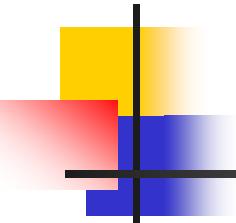


# Chapter 6. Multiphase Systems

고려대학교  
화공생명공학과

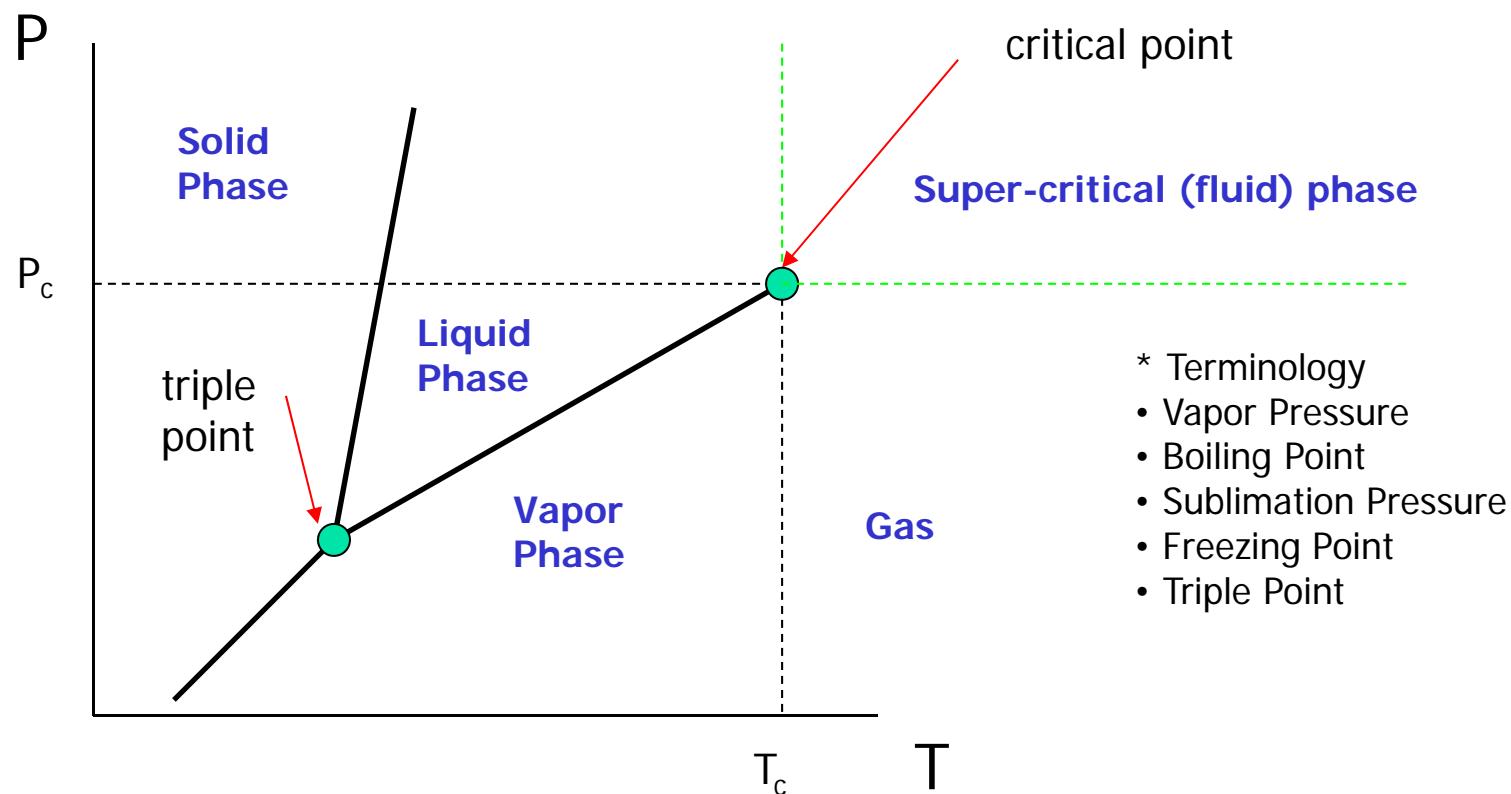


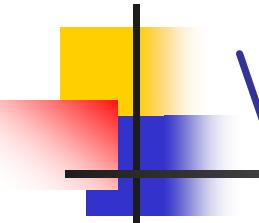
# Introduction

- Multiphase systems – Mainly involved in separation process
  - Distillation (증류) : vapor–liquid
    - Driving force of separation : Vapor pressure
  - Crystallization (결정화) : liquid – solid
    - Driving force of separation : solubility
  - Extraction (추출) : liquid – liquid
    - Driving force of separation : distribution coeff.

