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# Two Bacteria with Opposite Substrate Preferences (COMMENSA)



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# System

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Here is presented the batch growth of a two-organism culture on two substrates, in which both species can utilize both substrates (Kim et al., 1988), but where the organisms have opposing substrate preferences.

The two bacterial species involved are: *Klebsiella oxytoca* ( $X_A$ ) and *Pseudomonas aeruginosa* ( $X_B$ ). The two substrates are glucose (Y), which is preferred by *K. oxytoca*, and citrate (Z), which is preferred by *P. aeruginosa*.

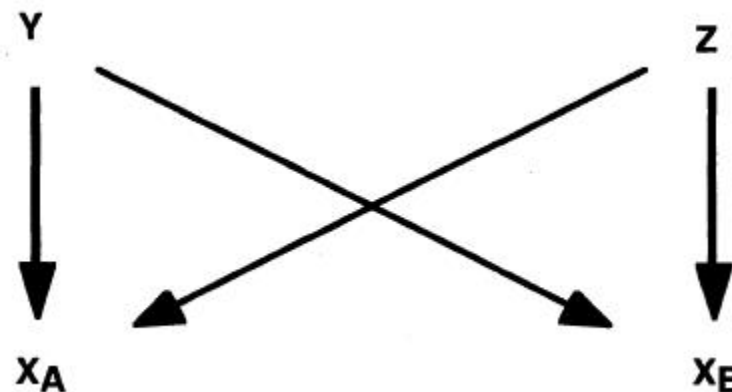


Fig. 1. Organism  $X_A$  prefers substrate Y, and organism  $X_B$  prefers substrate Z.



# Model Assumptions

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- ◆ The overall individual growth rate of each species at any time is the sum of the rate of growth on glucose plus the rate on citrate.
- ◆ The specific growth rate on each substrate depends on the concentration level of some key enzyme responsible for the rate-controlling step E.
- ◆ An inhibitor I is produced from the growth of *K. oxytoca* (Bacteria-A) on glucose and inhibits the growth of *P. aeruginosa* (Bacteria-B) on citrate. The inhibitor is thus a growth-associated product.



# Model

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The growth rates,  $\mu_{ij}$ , for each organism are the sums of the growth rates on glucose and citrate. The subscripts  $i$  and  $j$  have the following meaning:  $i$  refers to the organisms (*K. oxytoca* - A and *P. aeruginosa* = B) and  $j$  refers to the substrate (glucose = Y and citrate = Z). The levels of the key enzymes are denoted by E.

The biomass balances for the batch system are

$$\frac{dX_A}{dt} = (\mu_{AY} + \mu_{AZ}) X_A$$

$$\frac{dX_B}{dt} = (\mu_{BY} + \mu_{BZ}) X_B$$



# Model



The specific growth rate equations for the two organisms on each substrate are given by :

$$\mu_{AY} = \frac{\mu_{\max AY} S_Y E_{AY}}{K_{SA Y} + S_Y}$$

$$\mu_{AZ} = \frac{\mu_{\max AZ} S_Z E_{AZ}}{K_{SA Z} + S_Z}$$

$$\mu_{BY} = \frac{\mu_{\max BY} S_Y E_{BY}}{K_{SB Y} + S_Y}$$

$$\mu_{BZ} = \frac{\mu_{\max BZ} S_Z E_{BZ}}{K_{SB Z} + S_Z} \left( \frac{K_I}{K_I + I} \right)$$

The substrate balances are given by :

$$\frac{dS_Y}{dt} = -\mu_{AY} X_A \left( \frac{1}{Y_{SA Y}} + \frac{\alpha}{Y_I} \right) - \frac{\mu_{BY}}{Y_{SB Y}} X_B$$

$$\frac{dS_Z}{dt} = -\frac{\mu_{BZ}}{Y_{SB Z}} X_B - \frac{\mu_{AZ}}{Y_{SA Z}} X_A$$



# Model



Inhibitor ( I ) production is growth associated to organism A, and its decay is proportional to the cell concentration. The balance for the inhibitor is

$$\frac{dI}{dt} = \alpha \mu_{AY} X_A - \beta X_A$$

The balances for the key enzymes, which control the growth on secondary substrates are given by :

$$\frac{dE_{AZ}}{dt} = k_{PAZ} \frac{S_Z}{K_{E_{AZ}} + S_Z} \frac{K_{RAZ}}{K_{RAZ} + S_Z} - k_{PAZ} E_{AZ}$$

and

$$\frac{dE_{BY}}{dt} = k_{PBY} \frac{S_Y}{K_{E_{BY}} + S_Y} \frac{K_{RBY}}{K_{RBY} + S_Y} - k_{PBY} E_{BY}$$



# Model



The oxygen uptake rate (OUR), carbon dioxide evolution rate (CER) and dissolved oxygen tension (DOT) are given by :

$$\text{OUR} = \left( \frac{\mu_{AY}}{Y_{OAY}} + \frac{\mu_{AZ}}{Y_{OAZ}} + m_{OAY} + m_{OAZ} \right) X_A \\ + \left( \frac{\mu_{BY}}{Y_{OBY}} + \frac{\mu_{BZ}}{Y_{OBZ}} + m_{OBY} + m_{OBZ} \right) X_B$$

$$\text{CER} = \left( \frac{\mu_{AY}}{Y_{CAY}} + \frac{\mu_{AZ}}{Y_{CAZ}} + m_{CAY} + m_{CAZ} \right) X_A \\ + \left( \frac{\mu_{BY}}{Y_{CBY}} + \frac{\mu_{BZ}}{Y_{CBZ}} + m_{CBY} + m_{CBZ} \right) X_B$$

$$\text{DOT} = 100 \left( 1 - \frac{\text{OUR}}{K_L a C_0^*} \right)$$

The cell mass fractions are given by :

$$F_A = \frac{X_A}{X_A + X_B}$$

$$F_B = 1 - F_A$$





# Program



## Function file

```
1 function dy = commensa_func(t,y,UMAY,UMAZ,UMBY,UMBZ,KSAY,ksaz,ksby,ksbz,YSAY,ysaz,ysby,ysbz,yoay,yoaz,yoby,yobz,
2 dy = zeros(7,1);
3
4 UAY = UMay*y(6)*EAY/(KSAY+y(6));
5 UAZ = UMAZ*y(7)*y(4)/(ksaz+y(7));
6 UBY = UMBY*y(6)*y(5)/(ksby+y(6));
7 UBZ = (KI*UMBZ*y(7)*EBZ)/((ksbz + y(7))*(KI + y(3)));
8
9 dy(1) = (UAY + UAZ)*y(1);
10 dy(2) = (UBY + UBZ)*y(2);
11 dy(3) = ALPHA*UAY*y(1) - BETA*y(1);
12 if (y(3)<0)
13     y(3)=0; end;
14 dy(4) = (KPAZ*y(7)*KRAZ)/((KEAZ + y(7))*(KRAZ + y(7))) - (KPAZ*y(4));
15 dy(5) = (KPBY*y(6)*KRBY)/((KEBY + y(6))*(KRBY + y(6))) - (KPBY*y(5));
16 dy(6) = (-UAY*y(1))*(1/(YSAY) + (ALPHA/YI)) - y(2)*(UBY/YSBY);
17 if (y(6)<0)
18     y(6)=0; end;
19 dy(7) = -y(2)*(UBZ/YSBZ) - y(1)*(UAZ/ysaz);
20 if (y(7)<0)
21     y(7)=0; end;
```



