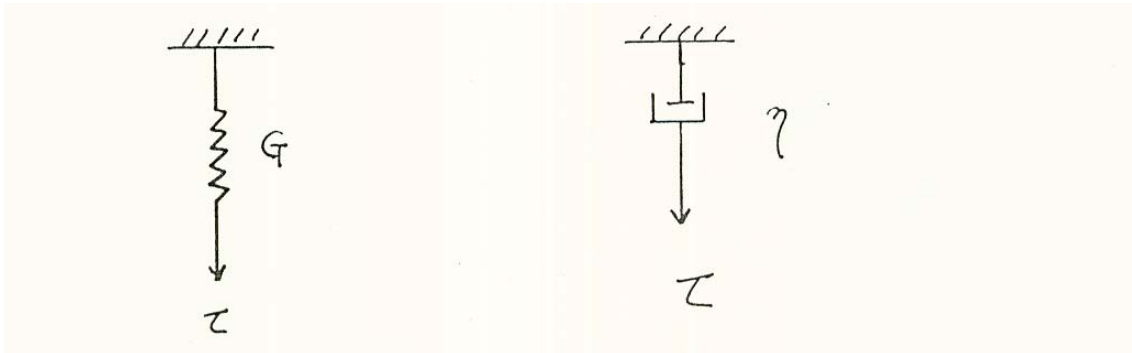


Chapter 18

- Linear viscoelasticity

- Two linear model
 - linear elastic
 - linear viscous



Linear elastic model
or Hookean solid

$$\tau = G \gamma$$

G = shear modulus

Linear viscous model
or Newtonian fluid

$$\tau = \eta \dot{\gamma}$$

η = viscosity

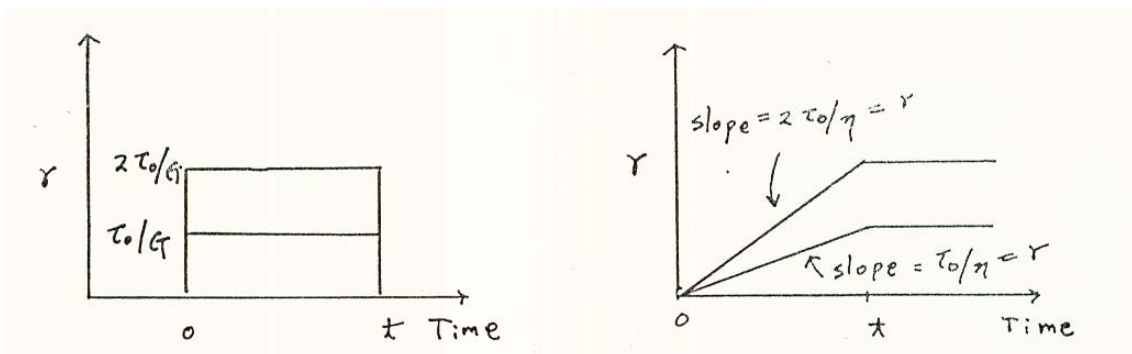


Fig. Response of spring

- the overall modulus is a function of time only, not the magnitude of stress of strain

Fig. Response of dashpot

- Doubling the stress doubles the slope of the strain-time line

$$G = \frac{\tau}{\gamma} = G \text{ (t only)}$$

for linear response

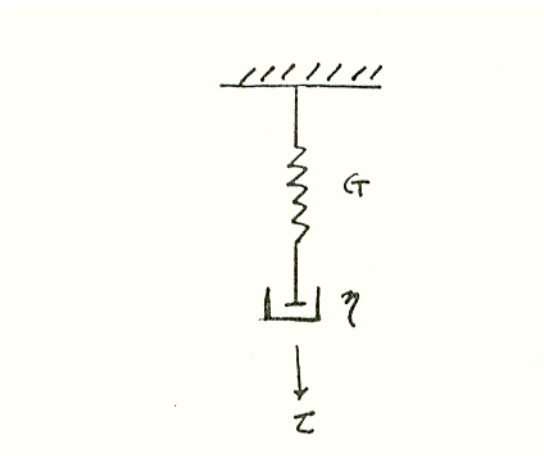
$$\dot{\gamma} = \tau / \eta$$

$$\gamma = (\tau / \eta)t$$

- Mechanical Models for Linear Viscoelastic Response.