## 6.1 Single-Component Phase Equilibrium

### ■ Phase Diagram

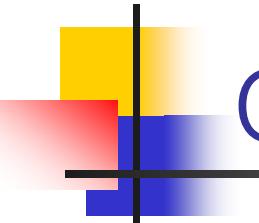




# Vapor Pressure

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- Source of Vapor Pressure Data
  - Experimental Data from Literature
    - Perry's Handbook
    - Journals ( J.Chem.Eng.Data, Fluid Phase Equilibria, ...)
  - Equations and Coefficients : Antoine,...
    - Perry's Handbook
    - Data Books, Databases , ...
  - From Cox chart (Fig. 6.1–4)
  - Estimation from Claisius – Clapeyron Equation



# Clausius–Clapeyron Equation

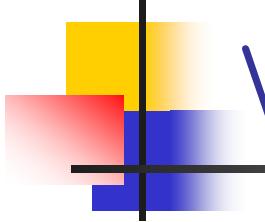
- Estimation of Vapor Pressure

$$\frac{dP}{dT} = \frac{\Delta H_v}{T\hat{\Delta V}} \longrightarrow \begin{array}{l} \text{Enthalpy of Vaporization} \\ \text{Volume change of Vaporization } (\text{V(gas)} - \text{V(liquid)}) \end{array}$$

$$\frac{d \ln P^*}{d(1/T)} = -\frac{\Delta H_v}{R}$$

↓  
**Integration**

$$\ln P^* = -\frac{\Delta H_v}{RT} + B \longrightarrow \text{This equation can be used as fitting equation for Vapor pressure data.}$$



# Vapor Pressure Equations

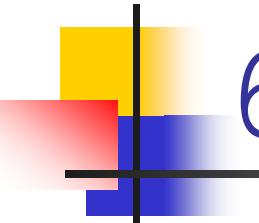
- Antoine equation (Table 6.1-1)

$$\log_{10} = A - \frac{B}{T + C}$$

- Wagner equation
  - “Properties of Gases and Liquids”

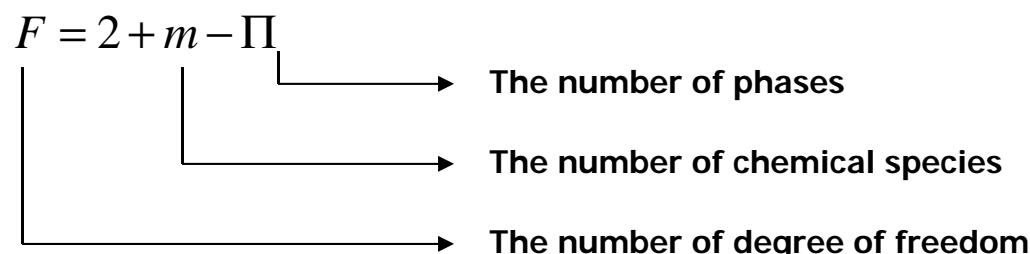
$$\ln P_{vpr} = (A\tau + B\tau^{1.5} + C\tau^3 + D\tau^6)/T_r$$

$$\tau = 1 - T_r$$

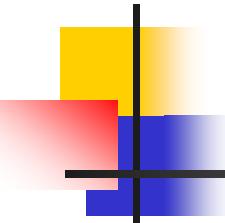


## 6.2 Gibbs Phase Rule

- Types of Process Variables
  - Extensive Variables – depend on the size of the system (N, V,...)
  - Intensive Variables – do not depend on the size of the system (T,P,...)
- Gibbs Phase Rule
  - Degree of freedom for intensive variables

$$F = 2 + m - \Pi$$


The number of phases  
The number of chemical species  
The number of degree of freedom



# Gibbs Phase Rule – Examples

$$F = 2 + m - \Pi$$

- Pure Water
  - $F = 2 + 1 - 1 = 2$
- Mixture of Ice and Water
  - $F = 2 + 1 - 2 = 1$
- VLE of acetone + nitrogen
  - $F = 2 + 2 - 2 = 2$

