Advanced Engineering Statistics

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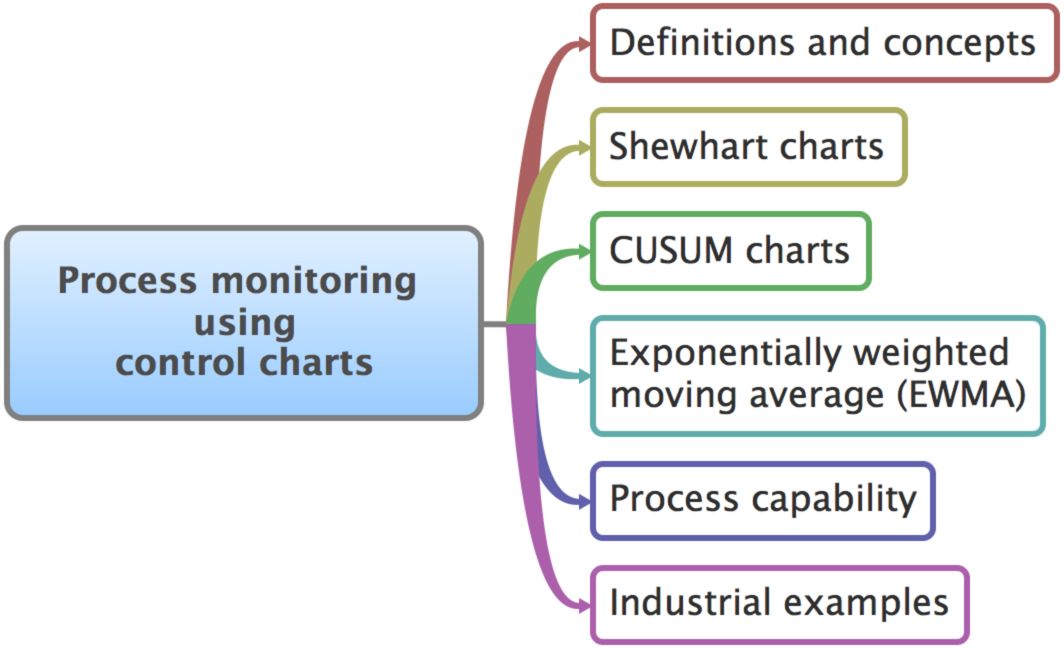
PKNU

Statistical Process Control (A.K.A Process Monitoring)

- What we will cover



Reading: Textbook Ch. ? ~ ?

What is process monitoring (or SPC) ?

- We know that quality is not optional; customers move onto suppliers that provide quality products
- Quality is not a cost-benefit trade-off either
- Car sales in North America: the steady rise of the Asian manufacturers

  Key point: apply monitoring at every step in the manufacturing line/system

  - Low variability early on; don't wait to the end
- Definition

  A collection of methods for controlling the quality of a product by collecting and interpreting data to determine the capability and current performance of a process.

  SPC methods make a distinction between what is called **common cause variation** and **special cause variation**.

Terminology

- **Common cause variation** (sometimes called **inherent variation)** is always present. It normally arises from several sources, each of which usually makes a relatively small contribution. Common cause is typically quantified using measures such as the sample standard deviation s or the range R.
- Processes exhibiting only common cause variation are said to be **in statistical control,** even if they not be meeting specifications. Such processes are stable, and hence predictable (within appropriate limits identified by confidence intervals). The magnitude of common cause variation determines the **system capability.**

Terminology

- In contrast, **special cause variation** is sporadic, sometimes upsetting a process when it occurs. Special cause variation can be distinguished from common cause variation by the size or pattern of change that occurs in process behaviour. Detection of special cause variation is often subjective, with guidance from objective techniques. Because special cause variation is **abnormal variation,** *it may be harmful or beneficial.*
- Detecting and acting upon special cause variation is a responsibility of everyone in an organization, from operators to management. Sytems exhibiting special cause variation which is not acted upon are **not in statistical control.** System capability has no meaning for such systems (i.e. it is important to ensure that a process is **stable** before evaluating its capability).

For Your Information

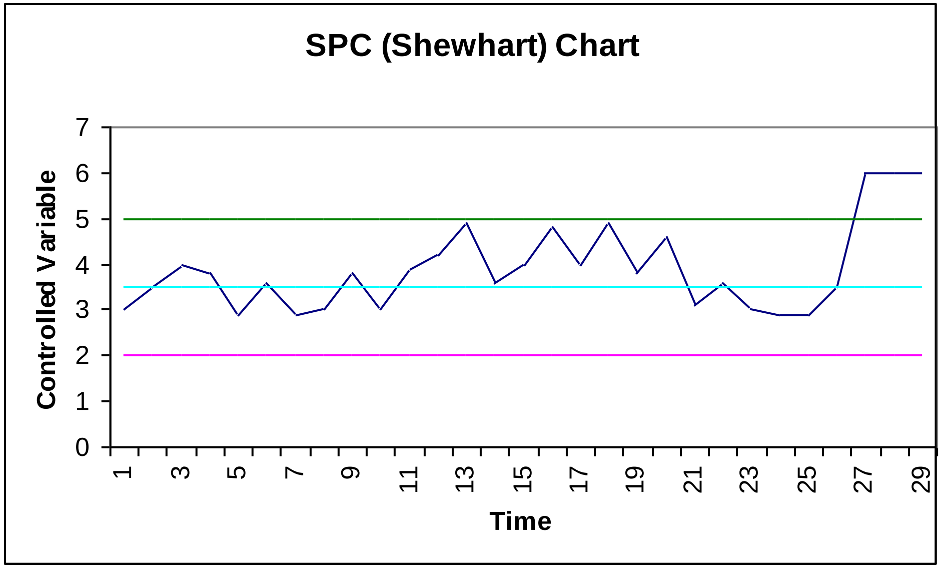
- Words from Quality Engineering
  - Reduction of common cause variation requires fundamental changes in an operation, requiring management authorization (i.e. fine tuning will have little effect).
  - Process Improvement comes about through the identification of special cause variation and then its deliberate elimination or persistence.