- The Maxwell Element

- a simple series combination of a linear viscous element(dashpot) and a linear elastic element(spring).



- The spring and dashpot support the same stress

$$\tau = \tau_{spring} = \tau_{dashpot}$$

- The overall strain of the element

$$\gamma = \gamma_{spring} + \gamma_{dashpot}$$

differentiation with time, t

$$\dot{\gamma} = \dot{\gamma}_{spring} + \dot{\gamma}_{dashpot}$$

$$\dot{\gamma} = \dot{\tau} / G + \tau / \eta$$

$$\tau = \eta \dot{\gamma} - \left(\frac{\eta}{G}\right) \dot{\tau} = \eta \dot{\gamma} - \lambda \dot{\tau}$$

(where $\lambda = \frac{\eta}{G}$: relaxation time)

- Creep Testing

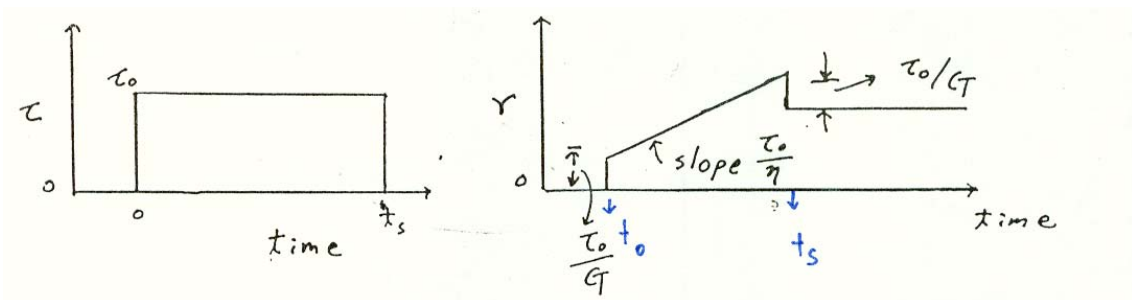


Fig. Creep response of Maxwell element

- creep test - a constant stress is instantaneously applied to the material,

and the resulting strain is followed as a function of time.

- Creep Recovery - deformation after removal of the stress
- τ_0/G - instantaneous stretching of the spring to an equilibrium value with the sudden application of stress (τ_0)

· Elastic Recovery - when the stress is released, the spring immediately contracts by an amount equal to its original extension.

• Stress Relaxation :

Test - suddenly applying a strain to the sample and following the stress as a function of time as the strain is held constant.

