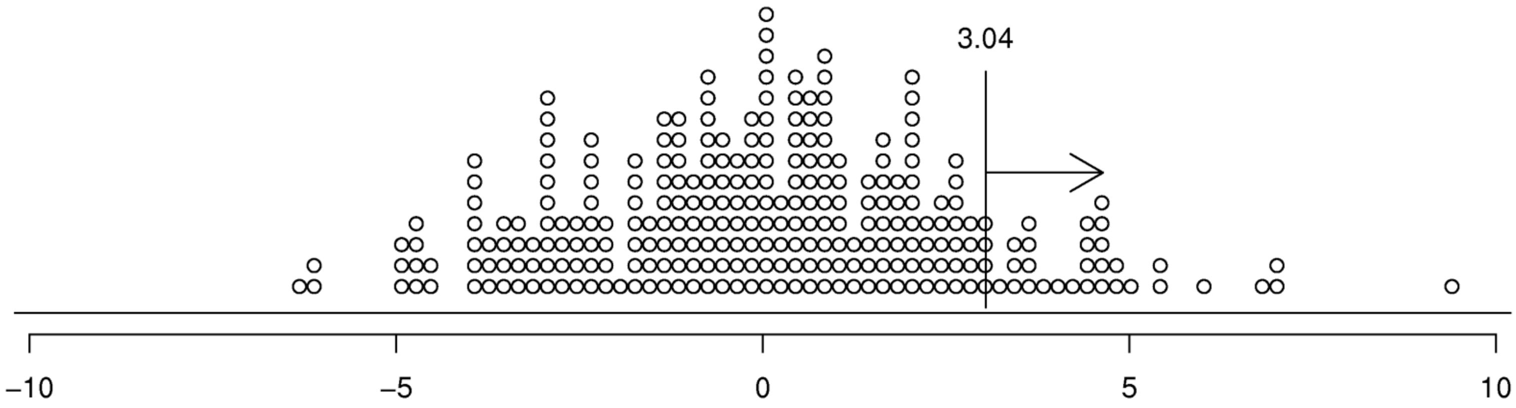


Test for differences/similarity

- What would be the simplest yet not a bad method?
  - Compare with (long-term) reference set.
    - Ex. 300 batches last 10 years.



    - 31 historical differences out of 300 had a difference value higher than 3.04.
    - 89.7 % of previous batches had lower yield.
- What if no reference set?

Test for differences/similarity

- Don’t have a suitable reference, e.g. just the 20 runs.
- But still want to know $\mu_A > \mu_B$ or the other.
  - Comparing μ_A & μ_B requires C.Is of μ_A & μ_B. Why?
- There are several cases to consider
  - Unpaired, variance known
  - Comparison of variances
  - Unpaired, variances unknown but equal
  - Unpaired, variances unknown and unequal
  - Paired

Case1: Unpaired data, σ^2_1 and σ^2_2 are known

- No pairing of data: there is no relationship between an individual value in one sample and an individual value in the other sample (i.e., independent)
- Compute:

$$\overline{X} - \overline{Y}$$

We ask the question, "is 0 a plausible value for $\bar{X} - \bar{Y}$?" i.e. is there no difference between the means?

Case1: Unpaired data, σ^2_1 and σ^2_2 are known

We can use confidence intervals to answer this question.

To do this we need to know the distribution of $\bar{X} - \bar{Y}$.

Since $\bar{x}$ and $\bar{y}$ are both normally distributed with variances σ_x^2 / n_x and σ_y^2 / n_y respectively.

$$\left(\bar{X} - \bar{Y}\right) \sim N\left(\mu_x - \mu_y, \frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}\right)$$

i.e. the variance of $\bar{X} - \bar{Y}$ is $\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}$ assuming that X and Y are independent

Case1: Unpaired data, σ^2_1 and σ^2_2 are known

$$1-\alpha = \Pr ob\left\{ -z_{\alpha/2} \le \frac{(\bar{x}-\bar{y})-(\mu_x-\mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x}+\dfrac{\sigma_y^2}{n_y}}} \le z_{\alpha/2} \right\}$$

and the 100(1-α) % confidence interval for the difference in population means is

$$\left[(\bar{X}-\bar{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}\ ,\quad (\bar{X}-\bar{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}} \right]$$

If the confidence interval contains zero, then there is no statistically significant evidence (at the 1-α level) of a difference in means.