Relationship to automatic feedback control

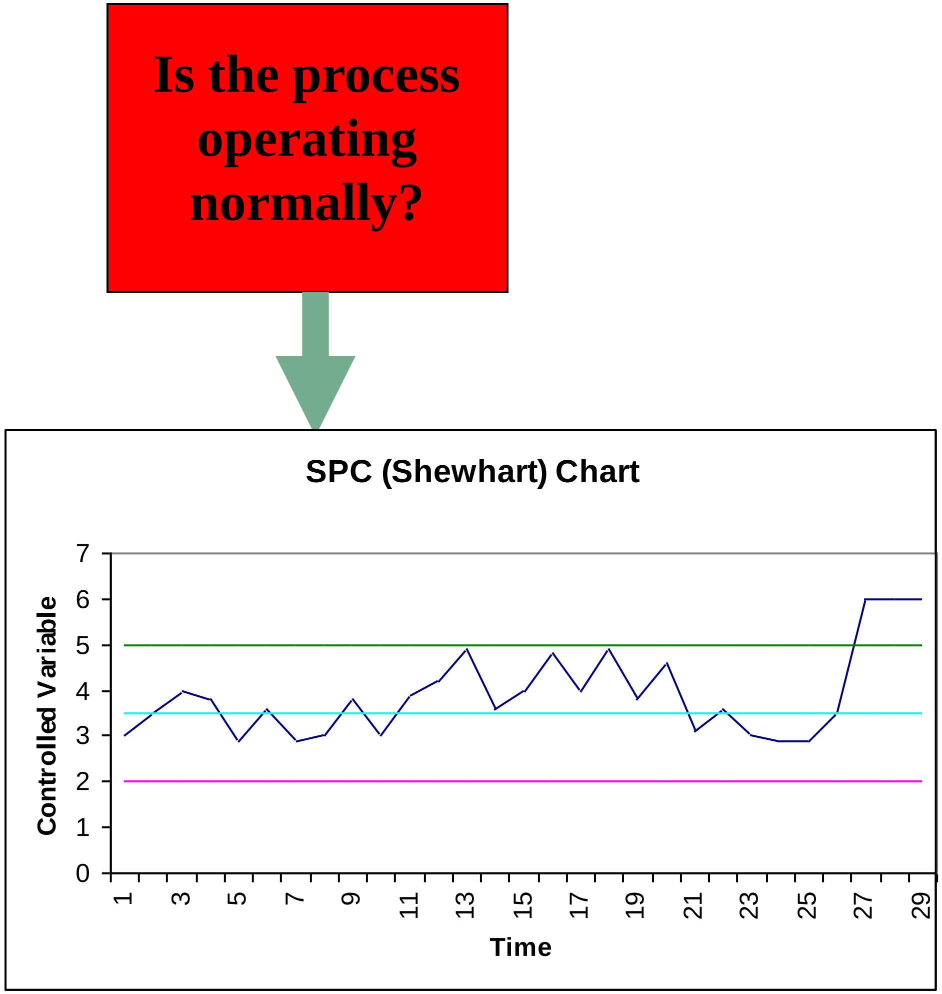
- Similar to automatic (feedback) control
  - continually applied
  - check for deviations (error)
- Different to automatic (feedback) control
  - adjustments are infrequent
  - usually manual
  - adjust due to special causes
  - aim is to make (permanent) adjustments to avoid that
  - variability from ever occurring again

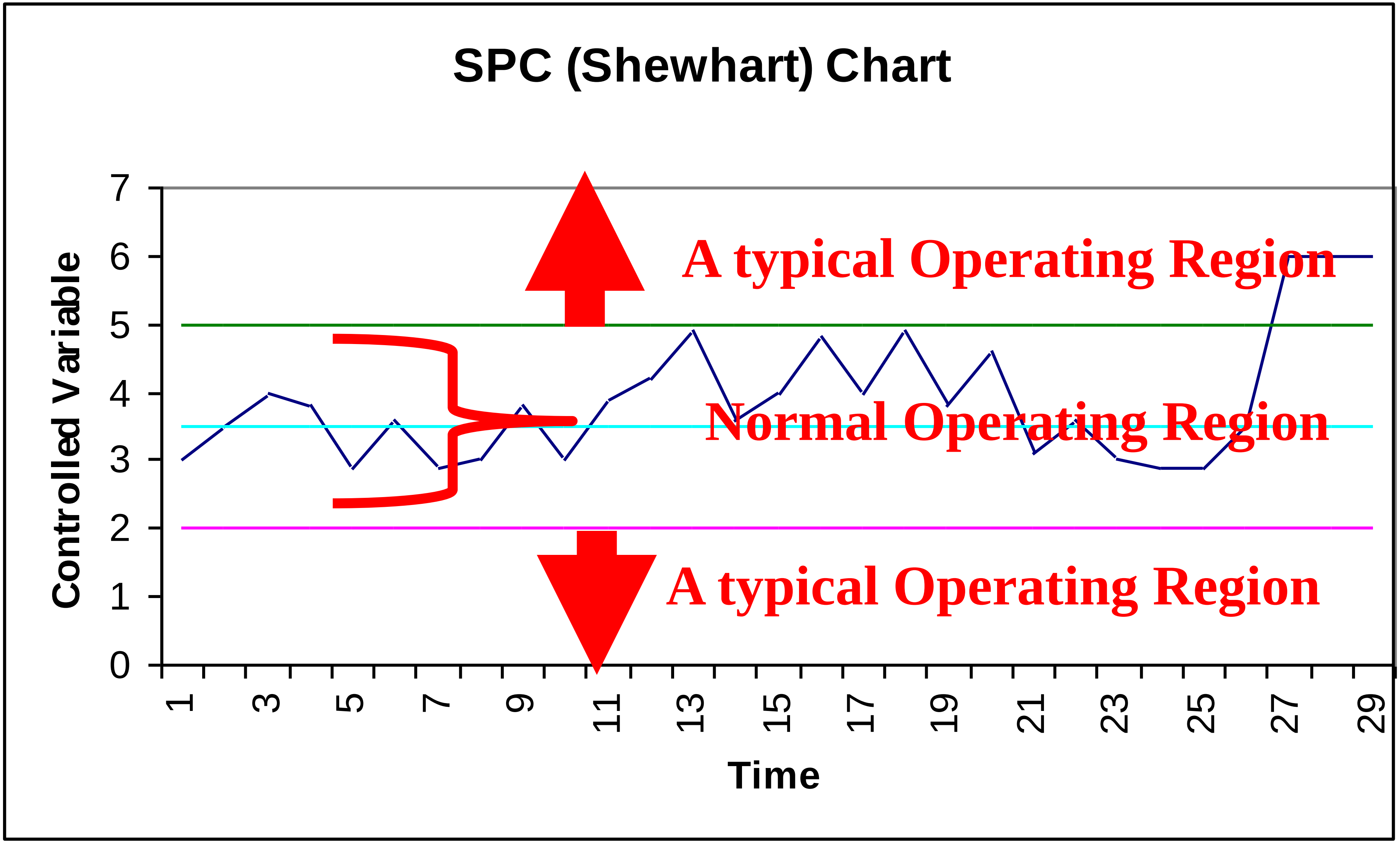
Control Charts

- Used to display and detect this unusual variability
  - it is most often a time-series plot, or sequence
  - a target value may be shown
  - one or more limit lines are shown
  - displayed in real-time, or pretty close to real-time



