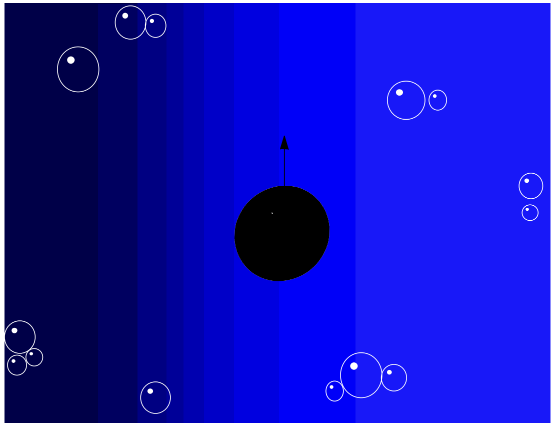
4. Flow of Particulates

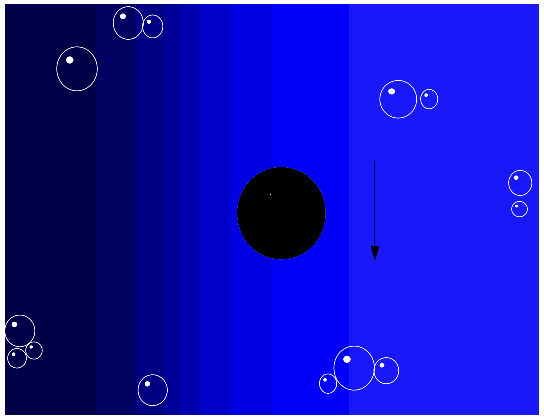
4.1 Introduction

Physical phenomena do not depend on the frame of reference of the observer and on the coordinate system used.



particle moves upward

≡



fluid flows downward

4.2 Flow past a Sphere

Drag coefficient:

Consider the steady motion of a sphere in an infinite expanse of an incompressible Newtonian fluid (or the uniform motion of an infinite fluid past a stationary sphere)

F_D = force, $ML\Theta^{-2}$

V_p = velocity, $L\Theta^{-1}$

D_p = sphere diameter, L

ρ = fluid density, ML^{-3}

η = fluid viscosity, $ML^{-1}\Theta^{-1}$

5 variables and 3 dimensions ⇒ 2 indep. dimensionless groups

$$\mathrm{Re} = \frac{D_p V_p \rho}{\eta}$$: Reynolds number

$$C_D = \frac{8}{\pi} \frac{F_D}{\rho V_p^2 D_p^2}$$: drag coefficient

$$\boxed{C_D = C_D(\mathrm{Re})}$$

Fig. 4-1. $C_D(\mathrm{Re})$ for flow past a sphere.

(from H. Schlichting, *Boundary Layer Theory*)

$\text{Re} < 1$: $C_D = \dfrac{24}{\text{Re}}$, inertialess Stokes flow

$\Rightarrow$ $F_D = 3\pi\eta V_p D_p$: Stokes' law

$1 < \text{Re} < 10^3$: $C_D \approx 18\,\text{Re}^{-0.6}$

$10^3 < \text{Re} < 2\text{x}10^5$: $C_D \approx 0.44$, Newton regime

$\Rightarrow$ $F_D \propto V_p^{\;2}$: postulated by Newton

* The sharp drop in C_D at Re of approximately 2×10^5 is discussed in Sec. 15.8 (boundary layer separation).

Settling velocity: (terminal velocity)

Fig. 4-2. Forces acting on a sphere falling in a viscous fluid.

gravity : $F_G = -\rho_p \frac{\pi D_p^3 g}{6}$

buoyancy : $F_B = +\rho \frac{\pi D_p^3 g}{6}$

drag : $F_D = \frac{\pi}{8} \rho V_p^2 D_p^2 C_D$

The force balance gives

$$0 = -\rho_p \frac{\pi D_p^3 g}{6} + \rho \frac{\pi D_p^3 g}{6} + \frac{\pi}{8} \rho V_p^2 D_p^2 C_D$$

$Re < 1$: $V_p = \dfrac{g D_p^2 (\rho_p - \rho)}{18\eta}$, Stokes settling velocity

$1 < Re < 10^3$: $V_p \approx [\frac{2g}{27} \cdot (\frac{\rho_p}{\rho} - 1)]^{5/7} D_p^{8/7} (\frac{\rho}{\eta})^{3/7}$

$10^3 < Re < 2x10^5$: $V_p \approx [3 D_p g (\frac{\rho_p}{\rho} - 1)]^{1/2}$

Ex. 4.1 Estimate the maximum spherical particle that will fall in Stokes flow in a given liquid.