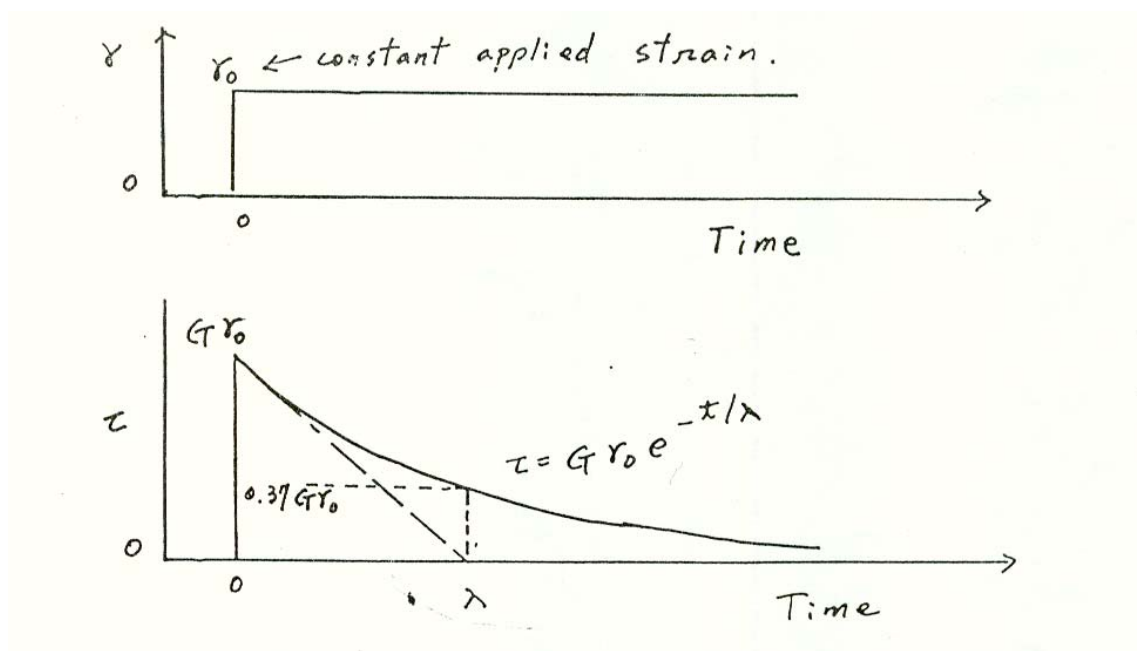
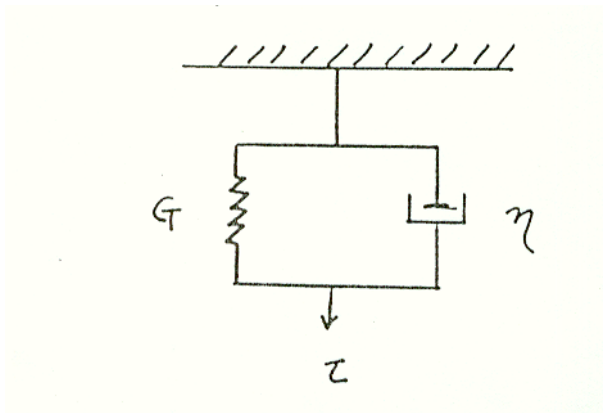


Fig. stress relaxation of Maxwell element

· The extended spring begins to contract, but the contraction is resisted by the dashpot

- λ (relaxation time) time constant for the exponential decay
 time required for the stress to decay to a factor of $\frac{1}{e}$ or 37% of its initial value

- The Voigt -Kelvin Element



- strain in each element is same:

$$\gamma = \gamma_{\text{spring}} + \gamma_{\text{dashpot}}$$

- the stress is the sum of the stresses :

