Chapter 1. Single Particles in a Fluid

1.1 Motion of Solid Particles in a Fluid: Drag Force

1) ***Drag Force***, F_D:

Net force exerted by the fluid on the spherical particle(diameter x) in the direction of flow

$$F_D = C_D\left(\frac{\pi}{4}d_p^2\right)\frac{\rho_f U^2}{2}$$

Area exerted friction	Kinetic by energy of unit mass fluid

where C_D : Drag coefficient

cf. for pipe flow

$$\tau_w = f\frac{\rho_f U^2}{2} \quad \rightarrow \quad F_w = f(\pi DL)\frac{\rho_f U^2}{2}$$

where f: Fanning friction factor

C_D vs. Re_p - Figure 1.1

$$Re_p = \frac{d_p U \rho_f}{\mu}$$

where U : the relative velocity of particle with respect to fluid

- For $Re_p < 1$ (creeping flow region)

$$F_D = 3\pi d_p \mu U$$

Stokes(1851)' law

$$= \left(\frac{24}{Re_p}\right)\left(\frac{\pi}{4}d_p^2\right)\frac{\rho_f U^2}{2}$$

$$\therefore\ C_D = \frac{24}{Re_p}$$

cf. $f = \frac{16}{Re}$

- For $500 < Re_p < 2\times10^5$

$$C_D \approx 0.44$$

- For intermediate range

$$C_D = \frac{24}{Re_p}(1 + 0.15 Re_p^{0.687})$$

or

Table 1.1

2) Motion in a Gas : *Non-continuum Effect*

Mean-free path of fluid

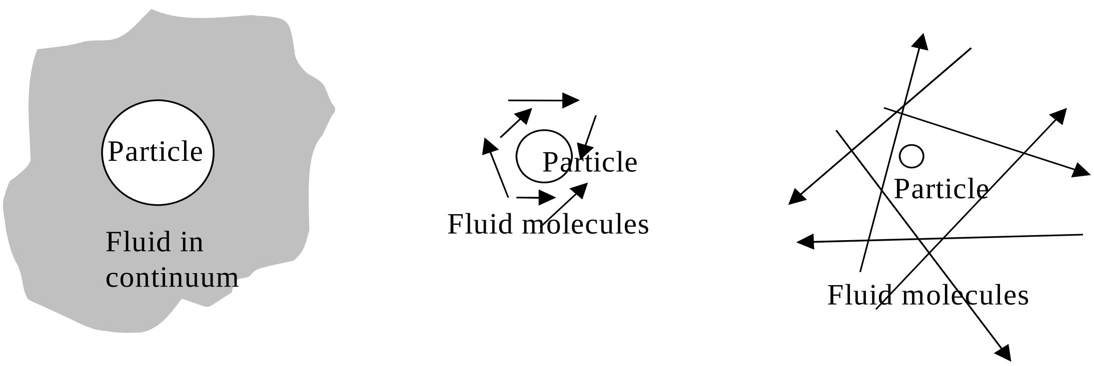
$$\lambda = \frac{1}{\sqrt{2} n_m \pi d_m^2}$$

where

n_m : number concentration of molecules

d_m : diameter of molecules

For air at 1 atm and 25℃ λ = 0.0651 μm



Continuum regime *transition regime* *free-molecule regime*

Knudsen number, Kn

$$Kn = \frac{\lambda}{d_p}$$

- Continuum regime : $Kn \sim 0$ (<0.1)
- Transition regime : $Kn \sim 1$ (0.1~10)
- Free molecule regime : $Kn \sim \infty$ (>10)

Corrected drag force

$$F_D = \frac{3\pi d_p \mu U}{C_c}$$

where C_c : Cunningham correction factor

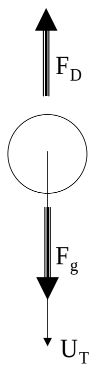
$$C_c = 1 + Kn[2.514 + 0.8\exp(-0.55/Kn)]$$

d_p, μm	C_c
0.01	22.7
0.05	5.06
0.1	2.91
1.0	1.168
10	1.017

In air at 1atm and 25℃

1.2 Particle Falling Under Gravity Through a Fluid

1) ***Terminal Settling Velocity***, U_T



The velocity of free falling particle when

$$F_D = F_g - F_B$$

$$\therefore\ C_D\left(\frac{\pi}{4}d_p^2\right)\frac{\rho_f U^2}{2} = \frac{\pi}{6}(\rho_p - \rho_f)d_p^3 g$$

$$\therefore U_T = \left[\frac{4}{3}\frac{g d_p}{C_D}\left(\frac{\rho_p - \rho_f}{\rho_f}\right)\right]^{1/2}$$