Stokes flow :

$$\mathrm{Re} < 1, \text{ or } \frac{D_p V_p \rho}{\eta} < 1, \text{ or } \frac{D_p^3 g(\rho_p - \rho)\rho}{18\eta^2} < 1$$

$$\therefore\ D_p < \left[\frac{18\eta^2}{g\rho(\rho_p - \rho)}\right]^{1/3}$$

In the case of water droplet in air,

$$\rho \approx 1.3\,\mathrm{kg/m^3},\ \eta \approx 2\mathrm{x}10^{-5}\,\mathrm{Pa \cdot s},\ \rho_p \approx 10^3\,\mathrm{kg/m^3}$$

$$\therefore\ D_p < \left[\frac{(18)(2\ \mathrm{x}10^{-5}\,\mathrm{Pa \cdot s})^2}{(9.8\ \mathrm{m/s^2})(1.3\,\mathrm{kg/m^3})(10^3\,\mathrm{kg/m^3})}\right]^{1/3} \approx 10^{-4}\,\mathrm{m}$$

and $V_p \approx 0.3\,\mathrm{m/s}$

Falling sphere viscometer:

In the Stokes regime, $\eta = \dfrac{gD_p^2(\rho_p - \rho)t_p}{18L}$

Ex. 4.2 A steel ball, with diameter D_p = 3mm and density ρ_p = 7.6 x 10^3kg/m^3 is dropped in a liquid with density ρ = 1.2 x 10^3 kg/m^3. The average time for the ball to drop a diatance of 0.5m is 10.0s. What is the viscosity of the liquid ?

$$\eta = \frac{(9.8\ m/s^2)(3x10^{-3}m)^2(7.6x10^3 - 1.2x10^3 kg/m^3)(10.0s)}{(18)(0.5\ m)}$$

$$= 0.6\ Pa\cdot s$$

Check if Re < 1.

Separation of particulates:

* Gravity settling chamber (gravitational force)
* Electrostatic precipitator (electrostatic force)
* Cyclone separator (centrifugal force)

Fig. 4-3. Schematic of a gravity settling chamber.

Fig. 4-4. Particle trajectories in a gravity settling chamber.

Fluid velocity : $V_f = \frac{Q}{WH}$

The time required for horizontal fluid motion from left to right :

$$t_H = \frac{L}{V_f} = \frac{WHL}{Q}$$

The time required for vertical particle motion from top to bottom :

$$t_V = \frac{H}{V_p}$$

To start at A and stop at B, $t_H = t_V$

Thus, we have $V_p = \dfrac{Q}{WL}$

Assuming that the particle Reynolds number is sufficiently small for Stokes' law to apply,

$$V_p = \frac{g D_p^2 (\rho_p - \rho)}{18\eta} = \frac{Q}{WL}$$

Therefore, the smallest particle diameter removed by the gravity settling chamber is

$$D_p = \left[\frac{18\eta Q}{g W L (\rho_p - \rho)} \right]^{1/2}$$

Sizing a distillation column:

To avoid liquid entrainment in the upward gas stream,

the settling velocity of liquid droplets, V_p

$\geq$ the upward gas velocity, V_{gas}

$D_p \approx 10^{-4}\text{m}$,

Newton regime,

$$V_p \approx [3D_p g(\frac{\rho_p}{\rho} - 1)]^{1/2}$$

$$\therefore\ V_{gas} \leq 0.055(\frac{\rho}{\rho_g} - 1)^{1/2}$$

Wall effects:

Suppose that the sphere is falling along the axis of a cylinder of diameter D_c, as shown in Fig. 4-5.

Fig. 4-5. Schematic of a sphere falling along the axis of a cylinder.