$$\tau = \tau_{\text{spring}} + \tau_{\text{dashpot}}$$

$$\tau = G\gamma + \eta \dot{\gamma}$$

Fig. Voigt -Kelvin element

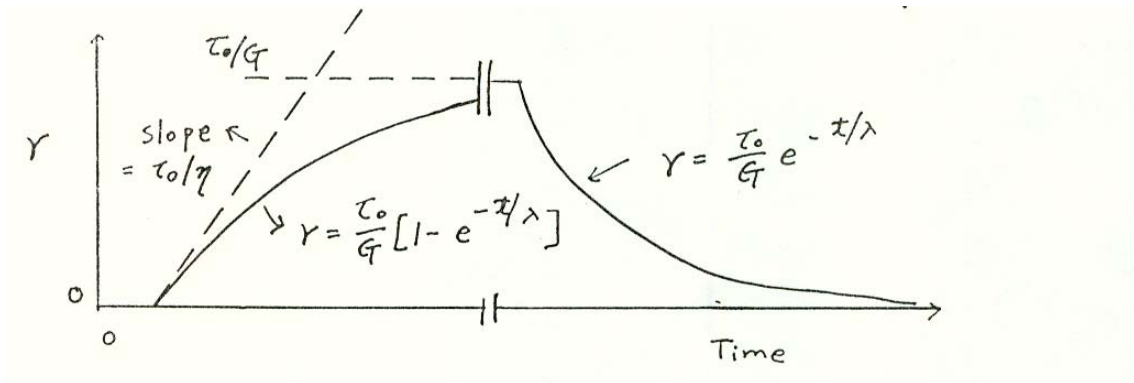
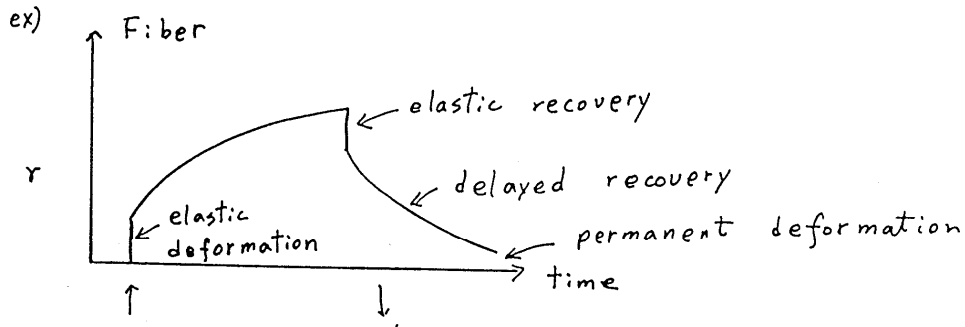
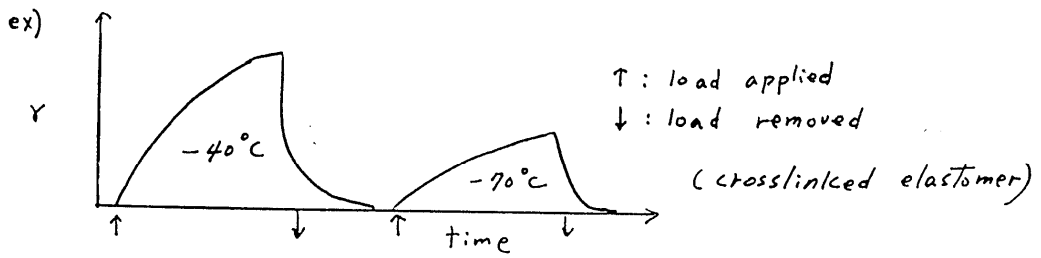
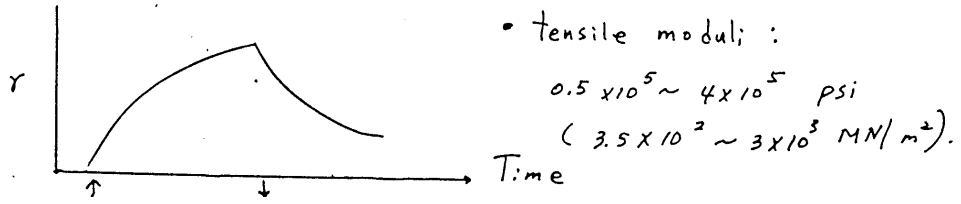


Fig. Creep response of a Voigt -Kelvin element

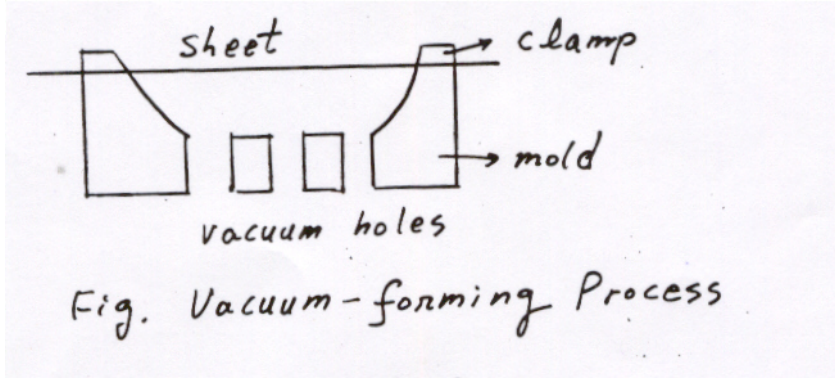
- initial slope of the strain vs time curve is τ_0/η
- as the element is extended, the spring provides an increasingly greater resistance to further extension, and so the rate of creep decrease.
- Eventually, the system comes to equilibrium with the spring alone supporting the stress (rate of strain $\rightarrow 0$, resistance of the dashpot $\rightarrow 0$)
- The equilibrium strain = τ_0/G
- Voigt -Kelvin model - a fair qualitative picture of the creep response of some crosslinked polymers.