Features of Control Charts




SPC (Shewhart) Chart
A typical Operating Region
Normal Operating Region
A typical Operating Region
Controlled Variable
Time
0
1
2
3
4
5
6
7
1
3
5
7
9
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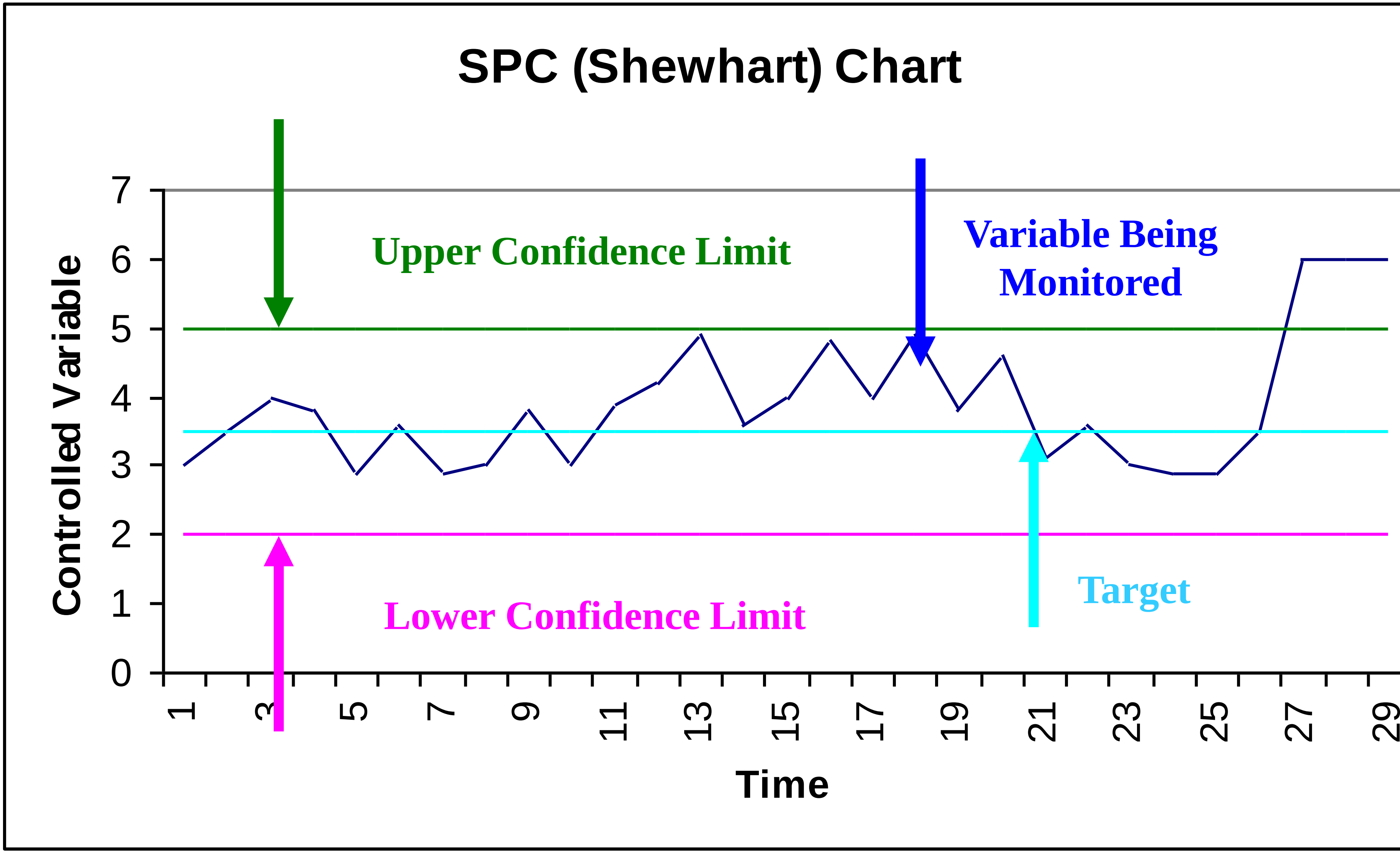
21
23
25
27
29


SPC (Shewhart) Chart
Upper Confidence Limit
Variable Being Monitored
Target
Lower Confidence Limit
Controlled Variable
Time
0
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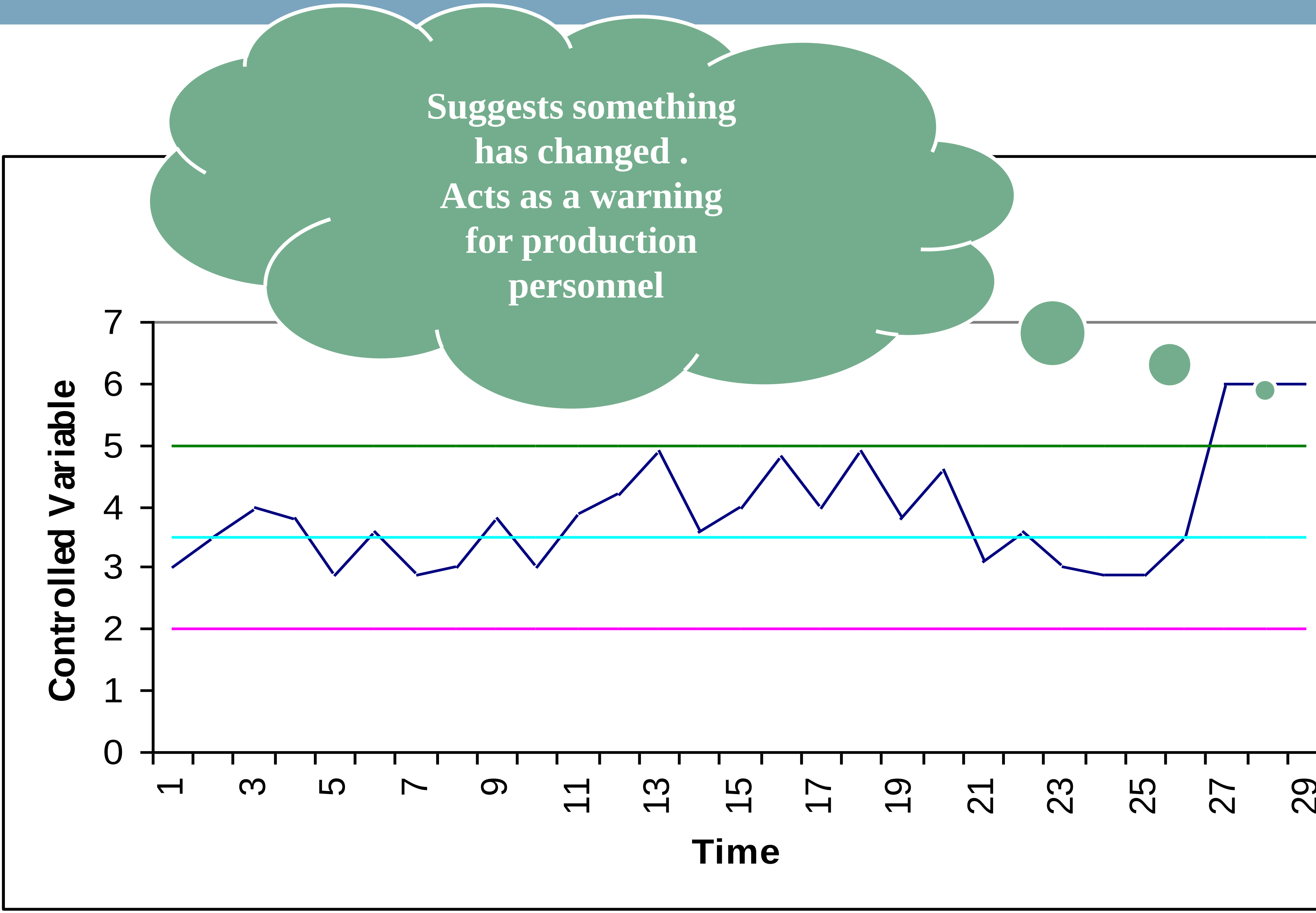