Dimensional analysis gives $C_D = C_D(\mathrm{Re}, \frac{D_p}{D_c})$

In the Stokes regime, $C_D = \frac{24}{\mathrm{Re}} \Phi(\frac{D_p}{D_c})$

Fig. 4-6. $\Phi(D_p/D_c)$ linear asymtote : $\Phi = 1 + 2.10\frac{D_p}{D_c}$

4.3 Other Submerged Objects

The general definition of the drag coefficient for an object of any shape is $C_D = \dfrac{F_D}{\frac{1}{2}\rho V_p{}^2 A_p}$ where

A_p : the projection of the solid object on a plane normal to the direction of flow

For a sphere, $A_p = \dfrac{\pi}{4} D_p^2$.

For a long cylinder of diameter D_p and length L_p, $A_p = D_p L_p$

$$\therefore\ C_D = \frac{F_D/L_p}{\frac{1}{2}\rho V_p{}^2 D_p}$$ and

$$\mathrm{Re} < 1 : \quad C_D = \frac{8\pi}{\mathrm{Re}} \frac{1}{\ln(8/\mathrm{Re}) - 0.077} \qquad : \text{no inertialess region}$$

Fig. 4-8. : C_D vs. Re for spheres, disks, and cylinders.

4.4 Bed of Particles

Porous media:

Consider a porous bed which is made up of unconsolidated spherical particles.

particle diameter D_p, and

void fraction ε = (empty volume available for fluid to pass)/(total volume of the bed) : depends on the manner of packing (hexagonal, square, etc.)

* Mean diameter of a mixture : $D_p = \left(\sum \frac{X_n}{D_{pn}} \right)^{-1}$

* Nonspherical particles : $D_p = 6 \times \frac{\text{volume of particle}}{\text{area of particle}}$

Friction factor-Reynolds number relation:

Consider the flow problem given in Fig. 4-9.

$|\Delta p|$ across a bed of cross-sectional area A and depth L when the flow rate is Q ?

The superficial velocity $\upsilon_{\infty} = \dfrac{Q}{A}$

: the velocity in the absence of particles

Dimensional analysis on $|\Delta p|/L, \upsilon_{\infty}, D_p, \rho, \eta, \varepsilon$:

$$\frac{D_p}{\rho \upsilon_{\infty}^2} \frac{|\Delta p|}{L} = \text{function of } \left(\frac{D_p \upsilon_{\infty} \rho}{\eta}, \varepsilon \right)$$

Consider an idealized model of a packed bed shown in Fig. 4-10.

υ_{eff} : the velocity in each cylinder

D_{eff} : diameter of cylinder

Then, we have $\dfrac{D_{eff}}{\rho \upsilon_{eff}^2} \dfrac{|\Delta p|}{L}$ = function of ($\dfrac{D_{eff}\upsilon_{eff}\rho}{\eta}$)

υ_{eff} = ?

$$Q = \varepsilon A \upsilon_{eff} \qquad \therefore \upsilon_{eff} = \frac{1}{\varepsilon}\frac{Q}{A} = \frac{\upsilon_{\infty}}{\varepsilon}$$

D_{eff} = ?

$$D_{eff} = \frac{4 \text{ x volume of fluid}}{\text{surface area wetted by fluid}}$$ (ref. D_H in Sec. 3.6)

volume of fluid = void volume

$$\frac{\varepsilon}{1-\varepsilon} = \frac{\text{void volume}}{\text{solids volume}}$$

$$\therefore \text{ void volume} = \frac{\varepsilon}{1-\varepsilon}\ (\text{solids volume})$$

$$= \frac{\varepsilon}{1-\varepsilon} N_p \frac{\pi D_p^3}{6}$$

$$\text{surface area} = N_p \pi D_p^2$$

$$\therefore \quad D_{\epsilon f} = \frac{4\frac{\varepsilon}{1-\varepsilon} N_p \frac{\pi D_p^3}{6}}{N_p \pi D_p^2} = \frac{2}{3} D_p \frac{\varepsilon}{1-\varepsilon}$$

$$\text{Therefore,} \quad \frac{\frac{2}{3} D_p \frac{\varepsilon^3}{1-\varepsilon}}{\rho \upsilon_\infty^2} \frac{|\Delta p|}{L} = \text{function of } \left(\frac{\frac{2}{3} \frac{D_p}{1-\varepsilon} \upsilon_\infty \rho}{\eta} \right)$$