ex) Amorphous plastic



● Sheet forming: heating the thermoplastic sheet above its softening point and forcing it to conform to a mold.



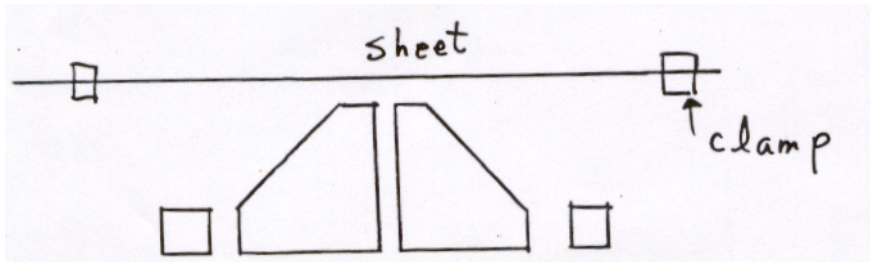


Fig. drape -forming (e.g) drinking cups, meat trays, cigarette packs

- Stamping: the sheet is heated in an infrared oven and then stamped in forming equipment, with a dwell time of about 8sec as the material cools in the mold. cycle time: 15~20 sec.

(e.g) battery tray, automobile bucket seat,etc.

- Solution casting: by dissolving the polymer in an appropriate solvent, spreading the viscous solution onto a polished surface, and evaporating the knife.

(e.g) membranes,etc

- reinforced thermoset molding: the composites have high strength -to -weight ratios and can be fabricated into a wide variety of complex shapes. Mostly with polyesters and epoxies.

- hand layup process -first sprayed of the liquid resin to provide a smooth surface finish.

Then followed by successive layer if reinforced fiber, either in the form of woven cloth or random matting, impregnated with the liquid resin, which is then cured (crosslinked) to give the finished product.

- filament winding -continuous filaments of reinforcing fiber are impregnated with liquid resin and then wound on a rotating mandrel.

(e,g) tanks and pipe for the chemical process industry, gun barrel -provide heat and abrasion resistance.

- pultrusion -to produce continuous lengths of objects with a constant cross section.

(e,g) structural beams

- Fiber spinning

- use spinnerette, a plate in which a multiplicity of holes have been formed to produce the individual fibers, which are then twisted together to form a thread for subsequent weaving operations.

- Fiber spinning :

- melt spinning : basically an extrusion process, the fibers are usually solidified by a crosscurrent blast of air.

The drawing step stretches the fibers, orienting the molecules in the direction of stretch and inducing high degrees of crystallinity.

(e.g) nylons, PET, etc.

- dry spinning : a solution of the polymer is forced through the spinnerette. There is considerable shrinkage as the solvent evaporates

(e.g) acrylic fiber, mainly polyacrylonitrile

- wet spinning : similar to dry spinning in that a polymer solution is forced through the spinnerette. The solution strands pass directly into a liquid bath.

(e.g) Rayon (cellulose) is common.

(ex) ultra-high molecular weight polyethylene with wax or paraffin oil : to yield high strength fiber.

- Compounding : used in combination with other ingredients.

(used in high molecular weight thermoplastics, because of the high viscosities.)

- Banbury mixer - the material is subjected to high shear rates and large power inputs in a closed, heated chamber containing rotating, intermeshing blades.

◦ Compounding :

- two-roll mill : generates high shear rates in a narrow nip between two heated rolls that counterrotate with slightly different velocities.

() 1 ft in diameter by 3ft in length.