(example)  
T and P

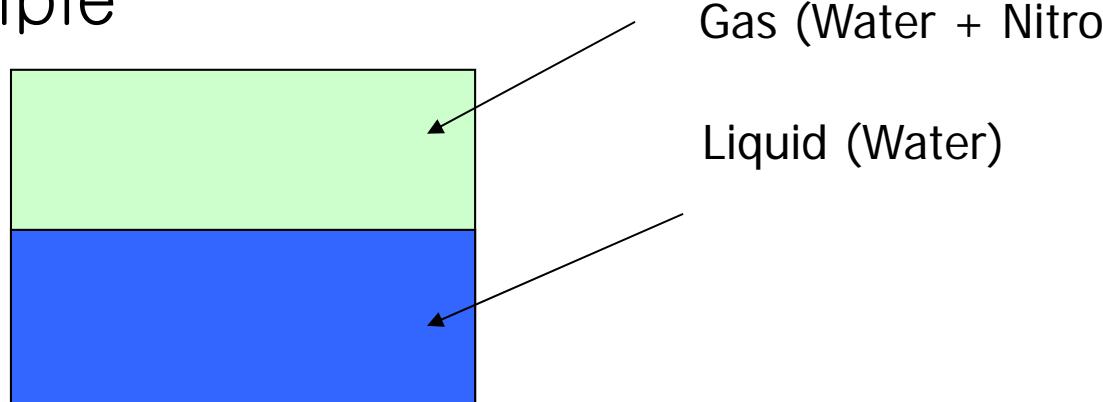
(example)  
T or P

(example)  
T and x  
P and x  
T and P  
:

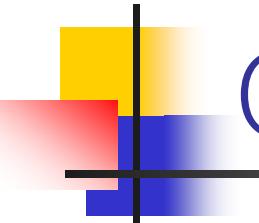
Other intensive variables can be calculated using thermodynamic relations

## 6.3 Gas–Liquid Systems

- Processes involving gas–liquid systems
  - Evaporation, drying, humidification
  - Condensation, dehumidification
- Example



The gas in GLE is called “**noncondensableThe gas phase is “**saturated****

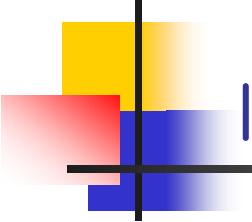


# GLE–Calculations

- For 2– component GLE
  - Saturation condition, single condensable species

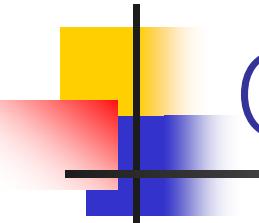
$$p_v = y_v P = p_v^*(T)$$

- Gibbs Phase Rule  $F = 2 + m - \Pi$ 
  - $F = 2 + 2 - 2 = 2$
  - Intensive Variables : y, T, P



## Important characteristics of GLE systems

- GLE → gas must be saturated with liquid
- The partial pressure cannot exceed the vapor pressure of the liquid
  - $p_v \leq p_v^*(T) \longrightarrow$  if  $p_v > p_v^*(T)$  then condensation starts.
- Superheated vapor (과열증기)
  - $p_v = y_v P < p_v^*(T)$
- Dew Point (이슬점)
  - $p_v = y_v P = p_v^*(T_{dp})$
- Degree of Superheat (과열도)
  - $= (T - T_{dp})$



# Quantities for GLE systems

- Special case of air+water systems → humidity
- Terminology
  - Relative Saturation (Relative Humidity)

$$s_r(h_r) = \frac{p_v}{p_v^*(T)} \times 100\%$$

- Molar Saturation (Molar Humidity)

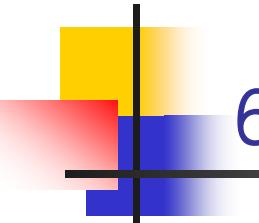
$$s_m(h_m) = \frac{p_v}{P - p_v} = \frac{\text{moles of vapor}}{\text{moles of vapor} - \text{free(dry) gas}}$$

- Absolute Saturation (Absolute Humidity)

$$s_a(h_a) = \frac{p_v M_v}{(P - p_v) M_{dry}} = \frac{\text{mass of vapor}}{\text{mass of vapor} - \text{free(dry) gas}}$$