Suggests something
has changed .
Acts as a warning
for production
personnel
Controlled Variable
Time
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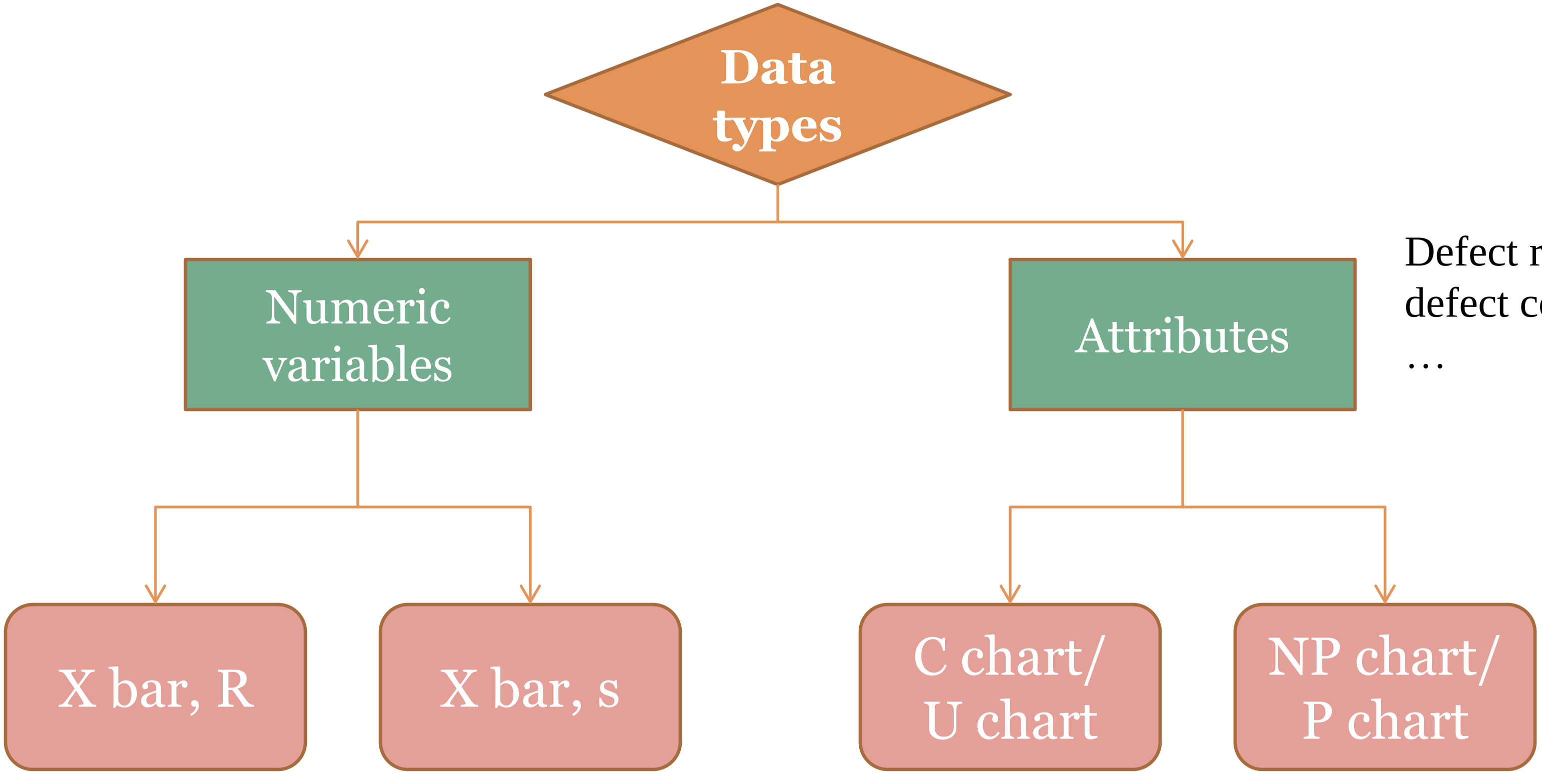
Type of control charts

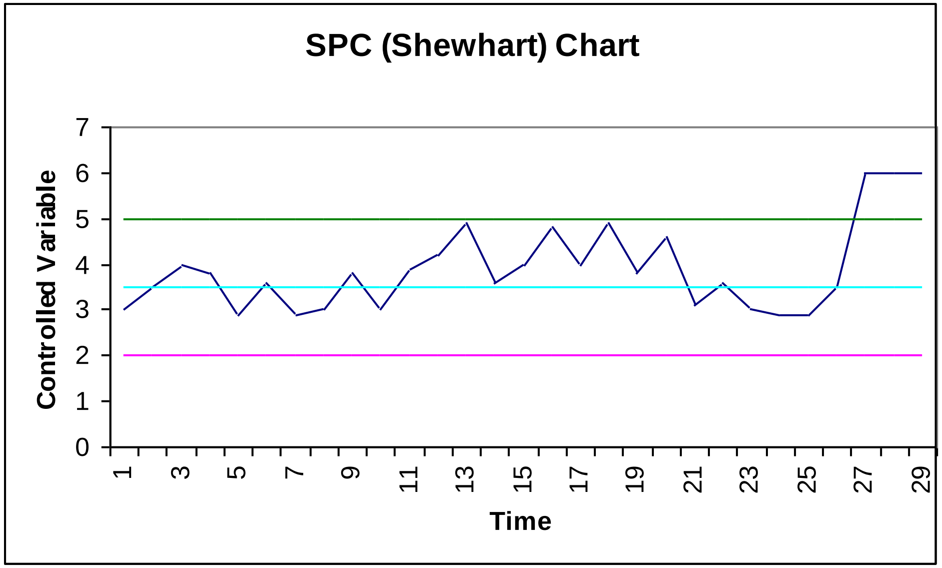


Defect rate
defect count
…

See related minitab command

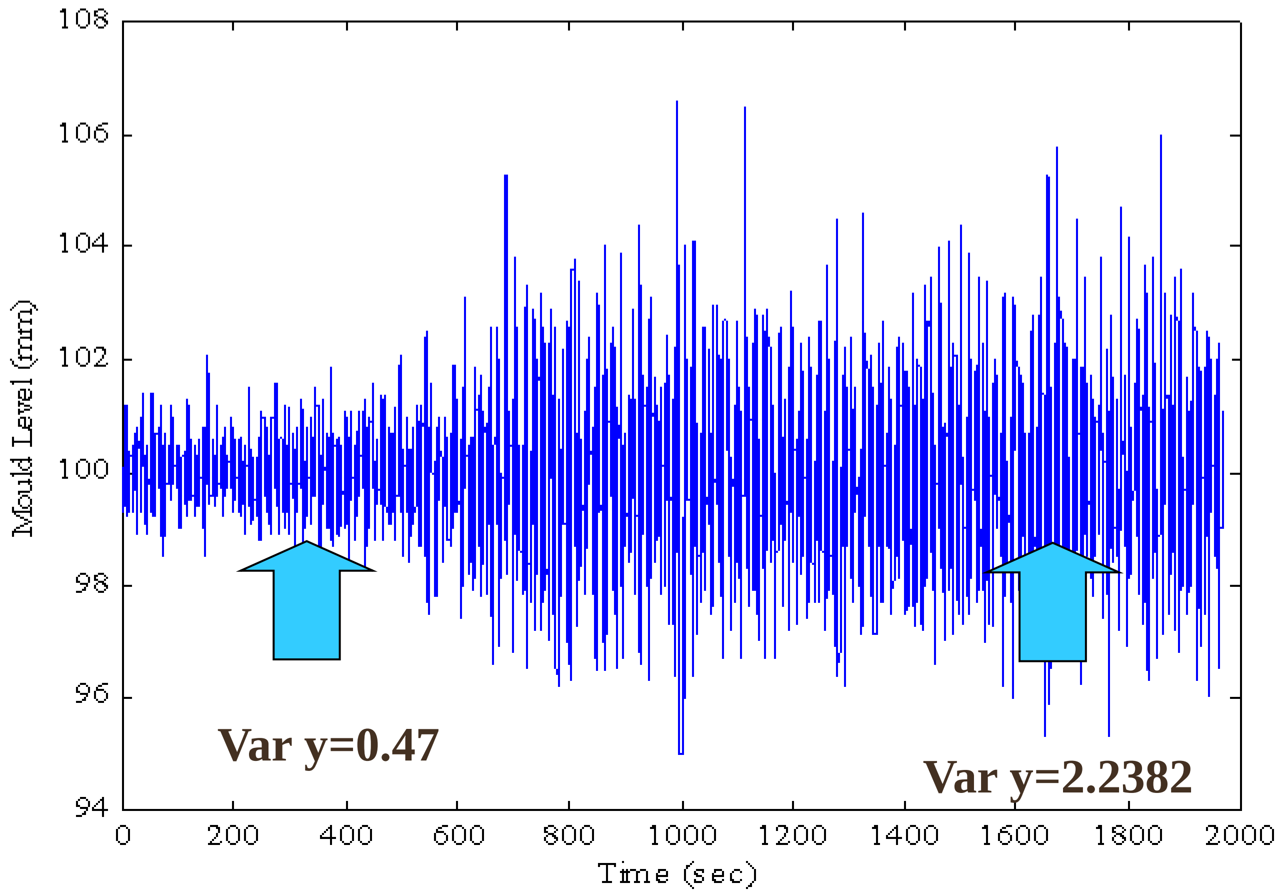
What do we want to see?

