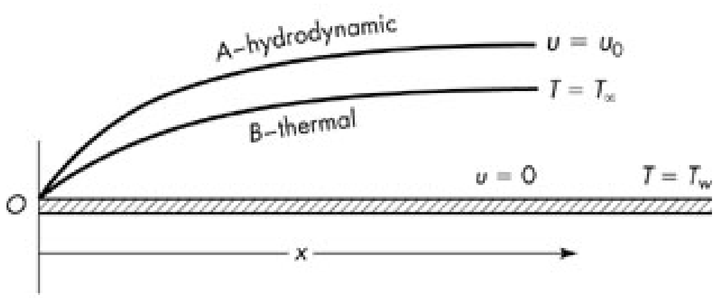
# Chapter 12. Heat Transfer to Fluids without Phase Change

In most cases, frictional heating may be neglected.

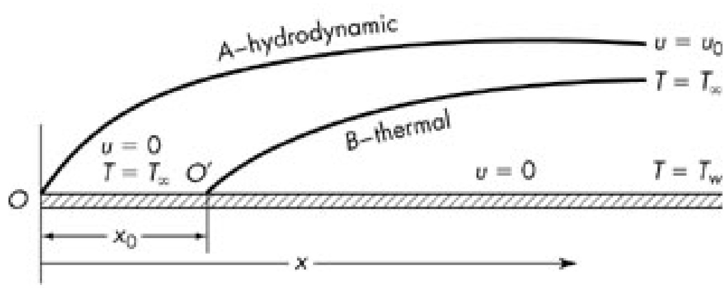
For highly viscous fluids, it may be important (ex. injection molding of polymers).

← Temperature & fluid property variations become large.

*** Thermal & hydrodynamic boundary layers (열경계층 및 유체동력학적 경계층)**




entire plate heated




unheated length = $x_0$

**Fig. 12.1.** Thermal & hydrodynamic boundary layers on flat plate.

- fully developed flow --- parabolic profile
- fully developed temperature profile --- plug (or rod-like) profile

. **Hydrodynamic boundary layer (유체동력학적 경계층):** a boundary layer developing within which the velocity varies from $u = 0$ at the wall to $u = u_0$.

. **Thermal boundary layer (열경계층):** a boundary layer developing within which the temperature varies from $T = T_w$ at the wall to $T = T_\infty$.

Relationship between the thickness of two boundary layers

→ **Prandtl number**

$$\boxed{\Pr \equiv \frac{C_p \mu}{k}} \Leftarrow \frac{\nu}{\alpha}$$

$\nu$ ← kinematic viscosity

$\alpha$ ← thermal diffusivity

: 즉, 운동량확산계수/열확산계수

- $\Pr > 1$ (TBL < HBL)
  for most liquids (2.5 for water, 600 for viscous liquids and concentrated solutions)
- $\Pr = 1$ (TBL = HBL)
  for gases (0.69 for air, 1.06 for steam)
- $\Pr < 1$ (TBL > HBL)
  for liquid metals (0.01 ~ 0.04)

# Heat Transfer by Forced Convection in Laminar Flow

In laminar flow, heat transfer occurs only by conduction.

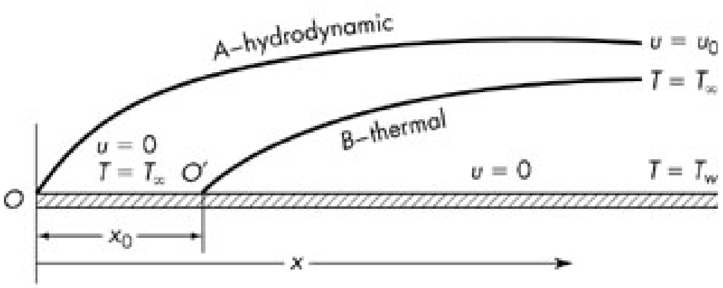
← no eddies to carry heat by convection

Basic assumptions:

. Fluid properties are constant & temperature independent.

. Flow is truly laminar with no eddies or crosscurrents.