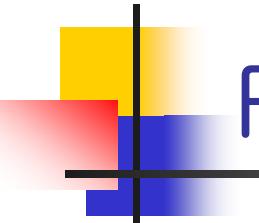
- Percentage Saturation (Percentage Humidity)

$$s_p(h_p) = 100 \frac{s_m}{(s_m)^*} = 100 \frac{p_v / (P - p_v)}{p_v^* / (P - p_v^*)}$$



## 6.4 Multicomponent Gas–Liquid Systems

- Transfer process
  - Gas → Liquid : absorption (흡수)
  - Liquid → Gas : stripping (탈기)
- VLE information
  - From tabulated VLE data
  - Raoult's Law and Henry's Law
  - VLE calculation assuming ideal solution
  - Rigorous VLE calculation using model equations



# Raoult's Law and Henry's Law

- Distribution of component between vapor and liquid phase

→ Phase equilibrium thermodynamics

- Simplifications

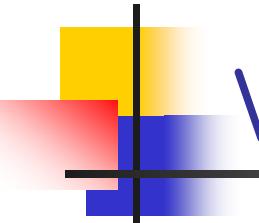
- Raoult's Law  $p_a \equiv y_a P = x_a p_a^*(T)$  ,  $x_a \rightarrow 1$

- Valid for almost pure component. Similar components

- Henry's Law  $p_a \equiv y_a P = x_a H_a^*(T)$  ,  $x_a \rightarrow 0$

- Valid for almost dilute component.

- Distribution coefficient  $K \equiv y_a / x_a$



# VLE calculations for ideal solutions

- Bubble Point temperature calculation

- Given  $P, x \rightarrow$  calculate  $T, y$

$$P = x_a p_a^*(T_{bp}) + x_b p_b^*(T_{bp}) + \dots$$

$$y_i = \frac{x_i p_i^*(T_{bp})}{P}$$

- Dew point temperature calculation

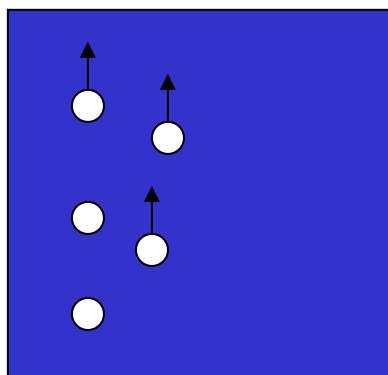
- Given  $P, y \rightarrow$  calculate  $T, x$

$$x_i = \frac{y_i P}{p_i^*(T_{dp})}$$

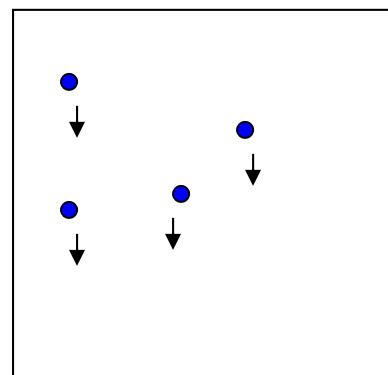
$$\sum_i x_i = \sum_i \frac{y_i P}{p_i^*(T_{dp})} = 1$$

# VLE calculations

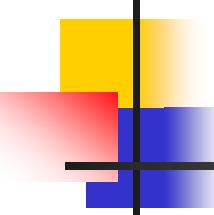
- Bubble T :  $P,x \rightarrow T,y$
- Bubble P :  $T,x \rightarrow P,y$
- Dew T :  $P,y \rightarrow T,x$
- Dew P :  $T,y \rightarrow P,x$