# Program



## Command File

```
1 - UMAX = 0.933;      UMAZ = 0.926;  
2 - UMBY = 0.497;      UMBZ = 0.532;  
3 - KSAY = 0.004;      KSAZ = 0.001;  
4 - KSBY = 0.004;      KSBZ = 0.001;  
5 - YSAY = 0.503;      YSAZ = 0.3;  
6 - YSBY = 0.33;       YSBZ = 0.37;  
7 - YOAY = 2.25;       YOAZ = 2.32;  
8 - YOBY = 1.58;       YOBZ = 2.1;  
9 - YCAY = 1.6;        YCAZ = 0.9;  
10 - YCBY = 0.85;      YCBZ = 0.93;  
11 - YI = 0.05;  
12 - KPAZ = 0.4;       KPBY = 0.12;  
13 - ALPHA = 0.7;      BETA = 0.001;  
14 - MOAY = 0.0;       MOAZ = 0.1;  
15 - MOBY = 0.06;      MOBZ = 0.0;  
16 - MCAY = 0.11;      MCAZ = 0.08;  
17 - MCBY = 0.1;       MCBZ = 0.0;  
18 - KI = 0.0001;  
19 - EAY = 1.0;        EBZ = 1.0;  
20 - KEAZ = 0.001;     KEYB = 0.001;  
21 - KRAZ = 0.00001;   KRBY = 0.3;  
22 - KLACO = 0.585;  
23 - ALGO = 0.0;
```



# Program



## Command File

```
29 - XAO = 0.001;      XBO = 0.01;
30 - IO = 0.0;
31 - EAZO = 0.1;      EBYO = 0.1;
32 - SYO = 0.74;      SZO = 0.8;
33 - Cinit = [XAO, XBO, IO, EAZO, EBYO, SYO, SZO];
34
35 - Tint = 0.1;
36 - Tfin = 8.2;
37 - Tspan = linspace(0, Tfin, Tfin/Tint);
38
39 - [T C] = ode45(@ commensa_func(t, y, UMay, UMaZ, UMBY, UMBZ, KSAY, KSAZ, KSBY, KSBZ, YSAY, YSAZ, YSBY, YSBZ, YOAY, YOAZ, YOB
40 - plot(T, C(:,1), '-r.', T, C(:,2), '-g.', T, C(:,3), '-b.', T, C(:,4), '-y.', T, C(:,5), '-k.', T, C(:,6), '-m.', T, C(:,7), '-c.')
41 - title(['Yield Constant for Inhibitor- ALPHA = ', num2str(ALPHA), ' kg/kg']);
42 - xlabel('Time, h', 'fontsize', 12, 'fontweight', 'b');
43 - ylabel('Concentrations XA, XB, I, EAZ, EBY, SY, SZ, kg/m^3', 'fontsize', 12, 'fontweight', 'b');
44 - h = legend('XA', 'XB', 'I', 'EAZ', 'EBY', 'SY', 'SZ', 0);
45 - set(h, 'fontsize', 11);
46
47 - FA = C(:,1) ./ (C(:,1) + C(:,2));
48 - FB = C(:,2) ./ (C(:,1) + C(:,2));
49
50 - figure
51
52 - plot(T, FA, '-rx', T, FB, '-gv')
53 - title(['Yield Constant for Inhibitor- ALPHA = ', num2str(ALPHA), ' kg/kg']);
54 - xlabel('Time, h', 'fontsize', 12, 'fontweight', 'b');
55 - ylabel('Cell Mass Fractions, FA, FB', 'fontsize', 12, 'fontweight', 'b');
56 - h = legend('Oxytoca Cell Mass Fraction FA', 'Aeruginosa Cell Mass Fraction FB', 0);
57 - set(h, 'fontsize', 11);
58 - set(title, 'fontsize', 12, 'fontweight', 'b');
```



# Nomenclature



## Symbols

<b>CER</b>	<b>Carbon dioxide evolution rate</b>	<b>kg/(m<sup>3</sup> h)</b>
<b>DOT</b>	<b>Dissolved oxygen tension</b>	<b>%</b>
<b>F</b>	<b>Cell mass fractions</b>	<b>-</b>
<b>I</b>	<b>Inhibitor concentration</b>	<b>kg/m<sup>3</sup></b>
<b>K</b>	<b>Saturation and inhibitions constants</b>	<b>kg/m<sup>3</sup></b>
<b>K<sub>L</sub>aC<sub>O</sub><sup>*</sup></b>	<b>Oxygen transfer rate</b>	<b>kg/(m<sup>3</sup> h)</b>
<b>OUR</b>	<b>Oxygen uptake rate (normalized)</b>	<b>kg/(m<sup>3</sup> h)</b>
<b>S</b>	<b>Substrate concentration</b>	<b>kg/m<sup>3</sup></b>
<b>X</b>	<b>Biomass concentration</b>	<b>kg/m<sup>3</sup></b>
<b>Y</b>	<b>Yield coefficients</b>	<b>kg/kg</b>
<b>E</b>	<b>Level of key enzyme</b>	<b>-</b>
<b>M</b>	<b>Specific maintenance rates</b>	<b>kg/(kg h)</b>
<b>α</b>	<b>Yield constant for inhibitor</b>	<b>kg/kg</b>
<b>β</b>	<b>Consumption rate constant for inhibitor</b>	<b>kg/(kg h)</b>
<b>μ</b>	<b>Specific growth rate</b>	<b>1/h</b>



# Nomenclature

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## Indices

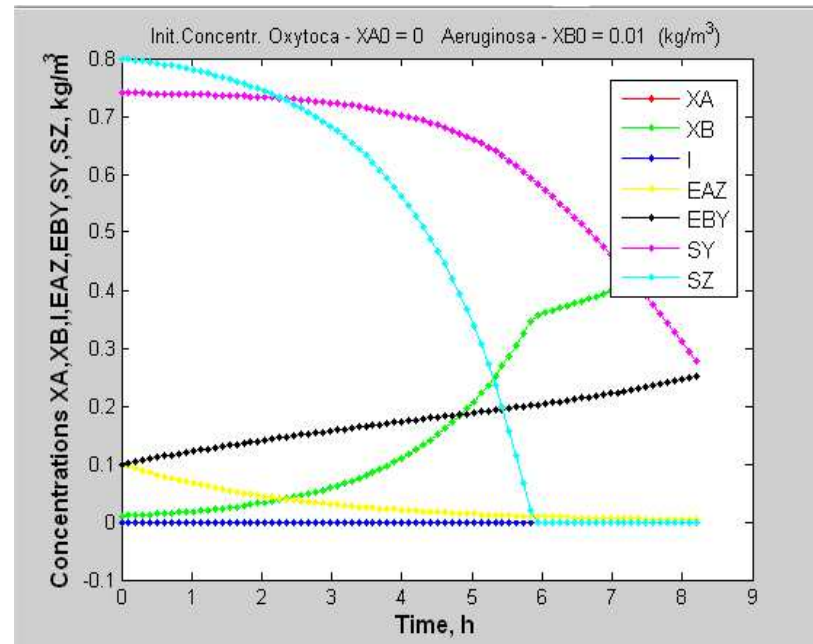
<b>A</b>	Refer to <i>K. oxytoca</i>
<b>B</b>	Refer to <i>P. aeruginosa</i>
<b>C</b>	Refer to carbon dioxide
<b>I</b>	Refer to inhibitor
<b>M</b>	Refer to maximum
<b>O</b>	Refer to oxygen
<b>P</b>	Refer to dilution due to cell division
<b>R</b>	Refer to repression
<b>S</b>	Refer to substrate
<b>Y</b>	Refer to glucose
<b>Z</b>	Refer to citrate