- Location
  - Close to target or not?



What do we want to see?

- Too much fluctuation?



$\overline{X}$-R / $\overline{X}$-s charts

- Also known as Shewhart chart
  - Named for Walter Shewhart from Bell Telephone and Western Electric
- In general, we want to know whether the quality meets the target and variation is also within certain range.
  - Sample mean ($\overline{X}$) for monitoring target
  - Range **R** or sample standard deviation **s** for monitoring variability

※Why R instead of s?

  - In 1920's when control charts were first introduced, calculations were carried out by hand, and so the sample range **R** was strongly preferred to the sample standard deviation as an estimate of dispersion because it was so much easier to calculate.

X-bar Chart

- To "build a control chart" is to determine values for the three lines on the chart
  - Centre line
  - Upper control limit
  - Lower control limit
- The **centre** line value for an X-bar chart may be
  - the target value for the performance characteristic of interest
  - Or the overall sample mean of $\bar{\bar{X}}$ values from recent samples of the measured characteristic, where

$$\bar{\bar{X}} = \frac{\text{Total of } \bar{X} \text{ values for all samples}}{\text{number of samples}}$$

X-bar Charts - Control Limits

- Upper and lower control limits for an X-bar chart are determined from the pdf of the individual sample means , which is $N(\mu,\sigma^2/n)$,
  - Where σ denotes the population standard deviation of individual X measurements
  - And n denotes the size of each sample
- Essentially, the limits represent 100(1-α)% confidence interval for the mean. The UCL and LCL are determined from the following.

$$\overline{\overline{X}} \pm z_{\alpha/2}\sigma_{\overline{x}} \qquad \sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{n}}$$

$$= \overline{\overline{X}} \pm z_{\alpha/2}\frac{\sigma_x}{\sqrt{n}}$$

X-bar Charts - Control Limits

- Of course, we generally don't know the true value of the variance and we have to use an estimate in its place. In this case, the confidence limits (and consequently, the control limits) are determined using the t distribution instead of the normal distribution.
- We can estimate the variance of the mean values as follows.

$$\bar{s} = s_X = \frac{\text{Total of s values for all individual samples}}{\text{Number of samples}}$$

- DOF=ν=k(n-1)
- Then we compute the control limits as:

$$\bar{\bar{X}} \pm t_{\nu.\alpha/2} s_{\bar{x}}$$
$$= \bar{\bar{X}} \pm t_{\nu.\alpha/2} \frac{s_x}{\sqrt{n}}$$

X-bar Charts - Control Limits

- In the past, all calculations were done by hand and were often done by personnel with very little background in statistics. Therefore, simple formulas were developed for confidence limits. Some of these are still in use today. One of the more common methodologies for X-bar and s charts is given below.

X-bar Chart - Control Limits

The **centre line value** for the $\overline{X}$ **chart** in this case is again

$$\overline{\overline{X}} = \frac{\text{Total of } \overline{X} \text{ values for all samples}}{\text{number of samples}}$$

and the lower and upper control limits for the $\overline{X}$ chart are again determined from the pdf of the individual sample means $\overline{X}$, which is N(μ,σ^2/n) . Using $\bar{s}/c_4$ as an estimate of σ, limits that contain approximately 99.73% of the possible values of $\overline{X}$, assuming that the population mean μ is equal to the centre line value $\overline{\overline{X}}$, **are**

$$\overline{\overline{X}} \pm 3\frac{\sigma_X}{\sqrt{n}} \approx \overline{\overline{X}} \pm 3\frac{\bar{s}}{c_4\sqrt{n}}$$

For easier use these limits are expressed as:

Lower control limit **=** $\overline{\overline{X}} - A_3\bar{s}$

Upper control limit **=** $\overline{\overline{X}} + A_3\bar{s}$

X-bar Chart - Control Limits

where

$$A_3 = \frac{3}{c_4 \sqrt{n}}$$

is a constant whose values are shown in the following table:

n	2	3	4	5	6	7	8
A_3	2.659	1.954	1.628	1.427	1.287	1.182	1.099

S Charts - Control Limits

The centre line value for the s chart is

$$\bar{s} = s_X = \frac{\text{Total of s values for all individual samples}}{\text{Number of samples}}$$

The lower and upper control limits are intended to contain 99.73 % of the possible value of s, assuming that the population standard deviation σ is equal to the centre line value $\bar{s}$.

<u>The pdf of the standard deviation s is such that</u>

$E(s) = c_4\sigma$

and

$Var(s) = \left(1 - c_4^2\right)\sigma^2$

where c_4 depends on the number of data used to calculate s.

S Charts - Control Limits

Even though s is not normally distributed, both the lower and upper control limits are set as

3 (standard deviation of $\bar{s}$ *) =* $\bar{s} \pm 3\left(\frac{\sqrt{1-c_4^2}}{c_4}\right)\bar{s}$

For easier use these limits are expressed as:

Lower control limit = $B_3\bar{s}$

Upper control limit = $B_4\bar{s}$

where

$$B_3 = 1 - 3\left(\frac{\sqrt{1-c_4^2}}{c_4}\right)$$

and

$$B_4 = 1 + 3\left(\frac{\sqrt{1-c_4^2}}{c_4}\right)$$

Constants used to Build s Charts

n	2	3	4	5	6	7	8
B_3	0	0	0	0	0.030	0.118	0.185
B_4	3.267	2.568	2.266	2.089	1.970	1.882	1.815
c_4	0.793	0.886	0.921	0.940	0.952	0.959	0.965

Example

- Using following measurement data, construct an Xbar-R chart.
  - Stat > control chart > variables chart for subgroups > Xbar-R

Sample group	x1	x2	x3	x4	x5
1	77	82	81	80	78
2	80	77	79	79	78
3	82	79	81	81	82
4	81	77	82	79	79
5	79	82	82	82	79
6	82	82	78	81	80
7	77	82	80	78	82
8	84	81	79	79	81
9	80	77	78	77	81
10	84	80	80	78	79

Overview of SPC

- General model for control charts
  - Let W be a sample statistic that measures some quality characteristics. μ_W and σ_W are mean and standard deviation of W. then the center line, upper control limit and the lower control limit become

$$UCL = \mu_W + k\sigma_W$$
$$CL = \mu_W$$
$$LCL = \mu_W - k\sigma_W$$

    Where k is the distance of the control limits from the center line, expressed in standard deviation units.