Case 2a: variances are unknown, but the same

- Use pooled variances

$$s_p^2 = \frac{\nu_x s_x^2 + \nu_y s_y^2}{\nu_x + \nu_y}, \qquad \nu_x \,\&\, \nu_y : \text{D.O.F of each samples}$$

$$\frac{\left(\overline{X} - \overline{Y}\right)}{\sqrt{\dfrac{s_p^2}{n_x} + \dfrac{s_p^2}{n_y}}} \sim t_\nu$$

$$\left[\left(\overline{X} - \overline{Y}\right) - t_{\nu,\alpha/2}\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}},\quad \left(\overline{X} - \overline{Y}\right) + t_{\nu,\alpha/2}\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}\right]$$

Case 2a: variances are unknown, but the same

- Comparison of sample variances

$$\frac{s_x^2 \sigma_y^2}{s_y^2 \sigma_x^2} \sim F_{\nu_x,\nu_y}$$

$$1-\alpha = \mathrm{Prob}\left\{ \frac{s_x^2}{s_y^2 F_{\nu_x,\nu_y,\alpha/2}} < \frac{\sigma_x^2}{\sigma_y^2} < \frac{s_x^2}{s_y^2 F_{\nu_x,\nu_y,1-\alpha/2}} \right\}$$

$$\left[\frac{s_x^2}{s_y^2 F_{\nu_x,\nu_y,\alpha/2}}, \quad \frac{s_x^2}{s_y^2 F_{\nu_x,\nu_y,1-\alpha/2}} \right]$$

If this contains 1, then…..

Case 2b: variances are unknown and different

-There is no exact procedure for this case but there are several approximate procedures.

-One such procedure is known as Cochran's procedure

-A crude approach is to construct a confidence interval for μ_x based on the estimated variance of the x sample, and then construct a confidence interval for μ_y using the estimated variance for the y sample. If the confidence intervals overlap, then there is no evidence at the 100(1-α) % confidence level of a difference.

-Many statistical S/W can handle this.

Case 3: Paired comparisons between two groups

- The idea is to remove extraneous sources of variation by taking measurements in pairs, each pair comprising one measurement from each of the two groups. Each pair should have as much in common as possible except the difference that is trying to be detected.

- Each pair is called a block. The idea of blocking will be discussed in more detail when we talk about Design of Experiments.

Paired comparisons between two groups

$$\mathrm{w_i = x_i - y_i}$$

$$\mathrm{\overline{w} = \frac{\sum_{i=1}^{n} w_i}{n}}$$

$$s_w^2 = \frac{\sum_{i=1}^{n}(w_i - \overline{w})^2}{n-1}$$ with ν_w=n-1 degrees of freedom

$$[\,\overline{w} - t_{\nu,\alpha/2}\, s_w/\sqrt{n}\,,\,\overline{w} + t_{\nu,\alpha/2}\, s_w/\sqrt{n}\,]$$

Example

- In an oxide coating operation on silicon chips, a series of test were conducted to determine whether position in the furnace had any appreciable effect on the resulting thickness of the oxide coating. Within each furnace run, chips were randomly assigned to two positions, denoted by A and B, within the furnace. 10 independent furnace runs were carried out and a single thickness measurement (in Angstroms) was made at the centre of each chip.