# Exercises



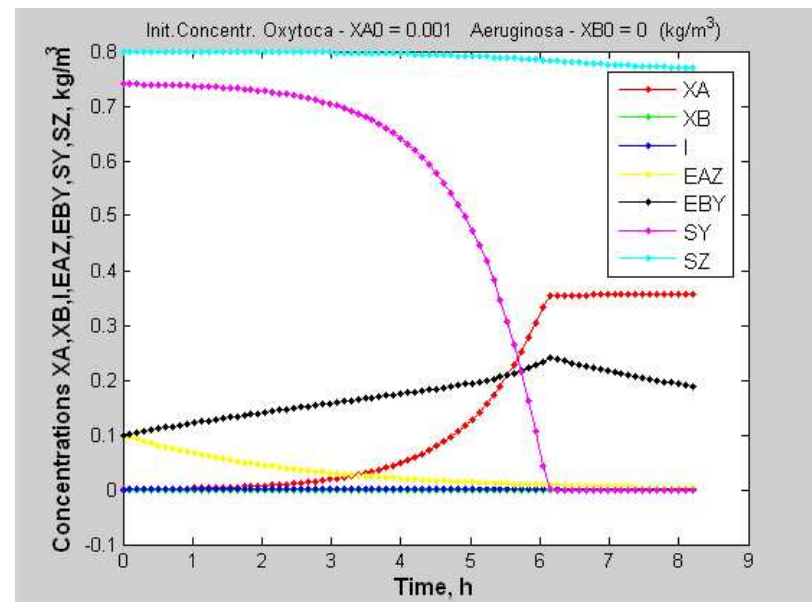
1) First test the model for each species separately. Run the simulation with the initial concentration of one species equal to zero. Graph the cell concentration, the substrate concentrations and the key enzyme levels for growth on the secondary substrate against time for the two runs. Notice the substrate preferences for each organism and the differences in key enzyme production.



# Exercises



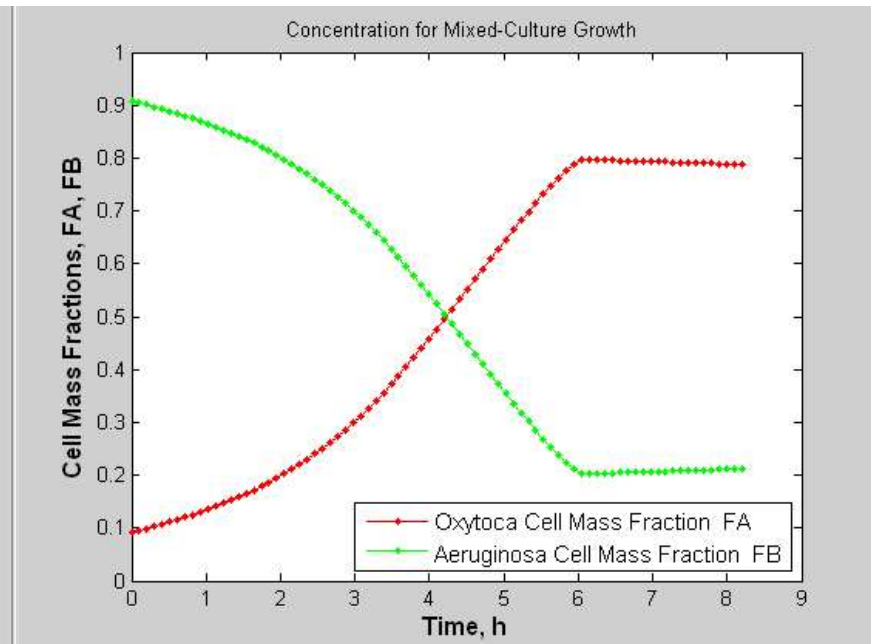
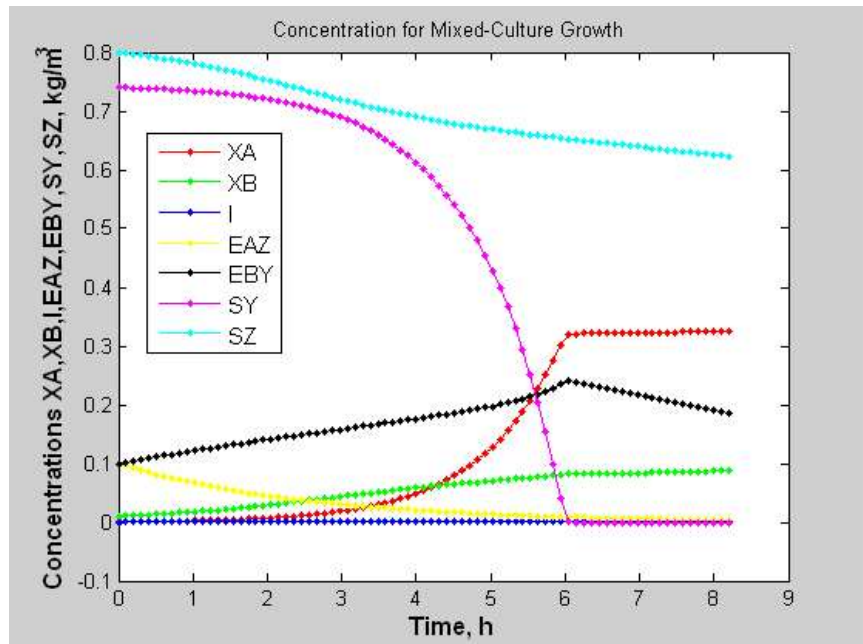
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# Exercises



2) Run the simulation for mixed-culture growth. Graph the parameters as above. What differences do you see in the production of the two key enzymes? Graph the cell fraction versus time; what are the final levels? What does this tell you about the possibilities of continuous culture under these conditions?

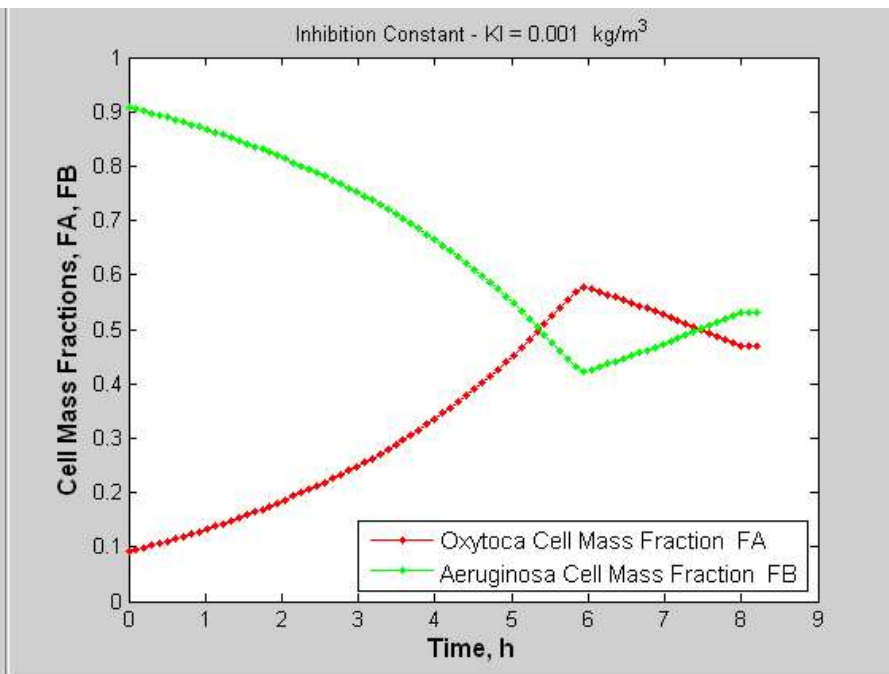
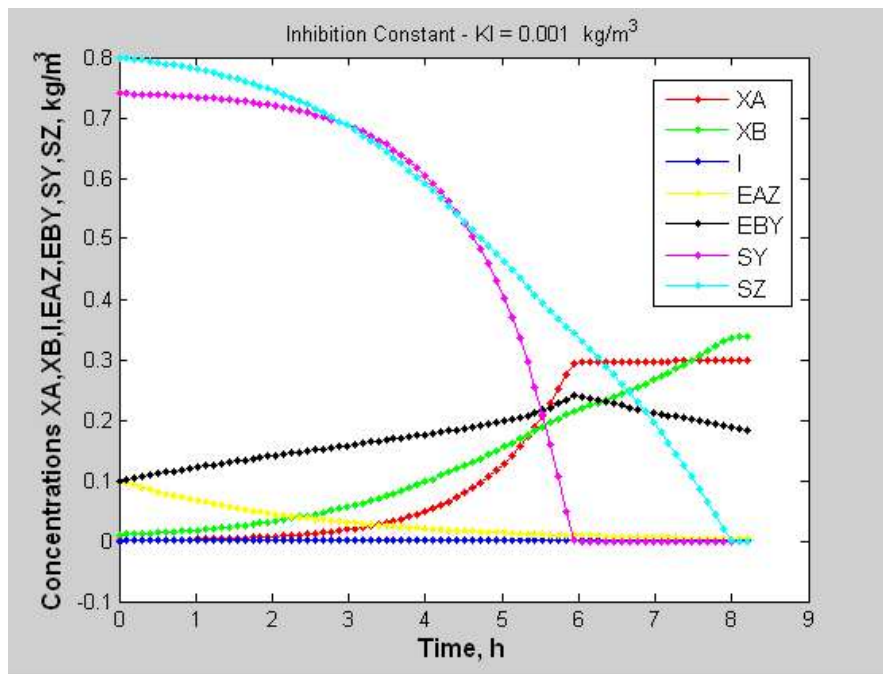