  Ex. For monitoring $\overline{X}$, $\Rightarrow \overline{\overline{X}} \pm 3\sigma_{\overline{X}}$

Overview of SPC

- General approach for SPC
  - Phase I: building and testing from off-line data
    - Data should be collected from “in-control” state
    - very iterative process
    - you will spend most of your time here
  - Phase II: using the control chart
    - on new, unseen data
    - implemented with computer hardware and software
      - usually for real-time display for operators
      - Line stop, or operator alarm depending on the errors
  - Control limits are reviewed periodically.

    → Continual process improvement.

Performance of SPC

- Correctness (or falseness)
  - Error probability
    - Type I error (α) : $\bar{x}$ typical of normal operation, but falls outside UCL or LCL limits. (ex. diagnose a normal person as a patient)
      - α = 0.0027 when using $\pm 3\sigma_{\bar{x}}$ limits
      - Synonyms: false alarm, false positive, overkill
    - Type II error(β): $\bar{x}$ is not in control but falls within UCL and LCL limits (ex. Diagnose a patient as being with no disease)
      - Synonyms: false negative, missing rate

※ Nothing makes a control chart more useless to operators than frequent false alarms.

Performance of SPC (Cont.)

- Promptness
  - average number of sequential samples we expect before seeing a point outside limits.
  - ARL = $1/\alpha$ (i.e., the probability that any point exceeds the control limits)

Ex. For an Xbar chart with 3-sigma limit, $\alpha = 0.0027$.

$$ARL = 1/a = 1/0.0027 \sim 370$$

That is, even if the process remains in control, an out-of-control signal will be generated every 370 points, on the average.

$\overline{X}$-R / $\overline{X}$-s charts _ Modification

- Basic Shewhart chart is not too sensitive to process shifts.

→ Western Electric Rules to enhance sensitivity.

  - 2 out of 3 consecutive values of X-bar on the same side of the centre line and more than 2 standard deviations from the centre line
  - 4 out of 5 consecutive values of X-bar on the same side of the centre line and more than 1 standard deviation from the centre line
  - 8 consecutive points are the same side of the centre line
  - 7 or more consecutive values is a consistently rising or falling pattern
  - a recurring cyclic pattern
  - abnormal clustering close to the centre line (signals a decrease in variation in the process)
  - clustering of values close to both control limits (suggests X-bar is following two distributions instead of one).

CUSUM chart

Cumulative Sum (CUSUM) Chart

- Shewhart chart takes a long time to detect shift in the mean, away from target, T.
- A Cumulative Sum Chart monitors $\mathbf{S_i}$, the **cumulative sum** ("cusum") of departures of sample mean values of measurements x, up to and including sample i , from their target value, T.

$$
\begin{aligned}
S_0 &= (x_0 - T) \\
S_1 &= (x_0 - T) + (x_1 - T) = S_0 + (x_1 - T) \\
S_2 &= (x_0 - T) + (x_1 - T) + (x_2 - T) = S_1 + (x_2 - T)
\end{aligned}
$$

$$S_i = \sum_{j=1}^{i} \left(\bar{X}_j - T \right)$$

- This definition of S_i includes the case of samples of size 1. Note that each value S_i includes all of the data collected up to that point.

※ In Shewhart charts, **only the current sample value is used as a basis for decision.**

CUSUM chart (Cont.)

- The following statistics are maintained separately:

  $SU_i = \max[0, SU_{i-1} + \overline{X}_i - (T + K)]$

  $SL_i = \max[0, SL_{i-1} - \overline{X}_i + (T - K)]$

If the value of either of these two statistics falls below zero, it is reset to 0.

$SU_0 = 0$ and $SL_0 = 0$

K is normally set to D/2, where D is the magnitude of the shift in population mean level away from the target value that is to be detected.

CUSUM chart (Cont.)

- As each new data value is acquired, both SU_i and SL_i are compared to a decision limit H
- Recommended values of K and H (by Montgomery and Runger)
  - If we define $H = h\sigma_{\bar{X}}$ and $K = k\sigma_{\bar{X}}$ where $\sigma_{\bar{X}}$ is the standard deviation of the samples, Using h = 4 or 5 and k = ½ will give good performance.
  - See related options in Minitab

Example

- Suppose you work at a car assembly plant in a department that assembles engines. In an operating engine, parts of the crankshaft move up and down a certain distance from an ideal baseline position. AtoBDist is the distance (in mm) from the actual (A) position of a point on the crankshaft to the baseline (B) position. To ensure production quality, take five measurements each working day, from September 28 through October 15, and then ten per day from the 18th through the 25th. (open cranksh.mtw in minitab)
- Construct Shewhart chart
  - Stat > control chart > variables chart for subgroups > Xbar-R
- Construct CUSUM chart
  - Stat > Control Charts > Time-weighted Charts > CUSUM.

EWMA (Exponentially Weighted Moving Average) chart

- **With a Shewhart chart,** decisions about departures of interest from a target value are made on the basis of **only the current value** of the measured performance characteristic. That is, **zero weight** is given to previous measured values. In contrast, the **cusum chart** assigns **equal weight to the current measured value and all previous values** of the performance characteristic.
- The **exponentially weighted moving average chart (EWMA chart)** provides a compromise between these two approaches.
  - heavier weights for recent observations
  - small weights old observations

EWMA chart (Cont.)

- The **EWMA chart** monitors **E_t**, the exponentially weighted moving average of all measured values of the performance characteristic, up to and including time t. The exponentially weighted moving average is defined as

$$E_t = \lambda x_t + (1-\lambda)E_{t-1}$$

which can also be expressed as

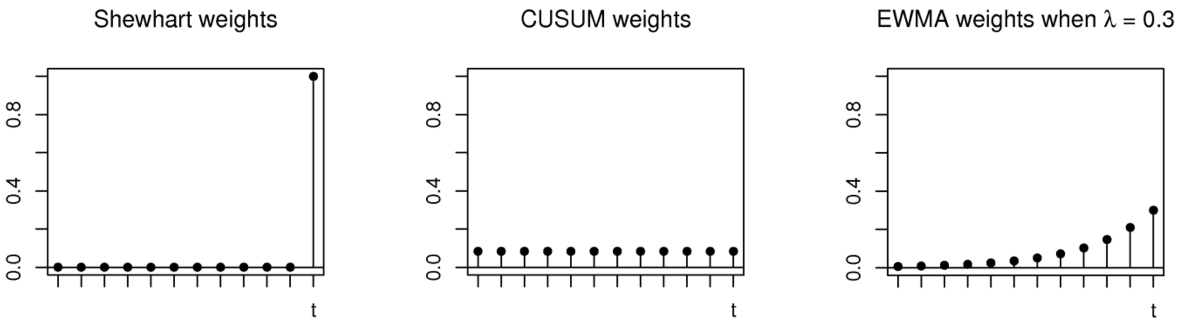
$$E_t = E_{t-1} + \lambda(x_t - E_{t-1})$$

By custom, **E_o** is usually set to the target value. This definition for **E_t** is equivalent to

$$E_t = \lambda x_t + \lambda(1-\lambda)x_{t-1} + \lambda(1-\lambda)^2 x_{t-2} + \cdots$$

EWMA chart (Cont.)