For Stokes law regime

$$\frac{3\pi d_p \mu U}{C_c} = \frac{\pi}{6}(\rho_p - \rho_f)d_p^3 g$$

$$\therefore U_T = \frac{(\rho_p - \rho_f) g d_p^2 C_c}{18\mu}$$

d_p, μm	U_T, cm/s[a]
0.1	8.8×10^{-5}
0.5	1.0×10^{-3}
1.0	3.5×10^{-3}
5.0	7.8×10^{-2}
10.0	0.31

[a]For unit density particle in air at 1 atm and 25℃

Worked Example(additional)

Example. In 1883 the volcano Krakatoa exploded, injecting dust 32Km up into the atmosphere. Fallout from this explosion continued for 15 months. If one assumes settling velocity was constant and neglects slip correction, what was the minimum particle size present? Assume particles are rock spheres with a specific gravity of 2.7.

$$\therefore U_T = \frac{(\rho_p - \rho_f) g d_p^2 C_c}{18\mu} = \frac{2.7 \cdot 980 \cdot d_p^2 \cdot 1}{18 \cdot 1.81 \cdot 10^{-4}}$$

$$= \frac{32 \cdot 10^3 \cdot 10^2}{15 \cdot 30 \cdot 24 \cdot 3600} = 0.0823\, cm/s$$

$$\therefore d_p = 3.19\mu m$$

2) Terminal Settling Velocity for NonStokesian particles

In Newton's regime

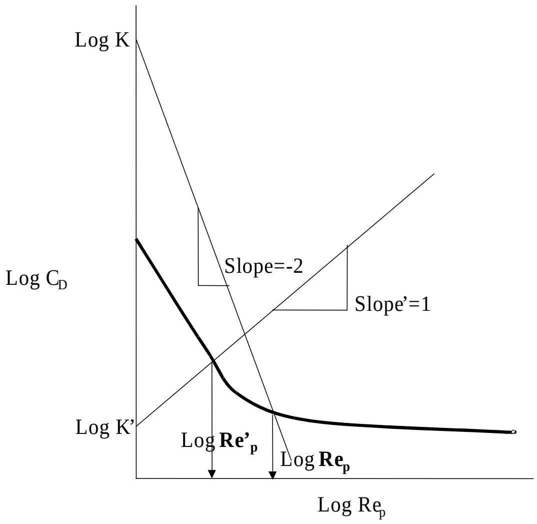
$$U_T = 1.74\left(\frac{d_p(\rho_p - \rho_f) g}{\rho_f}\right)^{1/2}$$

For intermediate regime : Trial and error(Numerical) or

$$C_D Re_p^2 \Rightarrow \frac{4}{3}\frac{d_p^3 \rho_f (\rho_p - \rho_f) g}{\mu^2} \equiv K \text{ (constant)}$$

$$\downarrow$$

$$\log C_D = \log K - 2 \log Re_p$$



From Re_p,

$$U_T = \frac{Re_p \mu}{d_p \rho_f}$$

Worked Example 1.6

* d_p? for given U_T

$$\frac{C_D}{Re_p} \Rightarrow \frac{4}{3}\frac{g\mu(\rho_p - \rho_f)}{U_T^3 \rho_f^2} \equiv K'$$

$$\log C_D = \log Re_p + \log K'$$

From the Figure above,

$$d_p = \frac{Re_p \mu}{U_T \rho_f}$$

Worked Example 1.5

http://www.processassociates.com/process/separate/termvel.htm ☞ U_T

http://www.processassociates.com/process/separate/termdiam.htm☞ d_p

3) Transient Response to Gravitational Field

In general,

Particle motion in gravity

$$m_p \frac{dU}{dt} = F_g - F_B - F_D$$

$$\rho_p \frac{\pi}{6} d_p^3 \frac{dU}{dt} = \frac{\pi}{6}(\rho_p - \rho_f) d_p^3 g - C_D\left(\frac{\pi}{4} d_p^2\right)\frac{\rho_f U^2}{2}$$

For Stokes' law regime, $F_D = 3\pi d_p \mu U$

Rearranging,

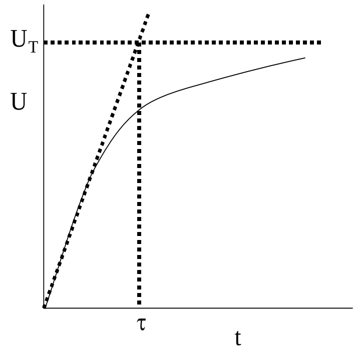
$$\tau \frac{dU}{dt} = \tau g - U$$

where $\tau \equiv \frac{(\rho_p - \rho_f) d_p^2 C_c}{18\mu}$

Relaxation time

Integration yields

$$U = U_T[1 - \exp(-t/\tau)]$$



* 여기서 τ는 외부의 변화에 대처하는 입자의 기민성과 관련한다.