# Exercises



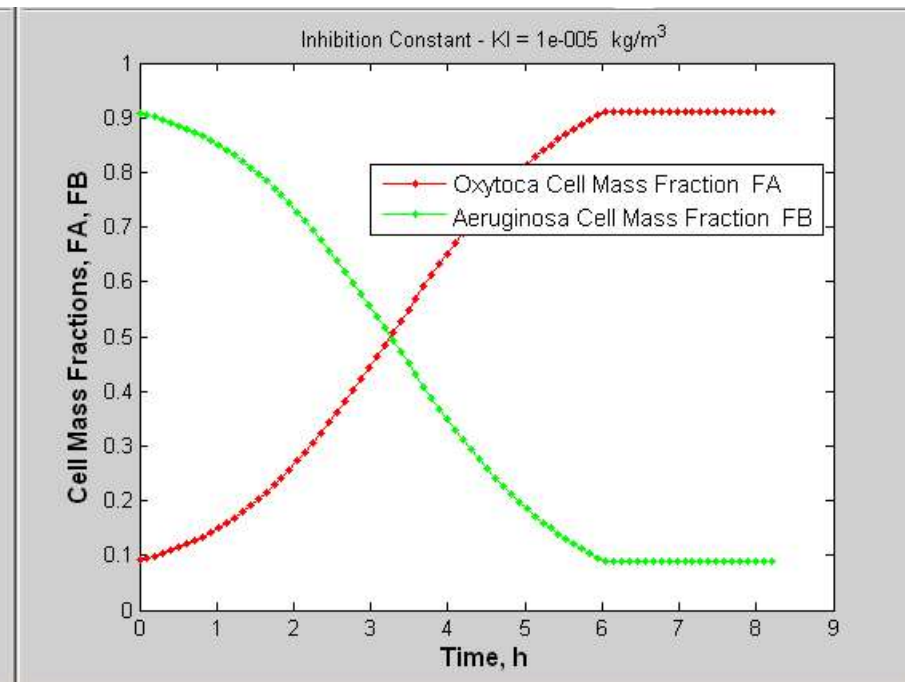
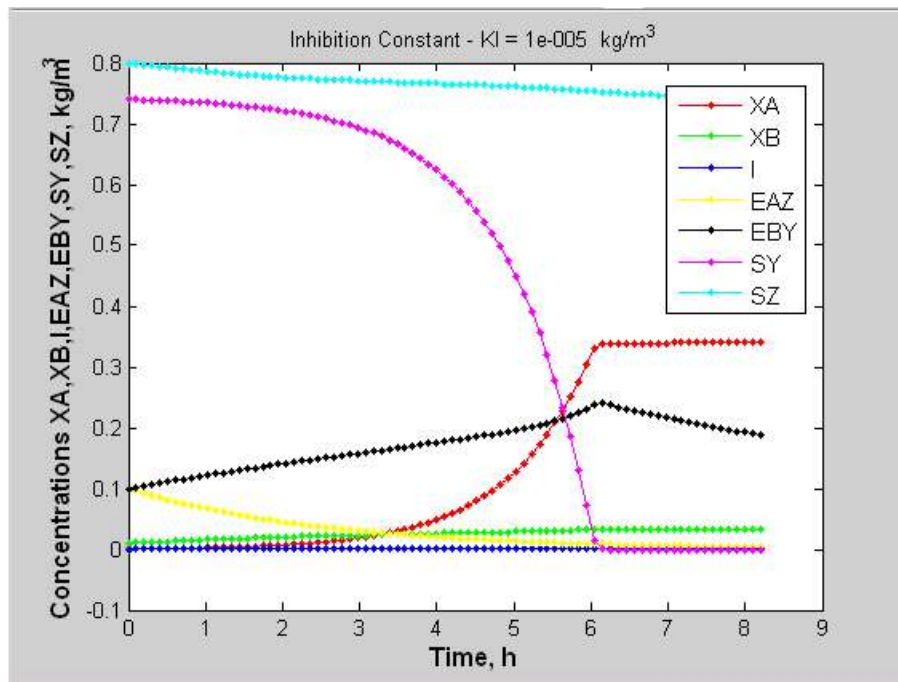
3) Vary  $K_i$  from 0.00001 to 0.001. What effect does this have on the final cell fractions for the two organisms?



# Exercises



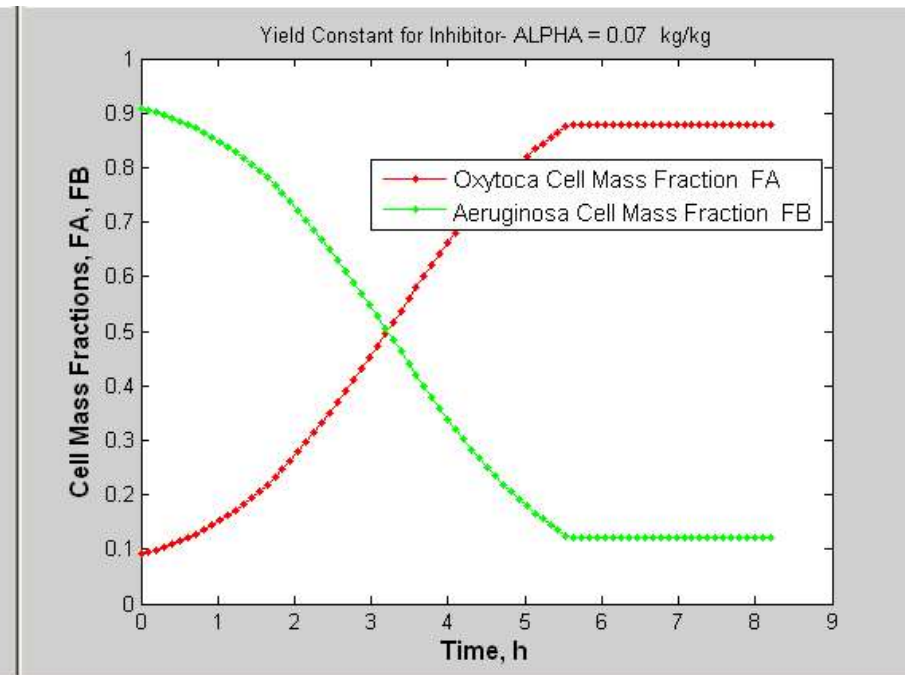
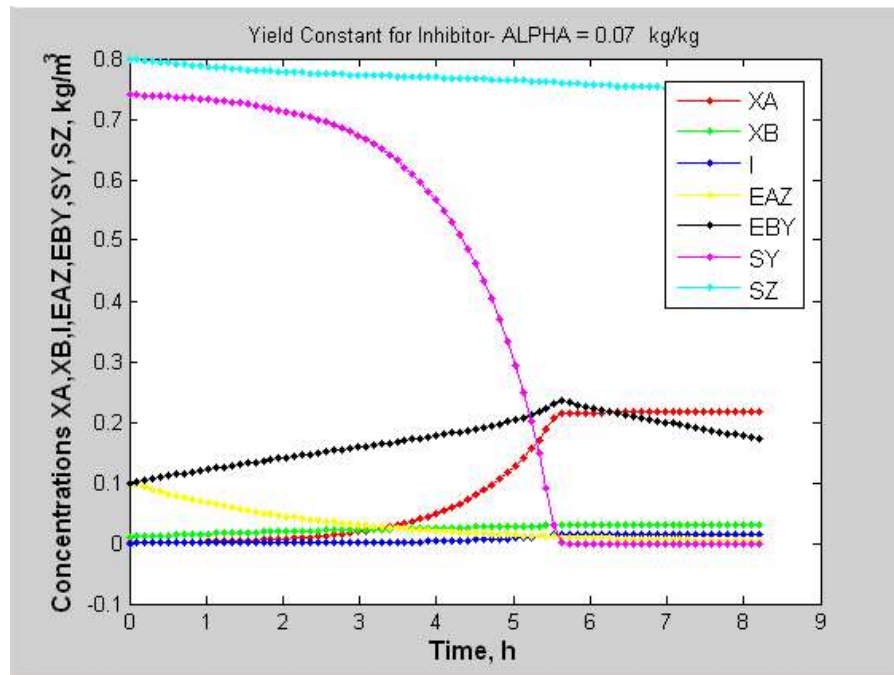
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# Exercises