## * Laminar flow heat transfer to flat plate




unheated length = $x_0$

local heat-transfer coefficient

: *h* at any distance *x* from the edge

$$Nu_x = \frac{h_x x}{k} = \frac{k}{y}\frac{x}{k} = \frac{x}{y}$$

layer thickness of TBL

local Nusselt number

: the ratio of the distance x to the thickness of the thermal boundary layer

When the plate is heated over the entire length (즉, $x_0 = 0$),

$$\boxed{\mathrm{Nu}_x = 0.332(\mathrm{Pr})^{1/3}(\mathrm{Re}_x)^{1/2}}$$

→ local Reynolds number = $\dfrac{u_0 x \rho}{\mu}$

$x_0$가 있는 경우,

$$\mathrm{Nu}_x = \frac{0.332}{\left(1-(x_0/x)^{3/4}\right)^{1/3}}(\mathrm{Pr})^{1/3}(\mathrm{Re}_x)^{1/2}$$

Average value of Nu over the entire length of the plate $x_1$,

$$h = 2h_{x_1}$$

(Average coefficient is twice the local coefficient at the end of the plate.)

$$\therefore\ \mathrm{Nu} = 0.664(\mathrm{Pr})^{1/3}(\mathrm{Re}_{x_1})^{1/2}$$

## * Laminar flow heat transfer in tubes

### For fully developed flow

Nu inside a pipe,

$$\mathrm{Nu} \equiv \frac{h_i D}{k}$$

$$q = \dot{m} c_p (\bar{T}_b - T_a)$$

$\bar{T}_b$: 출구평균온도, $T_a$: 입구온도

$$\frac{q}{A_i} = h_i \overline{\Delta T_L}$$

$A_i$ → $\pi DL$

$$\overline{\Delta T_L} = \frac{(T_w - T_a) - (T_w - \bar{T}_b)}{\ln\left(\dfrac{T_w - T_a}{T_w - \bar{T}_b}\right)}$$

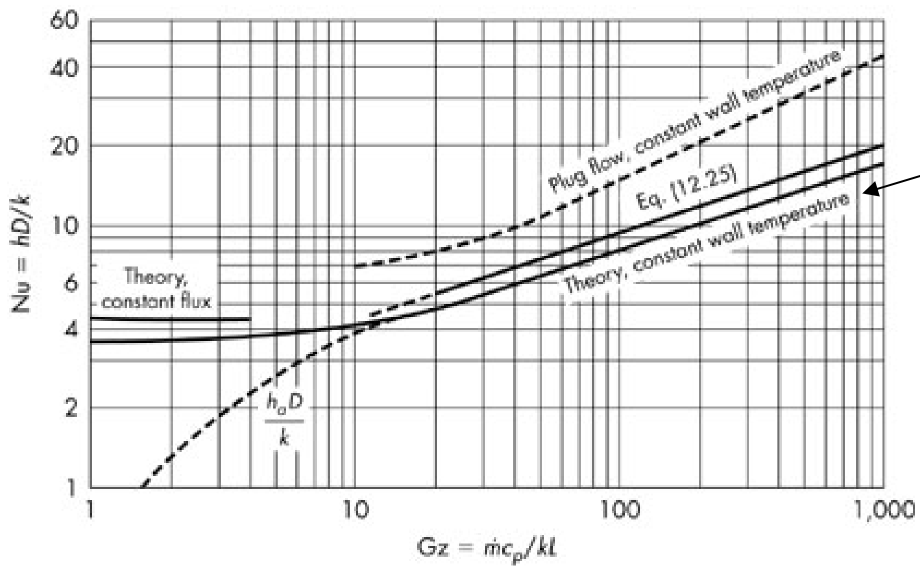
$$h_i = \frac{\dot{m} c_p (\bar{T}_b - T_a)}{\pi D L \overline{\Delta T_L}}$$

$$\Rightarrow \quad h_i = \frac{\dot{m} c_p}{\pi D L} \ln\left(\frac{T_w - T_a}{T_w - \bar{T}_b}\right)$$

$D/k$ 를 양변에 곱

$$\mathrm{Nu} = \frac{\mathrm{Gz}}{\pi} \ln\left(\frac{T_w - T_a}{T_w - \bar{T}_b}\right) \quad \text{--- Eq. (12.24)}$$

$\mathrm{Gz} \equiv \dfrac{\dot{m} c_p}{kL}$ : ***Graetz number*** (the ratio of the time to reach thermal equilibrium perpendicular to the flow direction to the residence time)




Eq. (12.24)의 plot

Eq. (12.18)을 대입

Heat transfer for laminar flow in tubes with a parabolic velocity profile

Gz > 20 인 경우의 실험식: $\mathrm{Nu} \cong 2.0\,\mathrm{Gz}^{1/3}$ --- Eq. (12.25)

## Correction for heating or cooling

← for very viscous liquids w/ large *T* drops

$$\mathrm{Nu} = 2\,\mathrm{Gz}^{1/3}\phi_v \quad \longleftarrow \quad \phi_v \equiv \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$\phi_v$: *viscosity correction factor*

