

# Chapter 4 Fluid Flow through Packed Bed of Particles

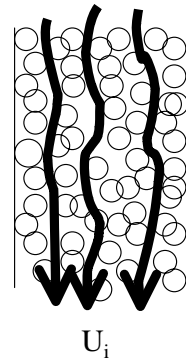
## 4.1 Pressure Drop - Flow Relationship

### (1) Laminar Flow

Fluid flow through a packed bed: simulated by fluid flow through a hypothetical tubes

$$\begin{aligned} \therefore \frac{(-\Delta p)}{H} &= \frac{32\mu U}{D^2} \\ \Rightarrow \frac{(-\Delta p)}{H_e} &= \frac{K_1 \mu U_i}{D_e^2} \end{aligned}$$

Hagen-Poiseuille equation



Substituting suitable relations

$$\therefore \frac{(-\Delta p)}{H} = 180 \frac{\mu U}{x^2} \frac{(1-\epsilon)^2}{\epsilon^3}$$

Carman-Kozeny equation

### (2) General Equation for Turbulent and Laminar Flow

*Ergun equation*

$$\frac{(-\Delta p)}{H} = 150 \frac{\mu U}{x^2} \frac{(1-\epsilon)^2}{\epsilon^3} + 1.75 \frac{\rho_f U^2}{x} \frac{(1-\epsilon)}{\epsilon^3}$$

Laminar

Turbulent

Laminar flow for  $Re^* = \frac{x U \rho_f}{\mu (1-\epsilon)} < 10$

Turbulent flow for  $Re^* = \frac{x U \rho_f}{\mu (1-\epsilon)} > 2000$

or

$$f^* = \frac{150}{Re^*} + 1.75$$

where  $f^* \equiv \frac{(-\Delta p)}{H} \frac{x}{\rho_f U^2} \frac{\epsilon^3}{(1-\epsilon)}$

*Friction factor*

Figure 4.1

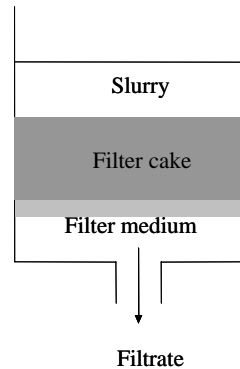
(3) Nonspherical Particles

$x_{sv}$  (surface-volume diameter) instead of  $x$

Worked Example 4.1

**4.2 Filtration**

For filtration theory and practice, see  
[http://coel.ecgf.uakron.edu/~chem/fclty/chase/FILTRATION%20FUNDAMENTALS\\_files/frame.htm](http://coel.ecgf.uakron.edu/~chem/fclty/chase/FILTRATION%20FUNDAMENTALS_files/frame.htm)



(1) Introduction

Filter media : Canvas cloth, woolen cloth, metal cloth, glass, cloth, paper, synthetic fabrics

Filter aids : To avoid cake plugging

- Diatomaceous silica, perlite, purified woolen cellulose, other inert porous solids
- By either adding slurry (increasing cake permeability) or precoating the filter media surface

Types of liquid filters

	Batch	Continuous
Pressure Filtration	Filter Press Shell-and-Leaf Filters	Automatic Belt Filters
Vacuum Filtration	Discontinuous Vacuum Filters	Rotary Drum Filters Horizontal Belt Filters
Centrifugal Filtration	Batch Types	Continuous Types

(2) Incompressible Cake

For cake filter

From laminar part of Ergun equation

$$\frac{(-\Delta p)}{H} = \frac{150\mu U(1-\varepsilon)^2}{x^2\varepsilon^3}$$

where  $L$  : cake thickness

$x$  : surface-volume diameter of particle

\* For *compressible* filter cake,

$$\frac{dp}{dL} = r_c \mu U$$

where  $r_c$  : a function of pressure difference

By defining cake resistance  $r_c$

$$r_c = \frac{150(1-\varepsilon)^2}{x^2\varepsilon^3},$$

$$\frac{(-\Delta p)}{H} = r_c \mu U$$

where  $U = \frac{1}{A} \frac{dV}{dt}$

$V$  : volume of slurry fed to filter

Also defining  $\phi$  (volume formed by passage of unit volume filtrate)

$$\phi = \frac{HA}{V},$$

$$\frac{dV}{dt} = \frac{A^2(-\Delta p)}{r_c \mu \phi V}$$

Including the resistance of filter medium,

since the resistances of the cake and the filter medium are in series,

$$(-\Delta p) = (-\Delta p_m) + (-\Delta p_c)$$

↓

$$\frac{1}{A} \frac{dV}{dt} r_c \mu H_c$$

By analogy for the filter medium

$$(-\Delta p_m) = \frac{1}{A} \frac{dV}{dt} r_m \mu H_m$$

$$\therefore (-\Delta p) = \frac{1}{A} \frac{dV}{dt} (r_m \mu H_m + r_c \mu H_c)$$

Defining equivalent height of filter cake and volume of filtrate

$$r_m H_m = r_c H_{eq} \text{ and } H_{eq} = \frac{\phi V_{eq}}{A}$$

where  $V_{eq}$ : volume of filtrate passing to create a cake of thickness

$H_{eq}$

$$\therefore \frac{1}{A} \frac{dV}{dt} = \frac{(-\Delta p)A}{r_c \mu (V + V_{eq})\phi}$$

### Constant rate filtration

$$\frac{1}{A} \frac{dV}{dt} = \frac{(-\Delta p)A}{r_c \mu (V + V_{eq})\phi} = \text{constant}$$

### Constant pressure filtration

Integrating

$$\frac{t}{V} = \frac{r_c \phi \mu}{A^2 (-\Delta p)} \left( \frac{V}{2} + V_{eq} \right)$$

### Worked Example 4.2

#### (3) Washing the Cake

Figure 4.2