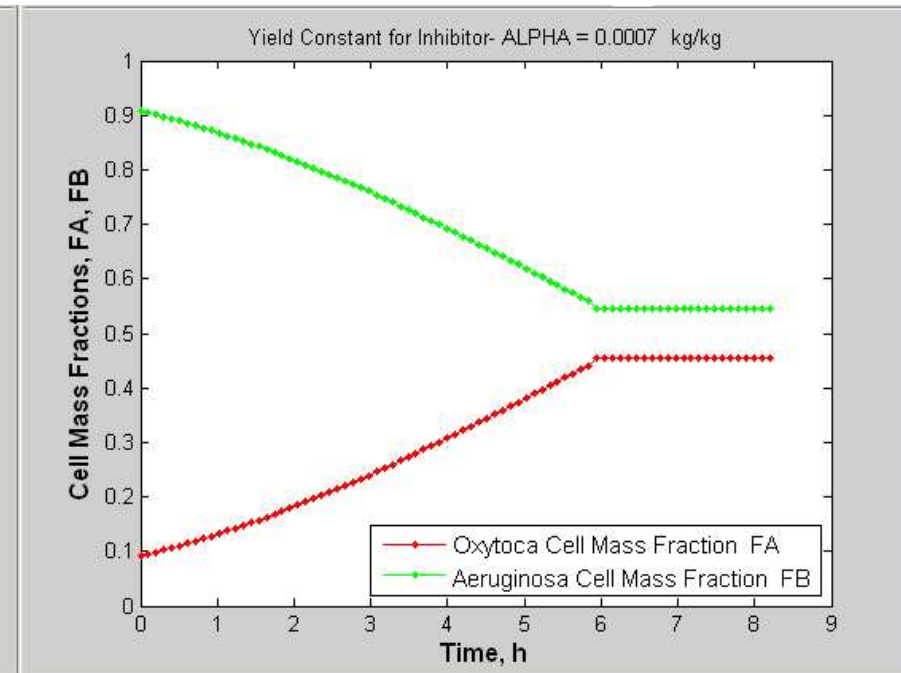
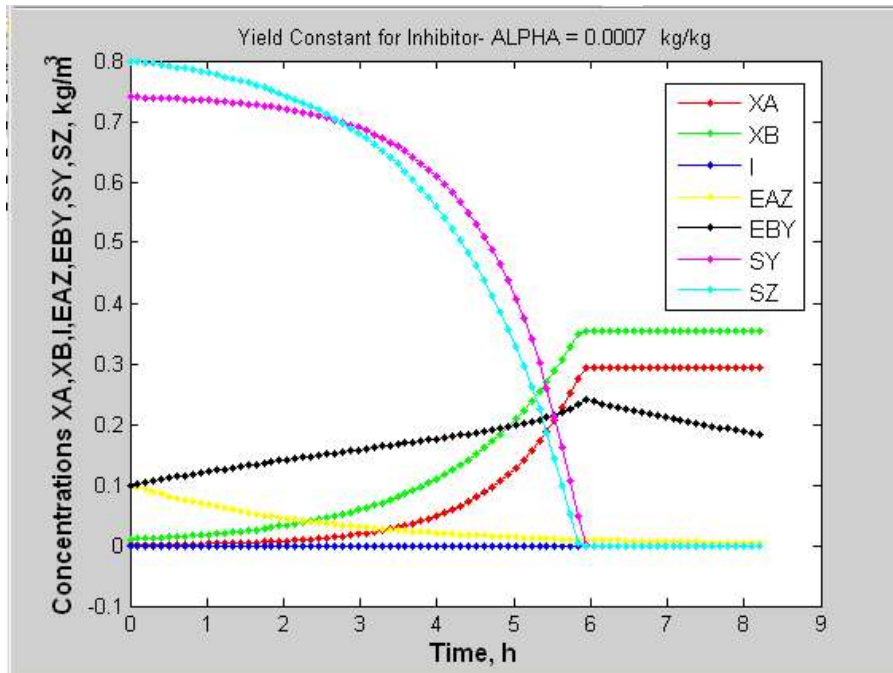
4) Vary  $\alpha$  between 0.0007 and 0.07. What happens to the cell fractions and the rate of inhibitor production? What effect does this have on key enzyme level production?



# Exercises



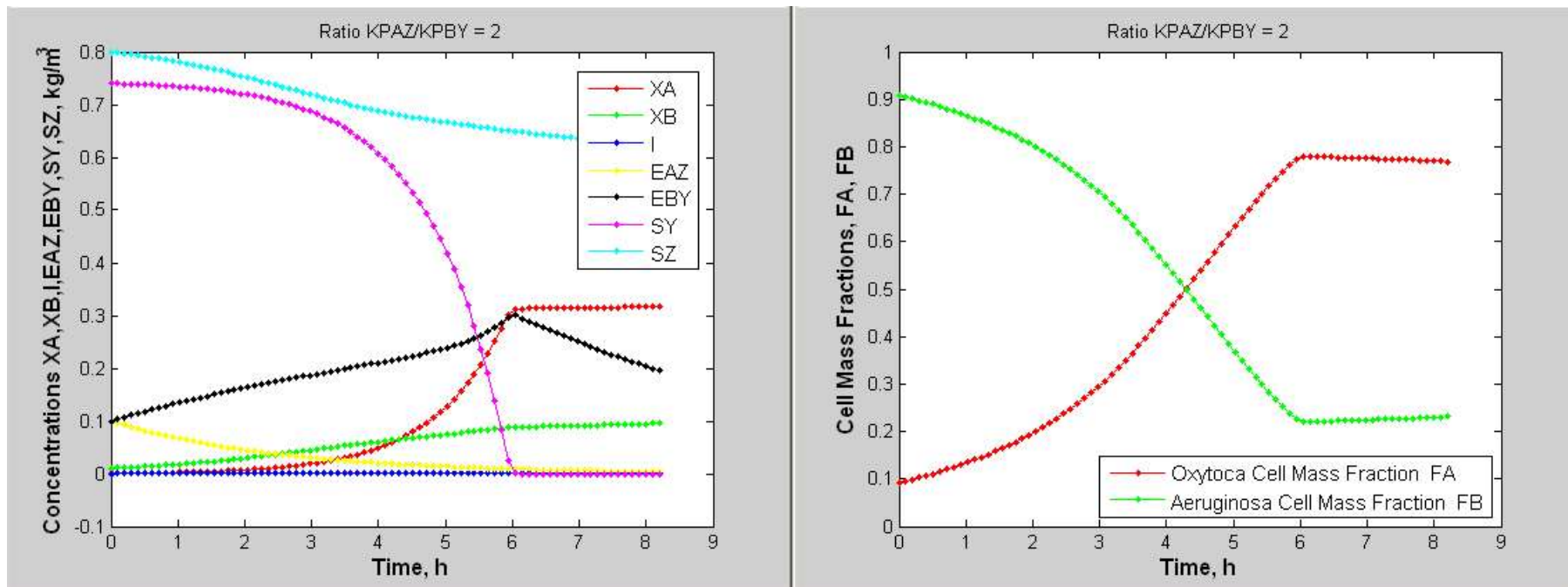
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# Exercises