Bubble point



Dew point

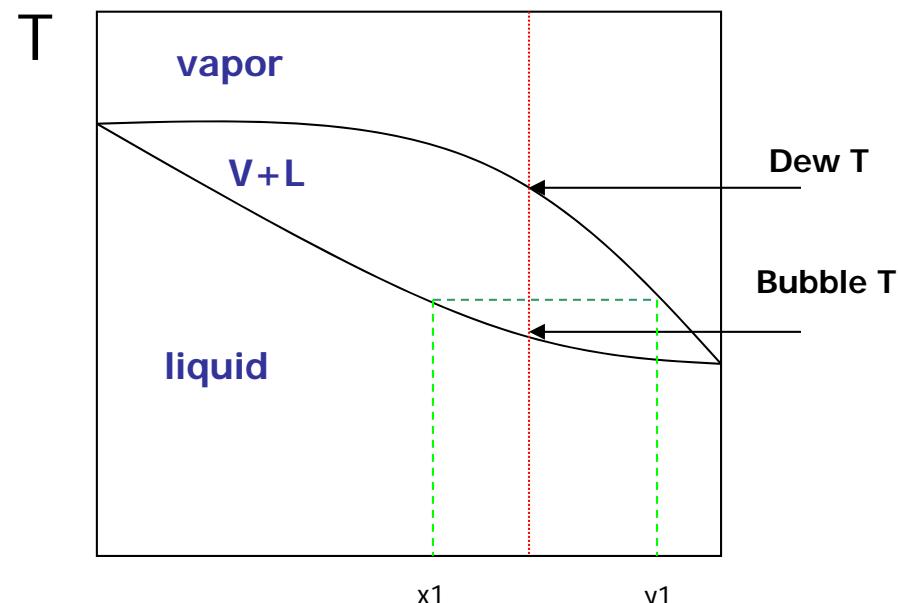


## Numerical Methods for the VLE calculations

- Newton – Raphson Method
- Secant Method
- ...
  
- ←Student presentation

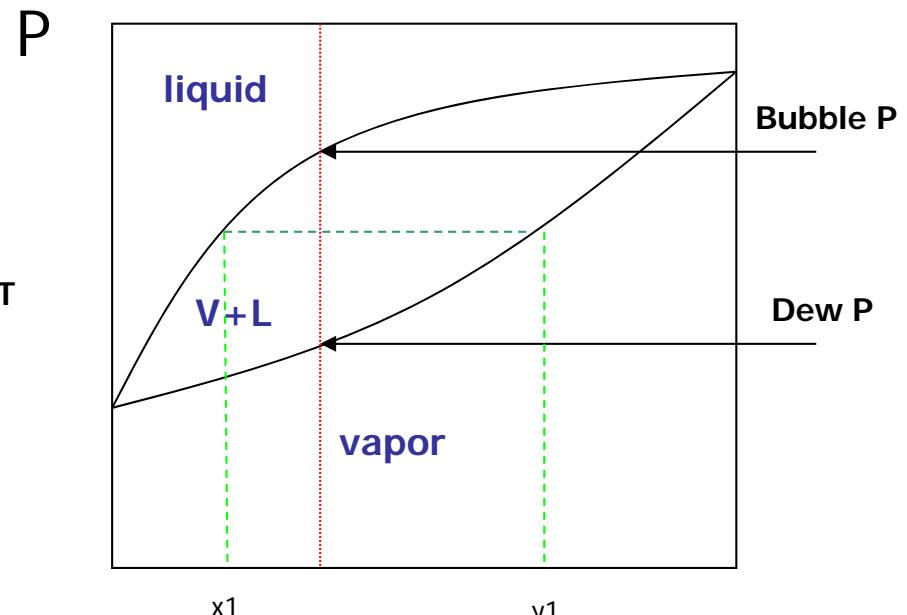
# Phase diagrams for binary VLE

- Txy diagram

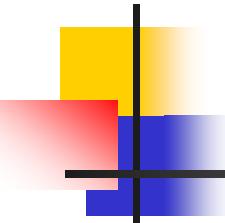


x or y

- Pxy diagram



x or y



# Solutions of Solids in Liquids

- Solubility
  - Limits on the amount of solids that can be dissolved
  - Solubility of a solid depends strongly on T
  - Ex)
    - 222 g AgNO<sub>3</sub> / 100 g H<sub>2</sub>O at 20 ° C
    - 0.003 g AgCO<sub>3</sub> / 100 g H<sub>2</sub>O at 20 ° C
    - 0.00002 g AgBr / 100 g H<sub>2</sub>O at 20 ° C
- Crystallization
  - Separation of solids and liquids
  - Driving force = solubility difference
  - A solute in equilibrium with a crystal must be saturated.