- Comparison in terms of weight



- Control limits for an **EWMA** chart are

$$\text{target value } \pm 3s\sqrt{\frac{\lambda}{2-\lambda}}$$

where s is an estimate of the standard deviation of the charted characteristic (e.g. $s/\sqrt{n}$ for sample means).

Other control charts

- Control charts for count data
  - np CHART: used for monitoring the occurrence of defective product or process behaviour when the sample size (number of items inspected at each sampling interval) is constant, and each item inspected is declared either acceptable or unacceptable.
  - p Chart: a tool for monitoring the occurrence of defective product or process behaviour when the sample size (number of items inspected at each sampling interval) may vary (because of fluctuating production levels or other reasons), and each item inspected is declared either acceptable or unacceptable.
  - Stat > control charts > attributes charts

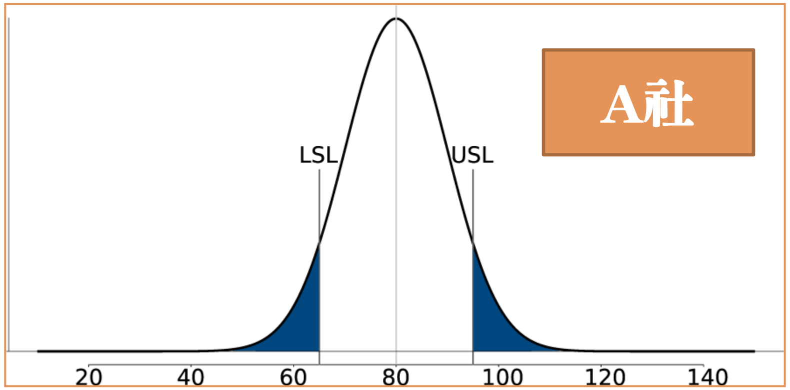
Examples

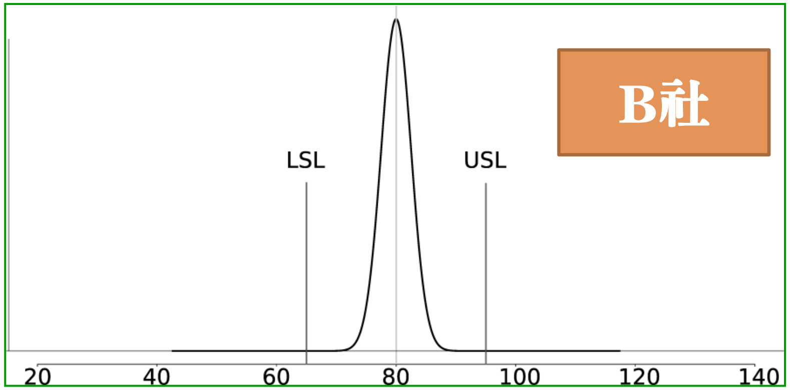
- Construct P chart using the following data

date	Sample size	defective	date	Sample size	defective
9/5	12	2	-	46	7
9/6	17	3	-	43	5
9/7	25	4	-	43	0
9/8	30	4	-	43	7
9/9	44	3	-	40	5
9/10	24	4	-	50	3
-	18	2	-	22	3
-	13	1	-	24	5
-	26	4	-	36	6
-	36	6	-	45	8
-	40	2	-	33	3

Process capability

- Suppose you need to choose a raw material supplier among company A and company B. You received a database containing quality of a raw material from each company and plotted them with spec. limits (LSL and USL) that you product requests. Which one would you choose?





- How to quantify this capability?

Process capability (Cont.)

- C_p (or PCR, process capability ratio)

$$C_P = \frac{USL - LSL}{6\sigma}$$

- C_{pk} (or PCR_k) for one-sided limit

$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$

μ and σ: calculated from data

- In general, C_p (or C_{pk}) = 1.33 is minimum requirement

※Stat > quality tools > capability analysis

※Note: Cpk and Cp are only useful for a process which is stable

Monitoring in industrial practice

- Widely used in industry, at all levels
- Management: monitor plants, geographic region, countries (e.g. hourly sales by region)
  - Dashboards, ERP, BI, KPI
- Challenges for you:
  - Getting the data out
  - Real-time use of the data (value of data decays exponentially)
  - Training is time consuming

General workflow

1. Identify variable(s) to monitor.
2. Retrieve historical data (computer systems, or lab data, or paper records)
3. Import data and just plot it.

   Any time trends, outliers, spikes, missing data gaps?
4. Locate regions of stable, common-cause operation.

   Remove spikes and outliers
5. Estimate limits by eye
6. Calculate control limits (UCL, LCL), using formula
7. Test your chart on new, unused data.

   Testing data: should contain both common and special cause operation
8. How does your chart work?

   Quantify the type I and II error. → Adjust the limits; repeat

General workflow

1. Run chart on your desktop computer for a couple of days

    Confirm unusual events with operators; would they have reacted to it? False alarm?