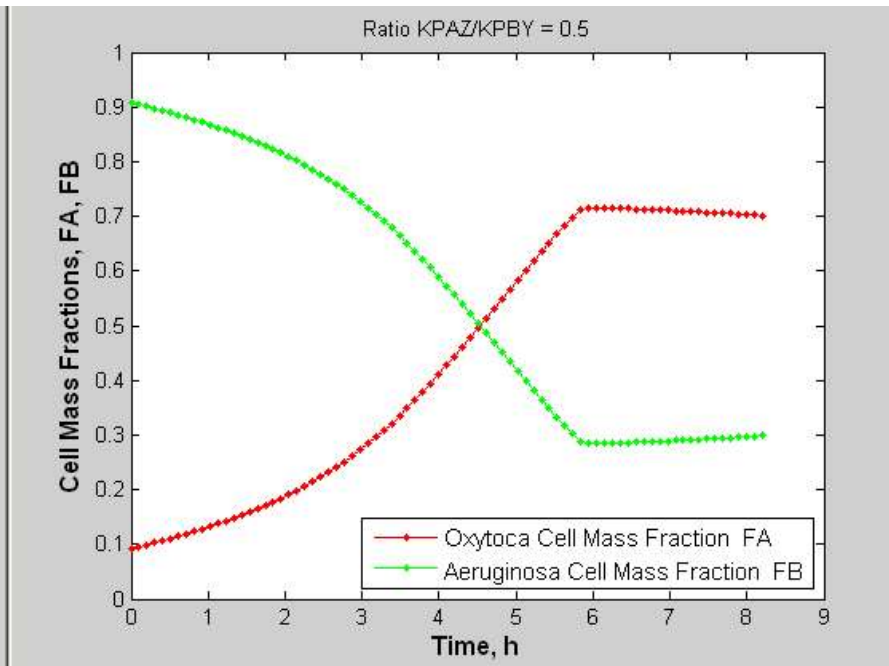
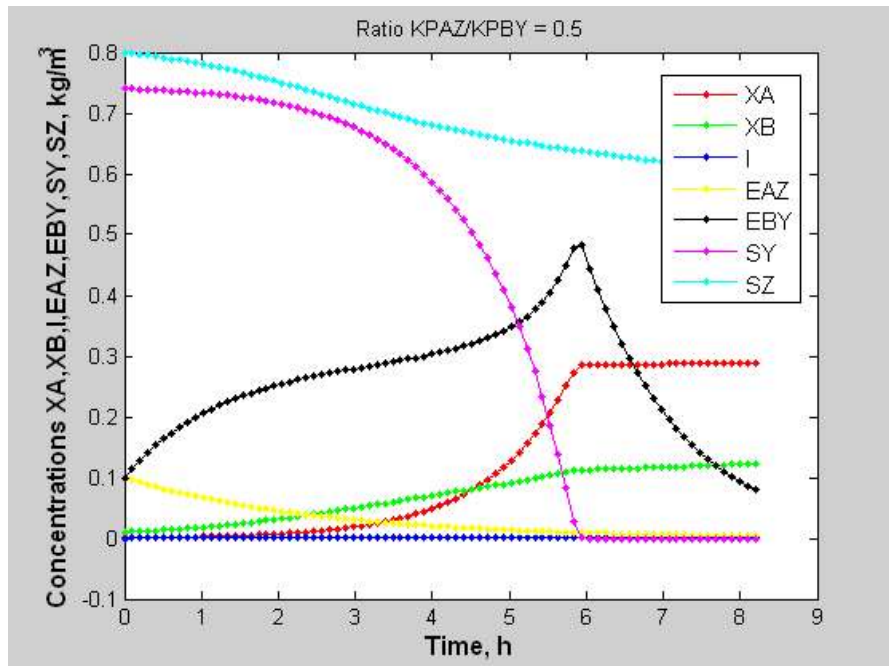
5) Vary the ratio  $k_{PAZ} / k_{PBY}$ . What effect does this have on the time necessary to exhaust citrate?



# Exercises



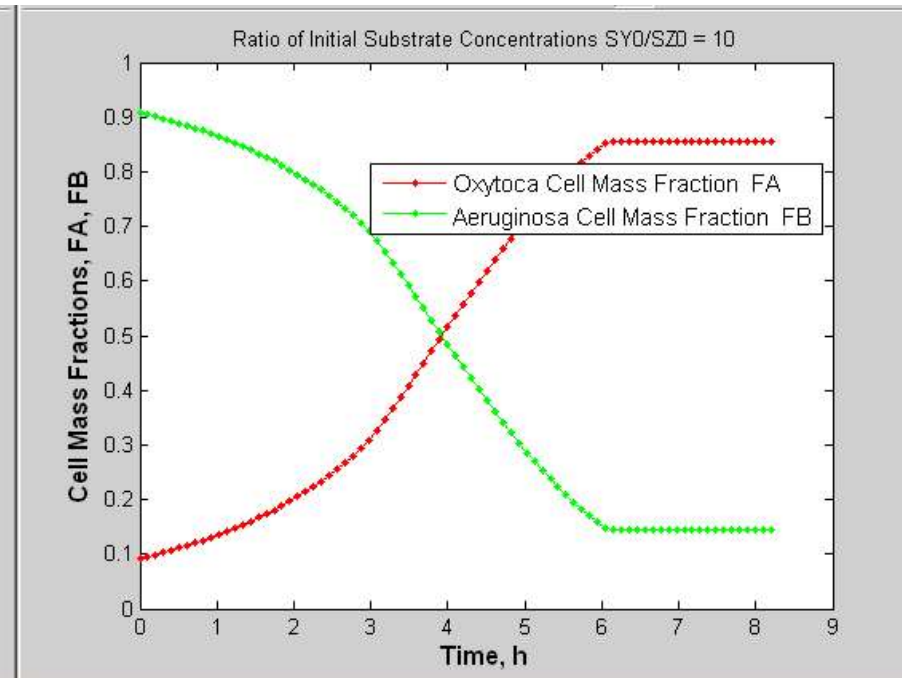
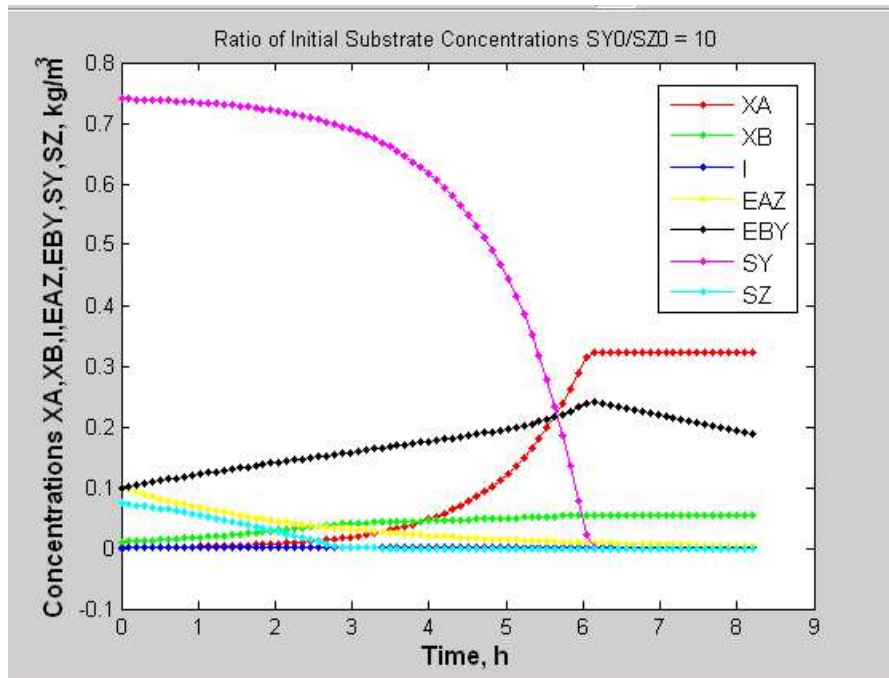
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# Exercises