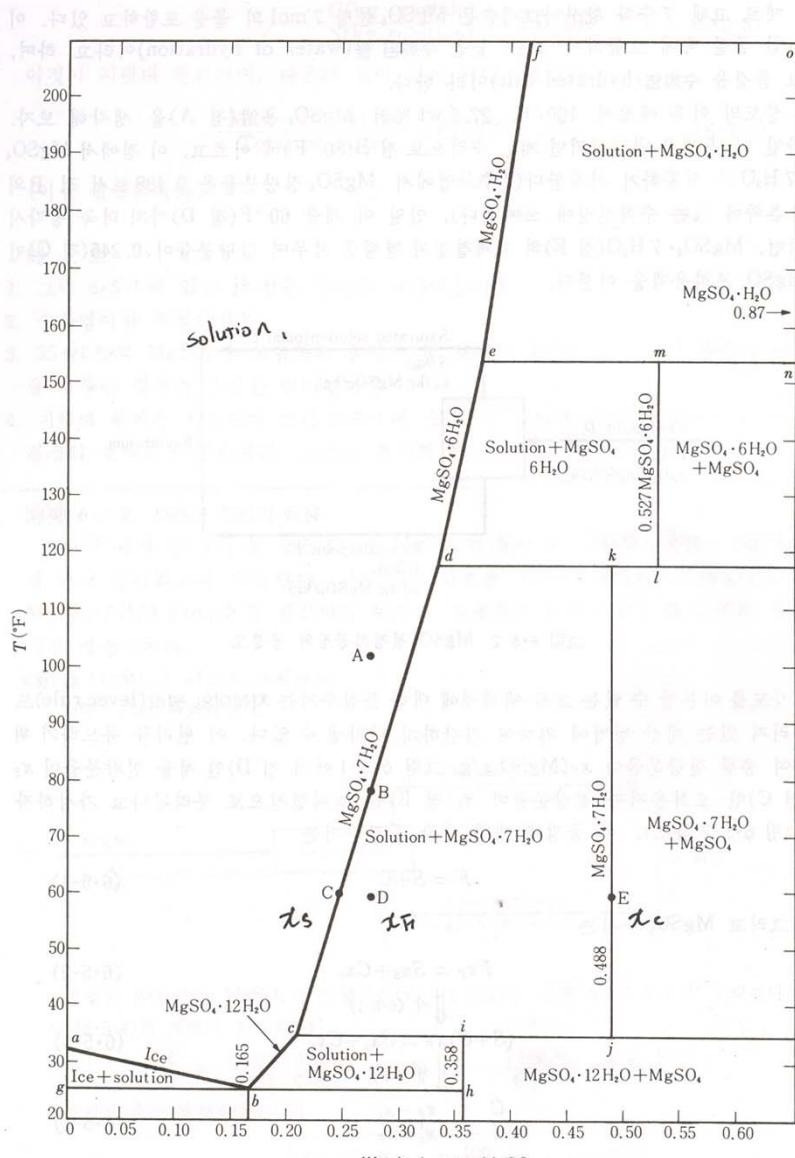
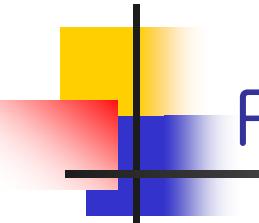


그림 6·5-1  $\text{MgSO}_4\text{-H}_2\text{O}$  계의 상도

# A Phase diagram for solid–liquid system



## Phase diagrams for solid–liquid systems

- See figure 6.5–1
- Lever rule (지렛대 원리)

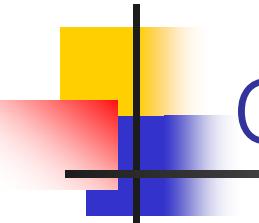
$$F(\text{feed}) = S(\text{solution}) + C(\text{crystal})$$

$$Fx_F = Sx_S + Cx_C$$

$$(S + C)x_F = Sx_S + Cx_C$$

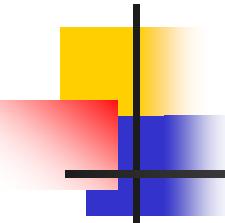
$$\frac{C}{S} = \frac{x_F - x_S}{x_C - x_F} = \frac{\overline{CD}}{\overline{DE}}$$

$$\therefore \frac{C(\text{kg crystal})}{S(\text{kg solution})} = \frac{\overline{CD}}{\overline{DE}}$$



# Colligative Solution Properties

- 용액의 총괄성
  - Properties change on a solution
    - Vapor pressure lowering
    - Boiling point elevation
    - Melting point depression
    - Osmotic pressure
  - Depend only on molar concentration  
(not on the solute and solvent)