Run	A	B	w
1	920	923	-3
2	914	924	-10
3	927	913	14
4	891	881	10
5	943	923	20
6	902	884	18
7	910	887	23
8	856	858	-2
9	937	916	21
10	857	857	0

Solution

$$[\,\overline{w} - t_{\nu,\alpha/2}\, s_w / \sqrt{n}\,, \overline{w} + t_{\nu,\alpha/2}\, s_w / \sqrt{n}\,]$$

$$[\,9.1 - t_{13,0.025}\, s_w / \sqrt{n}\,, 9.1 + t_{13,0.025}\, s_w / \sqrt{n}\,]$$

$$[\,9.1 - 2.26\frac{11.9}{\sqrt{10}}, 9.1 + 2.26\frac{11.9}{\sqrt{10}}\,]$$

$$[0.6,\ 17.6]$$

Advantages of Pairing

- Extraneous sources of variation are removed or minimized.
- The test of significance is more sensitive because the total variation is minimized.
- Measurements are paired based on proximity in time or space, or in some cases, blocking (pairing) is done by piece, person or animal
- Within each pair, the order of the measurements should be randomized.
- Pairing helps to remove one or more sources of variation.
  - Randomization helps to spread the effects of extraneous sources of variation over all measurements so as to avoid accidentally confounding the effect of interest with an extraneous source of variation.

Examples

- 1-sample Z / 1-sample t
  - Previously, process lead time (L/T) was 30 days. Recently, process L/T is surveyed (see the table below) after successful completion of a six sigma project. From this data, can you say that the six sigma project was successful to reduce process L/T?

Data: 23, 28, 35, 26, 22, 25, 28, 29, 24, 30, 18, 23, 24, 29, 28

(1) When σ^2 is known to be 4.09^2.

Stat > Basic statistics > 1-sample Z

(2) When σ^2 is unknown.

Stat > Basic statistics > 1-sample t

Examples

- 2-sample t
  - The components library was standardized in order to reduce set-up loss of SMD process. Following data were measured to know whether actual set-up loss time was reduced. What do you think?

Before	12.50	12.63	13.25	13.00	12.86	12.75	13.22	12.90	12.96	13.15
after	12.35	12.81	12.00	11.95	12.12	12.64	12.56	12.75	12.19	12.17

Stat > basic statistics > 2-sample t

Hypothesis test

- When a variable appears to have an effect, it is important to be able to state with confidence that the effect was really due to the variable and not just due to chance.

Ex. We want to introduce a new catalyst B. Does it improve our product properties over the current catalyst A?

Ex. Drug test

Experimental group

Given a new drug under test

Control group

Given a placebo

- How can one be sure that the drug treatment rather than chance occurrences were responsible for the difference between the groups?

[FYI] Vocabulary

- Confidence level, 1 - α (or 100(1 – α)%)
  - The confidence level tells you how sure you can be. It is expressed as a percentage and represents how often the true percentage of the population who would pick an answer lies within the confidence interval.
  - The 95% confidence level means you can be 95% certain.
- Significance level, α (or 100α%)
  - the criterion used for rejecting the null hypothesis.
- Null hypothesis (歸無/虛無 가설)
  - Ex. The increased yield of catalyst B is the result of pure chance.
- Alternative hypothesis
  - Ex. The null hypothesis is false. The increased yield of catalyst B is not the result of pure chance, but is .

[FYI] meaning of a p-value

- A measure of how much evidence we have **against** the null hypothesis.
- The general rule is that a small p-value is evidence against the null hypothesis while a large p-value means little or no evidence against the null hypothesis.
- The p-value is **not** the probability that the null hypothesis is true.
- In a court room,
  - H_0: the defendant is innocent. H_1: the defendant is guilty.

Judge's decision \ Fact	He is innocent	He is guilty
He is innocent (Accept H_0)	right	Wrong Type II error (2종 오류)
He is guilty (Reject H_0)	Wrong Type I error (1종 오류)	right

Examples

- Redo previous examples using (formal) hypothesis test.
  - What would be appropriate null & alternative hypotheses for each example?