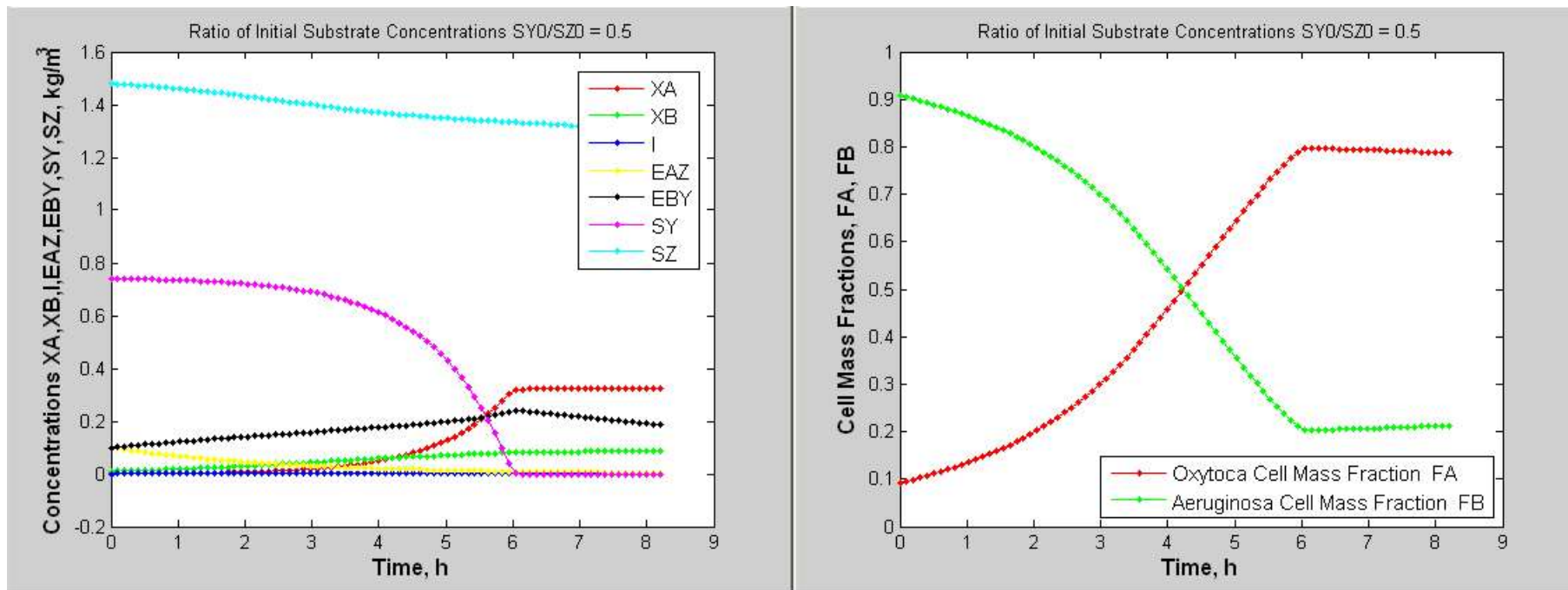
6) Vary the ratio of initial concentration of substrates from 0.1 to 10. At which values do the changes between runs become evident? Why?



# Exercises



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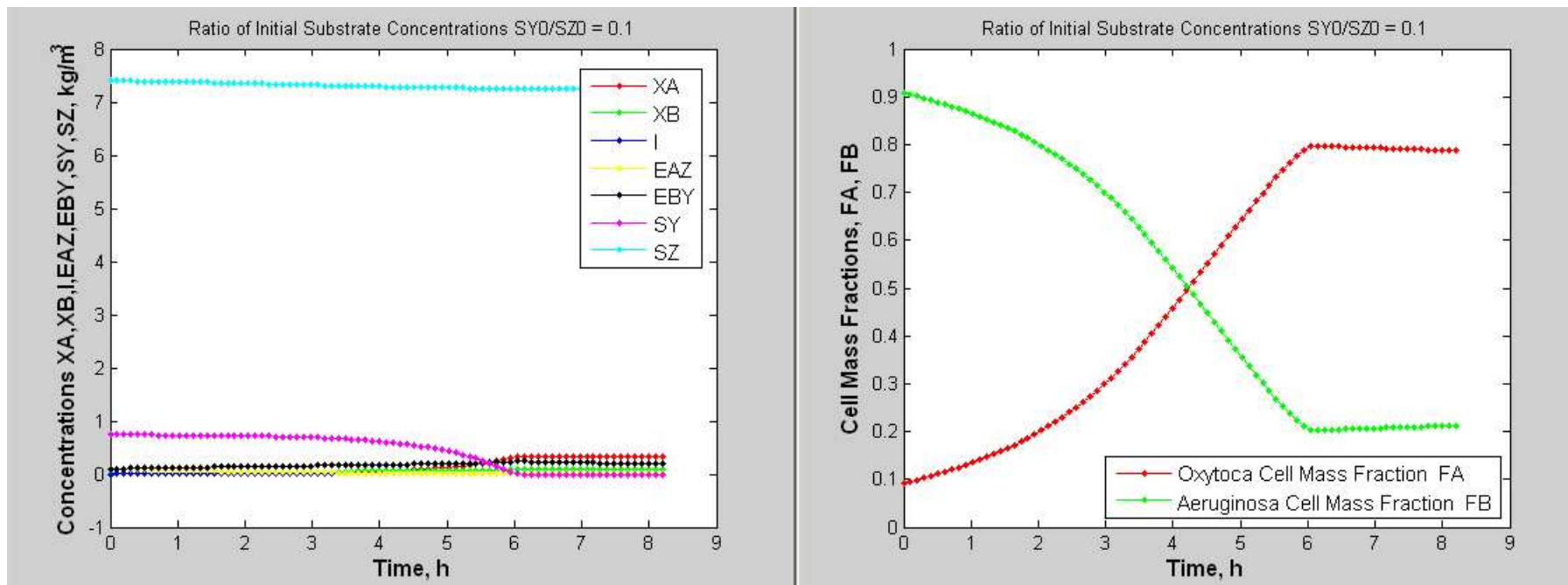