Let's define the packed bed friction factor f_p :

$$f_p = D_p \frac{\varepsilon^3}{\rho v_\infty^2 (1-\varepsilon)} \frac{|\Delta p|}{L}$$

and the packed bed Reynolds number Re_p :

$$Re_p = \frac{D_p v_\infty \rho}{(1-\varepsilon)\eta}$$

Then, we have $f_p = f_p \ (Re_p)$

Fig. 4-11. : $f_p = f_p \ (Re_p)$ for flow through granular packed bed

Ergun eq'n : $f_p = \frac{150}{Re_p} + 1.75$ $(10 \leq Re_p \leq 1000)$

Example 4.6 A sand pack is used to filter impurities from molten polyester downstream of the extruder and prior to spinning into filaments. The polymer at 280℃ is approximately a Newtonian liquid with $\eta = 600\,\mathrm{Pa \cdot s}$ and $\rho = 1300\,\mathrm{kg/m^3}$. The sand pack is 38mm in diameter and 16mm in depth, and the mass flow rate is 5 x 10^{-4}kg/s. Estimate the pressure drop through the pack if the particles have a mean diameter of 0.7mm and $\varepsilon = 0.38$.

$$\upsilon_\infty = \frac{Q}{A} = \frac{4(5\mathrm{x}10^{-4}\mathrm{kg/s})/(1300\mathrm{kg/m^3})}{\pi(38\mathrm{x}10^{-3}\mathrm{m})^2} = 3.4\mathrm{x}10^{-4}\mathrm{m/s}$$

$$\mathrm{Re}_p << 10 \Rightarrow f_p = \frac{150}{\mathrm{Re}_p}$$

$$\therefore\ |\Delta p| = \frac{150 L \upsilon_\infty \eta (1-\varepsilon)^2}{D_p^2 \varepsilon^3} = 7\mathrm{x}10^6\mathrm{Pa}$$

Fluidized beds:

$$\mathrm{Re}p \ll 10 \Rightarrow \quad \therefore \quad \frac{|\Delta p|}{L} = \frac{150 v_{\infty} \eta (1-\varepsilon)^2}{D_p^2 \varepsilon^3}$$

Force balance on the bed of particles:

Net upward force = $|\Delta p| A$

Net downward gravitational and buoyant force

$= (1-\varepsilon)(\rho_p - \rho) A L g$

$$|\Delta p| A = (1-\varepsilon)(\rho_p - \rho) A L g$$

: a state of weightlessness

: no further increase in $|\Delta p|$ as v_{∞} is increased.

: the bed will tend to expand, or become fluidized.

The minimum superficial velocity, υ_f is

$$\upsilon_f = \frac{(\rho_p - \rho) g D_p^2 \varepsilon^3}{150\eta(1-\varepsilon)}$$: the point of incipient fluidization

ε at incipient fluidization depends on the material and the particle size.

If data are not available, use the approximation of

$$\varepsilon^3/(1-\varepsilon) = 0.091$$

The maxumum limiting velocity, $\upsilon_{\max}$ is the settling velocity.

$$\upsilon_{\max} = \frac{(\rho_p - \rho) g D_p^2}{18\eta}$$

Example 4.7 Pulverized coal is to be burned at atmospheric pressure in a fluidized bed. The density of the coal is approximately 1000 kg/m^3. The mean particle diameter is 0.074mm and the gas, mostly air, has a viscosity $\eta = 10^{-4} Pa \cdot s$. Estimate the minimum fluidization velocity.

Assume that $\varepsilon^3/(1-\varepsilon) = 0.091$, and the gas density is negligible.

$$\upsilon_f = \frac{(1000)(9.8)(7.4\times10^{-5})^2(0.091)}{(150)(10^{-4})} = 3.2\times10^{-4} m/s$$

The entrainment velocity, υ_{max}, is

$$\upsilon_{max} = \frac{(1000)(9.8)(7.4\times10^{-5})^2}{(18)(10^{-4})} = 3.0\times10^{-2} m/s$$