## Colligative properties

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- Vapor Pressure Lowering

$$p_s(T) = (1-x)p_s^*(T)$$

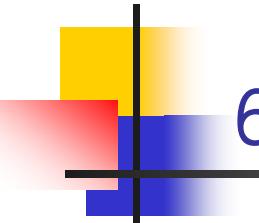
$$\Delta p_x^* = p_s^* - (p_s^*)_e = xp_s^*$$

- Boiling Point Elevation

$$\Delta T_b = \frac{RT_{b0}^2}{\Delta \hat{H}_v} x$$

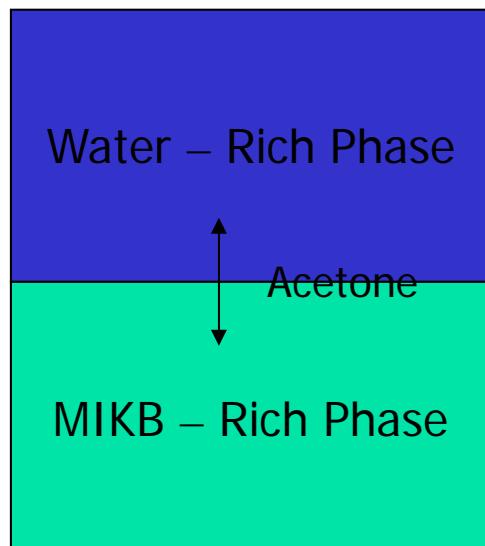
- Melting Point Depression

$$\Delta T_m = \frac{RT_{m0}^2}{\Delta \hat{H}_m} x$$



## 6.6 Immiscible and Partially Miscible Liquids

- Example ) Water + MIBK (Methyl Isobutyl Ketone) + Acetone System



Distribution Coefficient

$$K = \frac{(x)_{MIBK}}{(x)_{WATER}}$$

- Partially miscible liquids
- Immiscible System
- Liquid-Liquid Extraction

# Phase diagram for ternary LLE systems

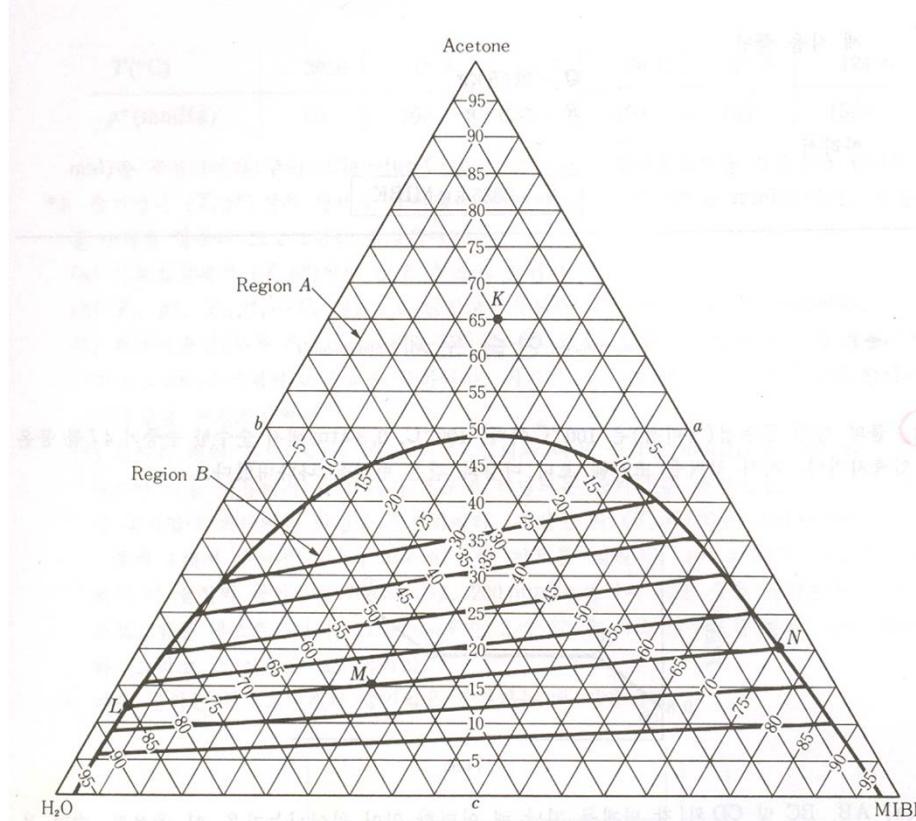


그림 6·6-1 25 °C에서 물-아세톤-메틸이소부틸케톤의 삼각상도