# Exercises



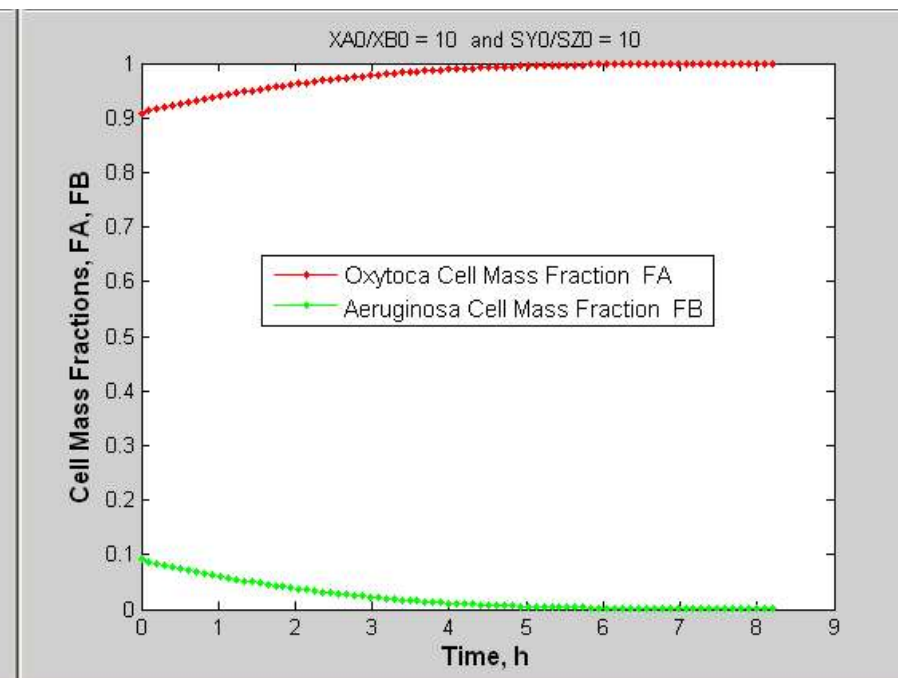
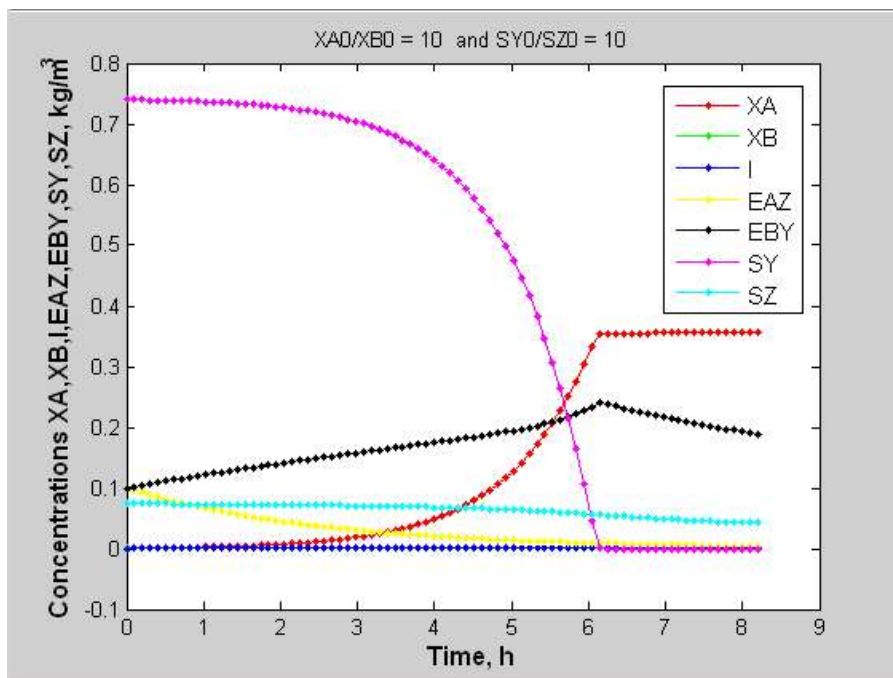
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# Exercises



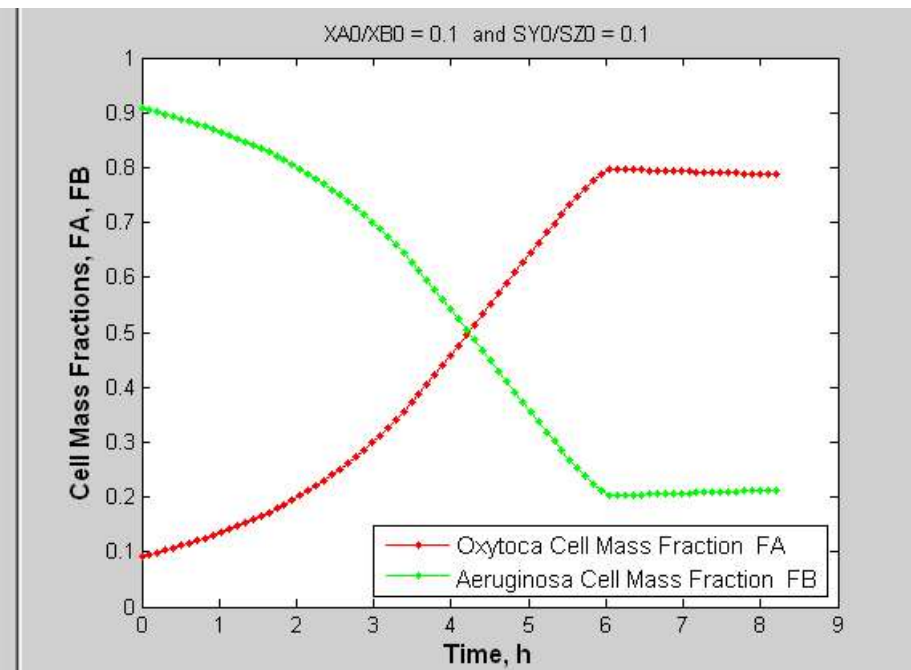
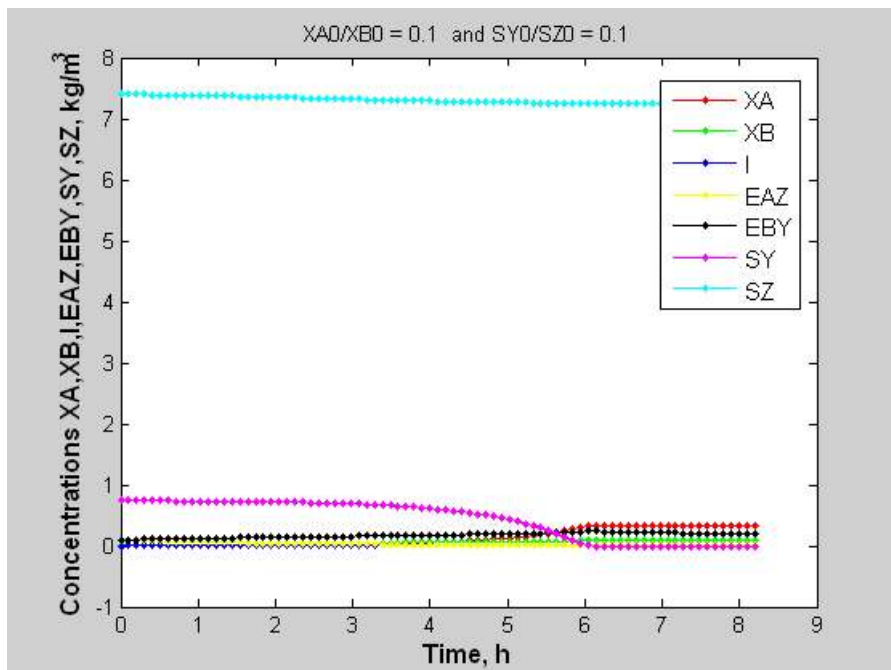
7) Vary the ratios of the initial cell concentrations and the initial substrate concentration.



# Exercises



7) Vary the ratios of the initial cell concentrations and the initial substrate concentration.



# Exercises



8) Alter the model for continuous culture.