Relaxation time for Unit Density Particles at Standard Conditions

Particle diameter, μm	Relaxation time, s
0.01	6.8×10^{-9}
0.1	8.8×10^{-8}
1.0	3.6×10^{-6}
10	3.1×10^{-4}
100	3.1×10^{-2}

Table 1.2

1.3 Nonspherical particles

※ C_D for nonspherical particles - Figure 1.3

1) ***Dynamic shape factor***, χ

$$\chi = \frac{F_D}{3\pi d_v \mu U}$$

따라서 비 구형 입자의 항력은 동적 형상인자의 값을 알면 구할 수 있다.

Dynamic shape factors of powders

Powders	Dynamic shape factor
sphere	1.00
cube	1.08
cylinder(L/D=4)	
axis horizontal	1.32
axis vertical	1.07
bituminous coal	1.05-1.11
quartz	1.36
sand	1.57
talc	2.04

2) Equivalent Diameters:

Spheres with the same properties

- Volume-Equivalent diameters

  Surface-area equivalent diameters

- Terminal velocity-equivalent diameters

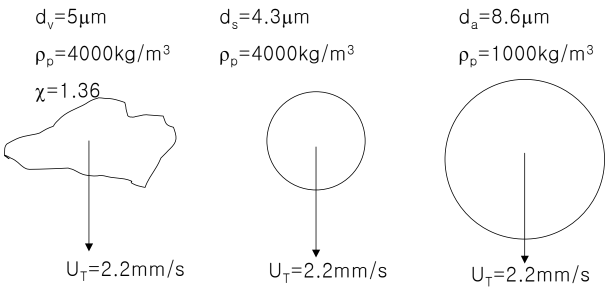
  Stokes diameter : $d_{St} = \left[\frac{18\mu U_T}{(\rho_p - \rho_f) g} \right]^{1/2}$

  Aerodynamic diameter : $d_a = \left[\frac{18\mu U_T}{(\rho_0 - \rho_f) g} \right]^{1/2}$

- Shape factors

  Sphericity, ψ

$$\psi = \left(\frac{d_v}{d_{St}} \right)^2$$



Irregular particles and its equivalent spheres

1.S Migration of Particles by Other External Force Fields

입자는 외부력 장(external force field)하에서 영향을 받아 움직인다. 이 때 움직임을 migration, 그 속도를 *migration velocity*라 부른다.

In Stokes' law regime

$$F_{ext} = F_D \left(= \frac{3\pi\mu d_p}{C_c} U_{mig} \right)$$

where U_{mig} : migration or drift velocity in the fields

e.g., For gravitational field,

$$F_{ext} = \frac{\pi}{6} d_p^3 (\rho_p - \rho) g \rightarrow U_T = \frac{\rho_p g d_p^2}{18\mu C_c \rho_p} (\rho_p - \rho_f)$$

terminal settling velocity

Flux by migration

$$\vec{J} = C \overrightarrow{U_{mig}}$$

where C: particle concentration

1) Centrifugal migration

$$F_c = m_p \left(1 - \frac{\rho_f}{\rho_p} \right) \frac{U_t^2}{r} = m_p \left(1 - \frac{\rho_f}{\rho_p} \right) r\omega^2$$

↑ ↑

Acceleration of centrifugation,

cf. g

$$\therefore\ U_{cf} = \frac{(\rho_p - \rho_f) d_p^2 U_t^2 C_c}{18\mu r}$$

단순 중력에 의한 것보다 더 높은 가속도를 얻을 수 있으므로 작은 입자의 침강을 보다 빠르게 얻을 수 있다.

2) Electrical Migration

$$\vec{F} = q\vec{E} = n_e e E$$

where q : charge of particles

E : strength of electric field

e : charge of electron (elementary unit of charge)

n_e : number of the units

$$U_e = \frac{n_e e E C_c}{3\pi\mu d_p}$$

전기적 방법에 의한 입도 측정이나 전기집진기의 원리가 된다.