    Refine your limits

2. Not an expert system - will not diagnose problems:

    use your head; look at patterns; knowledge of other process events

3. Demonstrate to your colleagues and manager

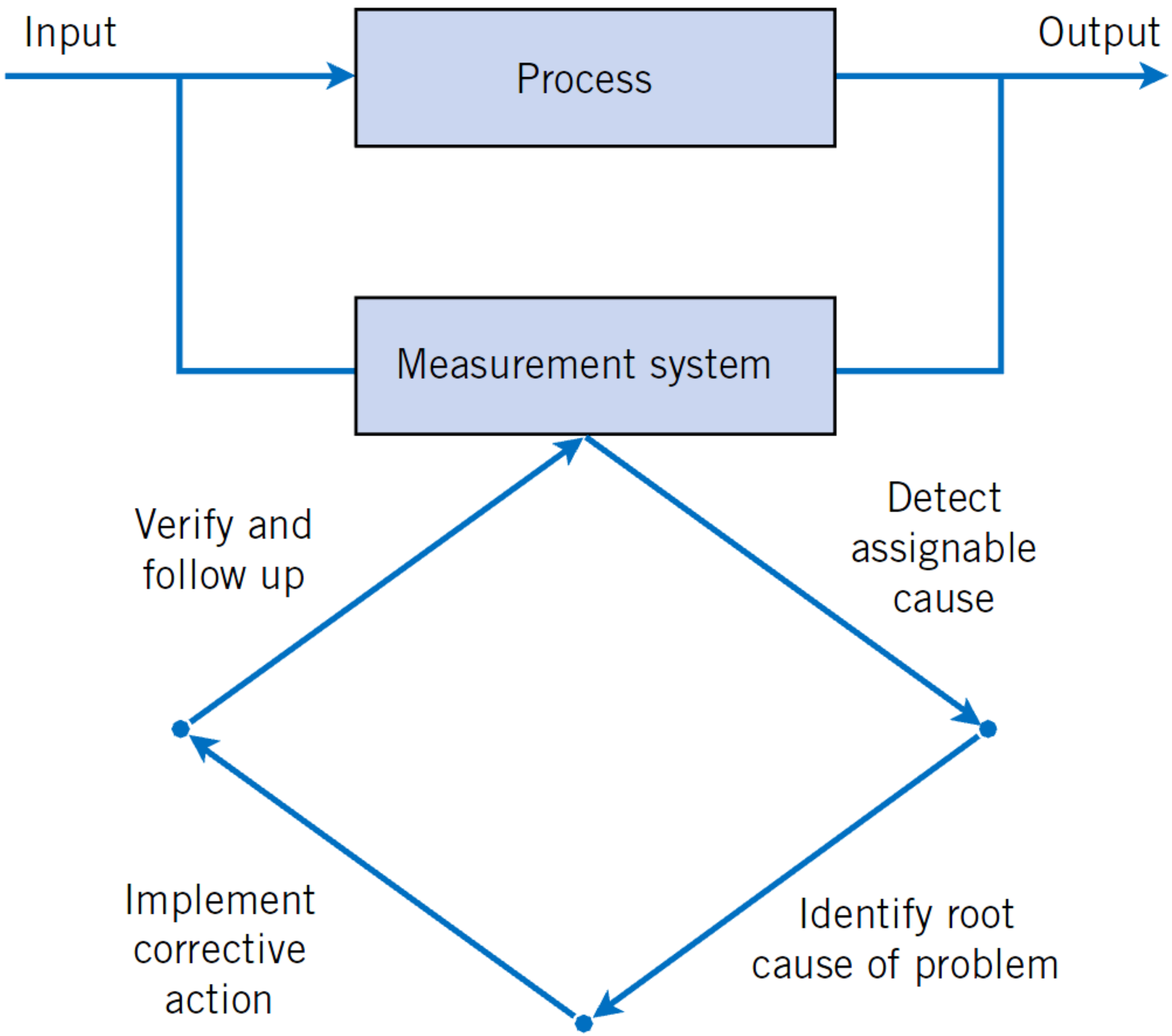
    But go with dollar values

4. Installation and operator training will take time

5. Listen to your operators

    make plots interactive - click on unusual point, it drills-down to give more context

Process improvement using the control charts



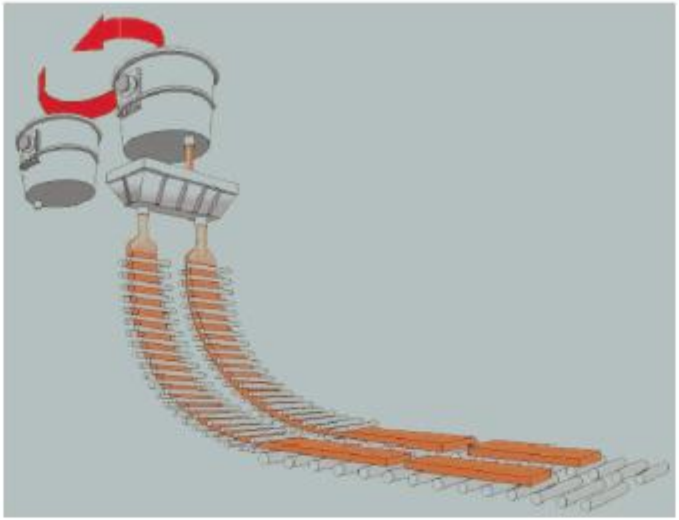
Industrial case study: Dofasco

- ArcelorMittal in Hamilton (formerly called Dofasco) has used multivariate process monitoring tools since 1990's
- Over 100 applications used daily
- Most well known is their casting monitoring application, Caster SOS (Stable Operation Supervisor)
- It is a multivariate monitoring system.

We will briefly review multivariate statistics later!

Dofasco case study: slabs of steel

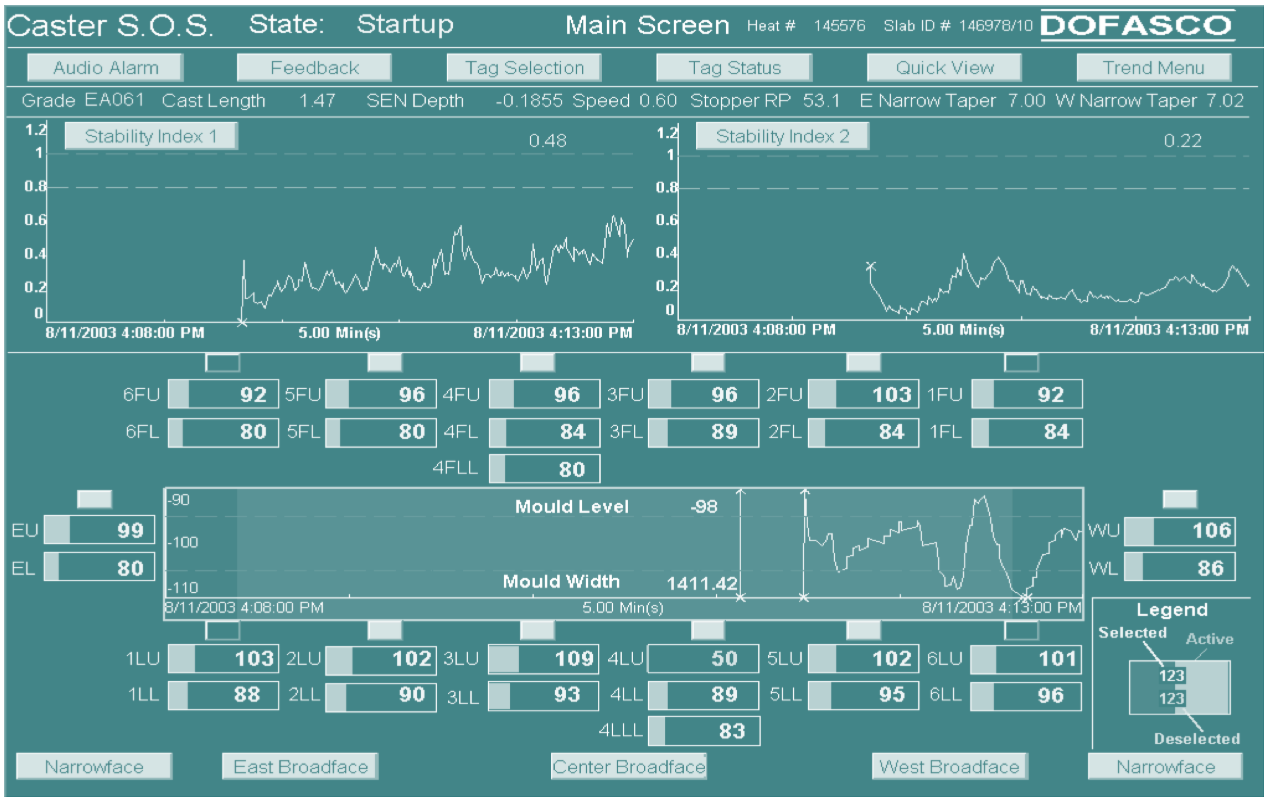
Dofasco case study: casting



Dofasco case study: breakout

Dofasco case study: monitoring for breakouts

- Screenshot of caster SOS



Warning limits and the action limits.

Lots of other operator-relevant information

Dofasco case study: economics of monitoring

- Implemented system in 1997; multiple upgrades since then
- Economic savings: more than $ 1 million/year
  - each breakout costs around $200,000 to $500,000
  - process shutdowns and/or equipment damage