

Chapter 10 Particle Size Reduction

10.1 Introduction

- To create particles in a certain size and shape
 - To increase the surface area available for next process
 - To liberate valuable minerals held within particles
- * Size reduction process : extremely energy-intensive
- 5 % of all electricity generated is used in size reduction
 - Efficiency of size reduction : 1 %

10.2 Particle Failure Mechanisms

Stress-strain behavior

Interatomic force vs. interatomic distance

Figure 10.1

yield strength - tensile strength

brittle vs. ductile(tough)

Strain energy : energy stored in a body under tension

- not uniform but concentrated in
splits, cracks, hollow parts, foreign inclusions,
displacement

Inglis (1913)

Stress concentration factor, K

$$K \equiv \frac{\text{local stress}}{\text{mean stress in body}}$$
$$= 1 + 2\sqrt{L/R}$$

where L : half the length of the crack

R : the radius of the crack tip or hole

Griffith (1921)

For crack to propagate

Strain energy > surface energy created

Requires appropriate crack propagation mechanism

* Critical minimum crack length, L_c

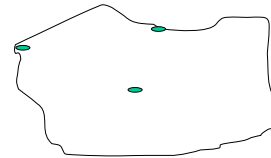
If $L > L_c$, the crack propagate

Dissipation velocity of excess strain energy = sonic

> crack propagation velocity → multiple fracture

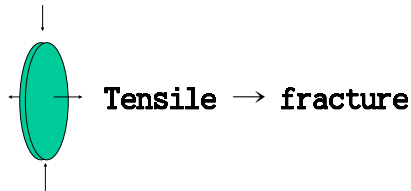
Gilvary (1961)

Volume, facial, edge cracks → different size distribution
size reduction



Evans(1961)

Compressive



* For small particles

Needs more difficult to break, $K \downarrow$

- Small L

- Less space for stress distribution → overestimate of K

e.g. Grindability limit

$4 \mu m$ for quartz

$1 \mu m$ for calcite

10.3 Models Predicting Energy Requirements and Product Size Distribution

(1) Energy Requirement

Rittinger(1867)

$$E = C_R \left[\frac{1}{x_2} - \frac{1}{x_1} \right] \text{ or } \frac{dE}{dx} = - C_R \frac{1}{x^2}$$

where x_1, x_2 : **diameters of initial and final particles**

C_R : **a constant**

Kick(1885)

$$E = C_K \ln \left(\frac{x_1}{x_2} \right) \text{ or } \frac{dE}{dx} = C_K \frac{1}{x}$$

where C_K : **a constant**

Bond(1952)

$$E = C_B \left(\frac{1}{\sqrt{x_2}} - \frac{1}{\sqrt{x_1}} \right) \text{ or } \frac{dE}{dx} = C_B \frac{1}{x^{3/2}}$$

or

$$E_B = W_1 \left(\frac{10}{\sqrt{x_2}} - \frac{10}{\sqrt{x_1}} \right)$$

where x_1, x_2 : **top particle sizes before and after, or the sieve sizes in μm through which 80% powders in the feed and product, respectively.**

W_1 : **Bond work index**

e.g. $W_1 = 9.45 \text{ kWh/ton}$ **for bauxite**

$= 20.7$ **for coke from coal**

$= 8.16$ **for gypsum rock**

In general,

$$\frac{dE}{dx} = - \frac{C}{x^N}$$

where $N = 2$ **for Rittinger**

$= 1$ **for Kick**

$= 1.5$ **for Bond**

Figure 10.2

Kick \rightarrow Bond \rightarrow Rittinger as $x \downarrow$

Worked Example 10.1

(2) Prediction of the Product Size Distribution

Definitions

S_j : the specific rate of breakage

- probability of a particle of size j being broken in unit time

$b(i, j)$: breakage distribution function

- fraction of size i from the breakage of mother particle j

Then population balance:

$$\frac{dm_i}{dt} = \sum_{j=1}^{i-1} b(i, j) S_j m_j - S_i m_i$$

where $i < j$

Figure 10.3

* $B(i, j)$: $j \rightarrow i$ to n

In terms of mass fraction

$$\frac{dx_i}{dt} = \sum_{j=0}^{i-1} b(i, j) S_j x_j - S_i x_i$$

Worked Example 10.2

10.5 Types of Comminution Equipment

(1) Factors Affecting Choice of Machine

- Stressing mechanism
- Mode of operation : batch/continuous or open/closed circuit
- Capacity
- Size of feed and product

- Material properties
- Carrier medium : air/inert gas/water/oil
- Integration with other unit operation : drying, classification, mixing, transportation, storage

(2) Stressing Mechanisms

Stressing between two solid surfaces : *Crushing*

Figure 10.4

- 0.01 - 10m/s
- For coarse(< 100mm) and medium-coarse size reduction (< 10mm)
- For medium-hard(Moh's:4-6) to medium materials(Moh's:7-10)

Jaw crusher(Figure 10.6)

Gyratory crusher(Figure 10.7)

Crushing roll(Figure 10.8)

Horizontal table mill(Figure 10.9)

Stressing against solid surface : *High velocity impact*

- Medium-fine to ultrafine comminution

Hammer mill(Figure 10.10)

Pin mill(Figure 10.11)

Fluid energy mill(Jet mill)(Figure 10.12)

Stressing by *Crushing and impact* (or using carrier medium)

Sand mill(Figure 10.13)

Colloid mill(Figure 10.14)

Ball mill(Figure 10.15)

* Wet size reduction

- Stressing between two surfaces + shearing forces of the medium
- finer products/lowering dust emission/30% energy saving
- higher wear/needs wastewater treatment

(3) Particle Size

Terminologies of comminution according to particle size

Table 10.1

Comminution equipment according to particle size

Table 10.2

(4) Material Properties

- Toughness, abrasiveness
- Toughness
- Co-Adhesivity
- Fibrous nature
- Low melting point
- Thermally sensitive materials, flammability
- Toxic/radioactive materials

(5) Carrier Medium

Air, inert gas

Water, oil

(6) Mode of Operation

Batch vs. continuous

(7) Types of Milling Circuits

Open circuit vs. closed circuit

Figure 10.16 Figure 10.17, 10.18