$\mu_w$: viscosity at wall *T*

# Heat Transfer by Forced Convection in Turbulent Flow

Turbulence in tubes ----- Re > 2,100

(엄밀하게는 Re > 4,000인 경우

2,100 < Re < 4,000인 경우는 transition region)

Heat transfer rate in turbulent flow > that in laminar flow

. Empirical correlation for long tubes with sharp-edged entrances:

$$\frac{h_i D}{k} = 0.023\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{c_p \mu}{k}\right)^{1/3}$$

⟶ $G$: mass velocity (= $\bar{V}\rho$ ) or mass flux

$$\rightarrow \mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$ : ***Dittus-Boelter equation***

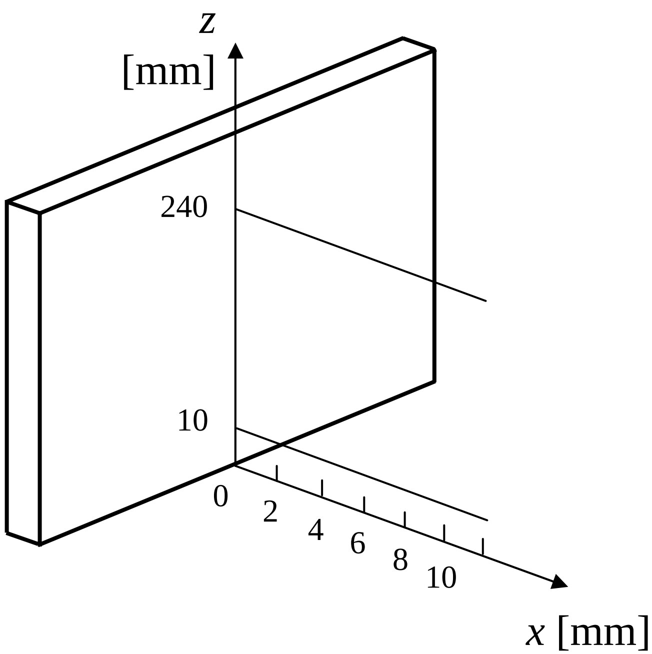
. Modified relationship:

$$\frac{h_i D}{k} = 0.023\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{c_p \mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$\rightarrow \mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\,\phi_v$$ : ***Sieder-Tate equation***

# Natural Convection

## Example of natural convection: A hot, vertical plate in contact with air



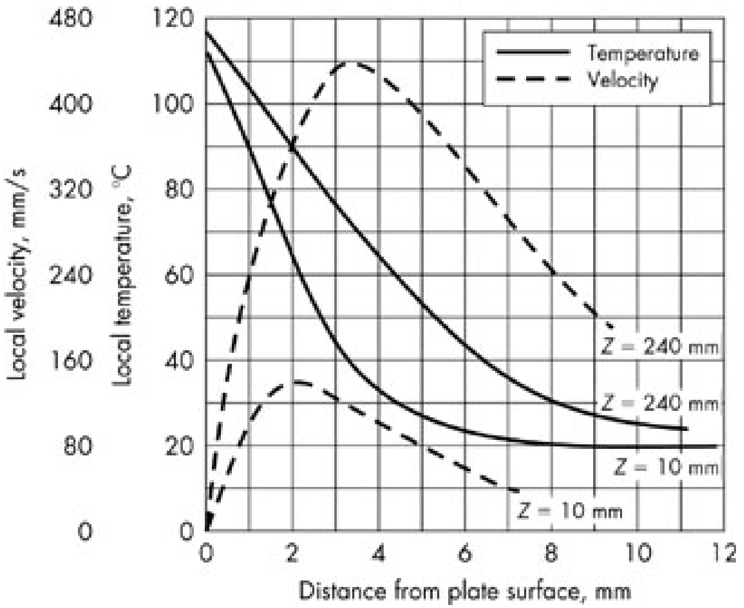
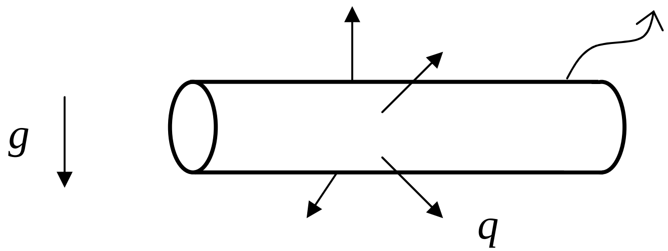




**Fig. 12.7.** Velocity and temperature gradients, natural convection from heated vertical plate.

$z$ > 600 mm: $T$ vs. $x$ curves do no change with further increase in height.

*** Natural convection to air from a hot, horizontal pipe**




natural convection surrounding a hot, horizontal pipe

thermal expansion coefficient

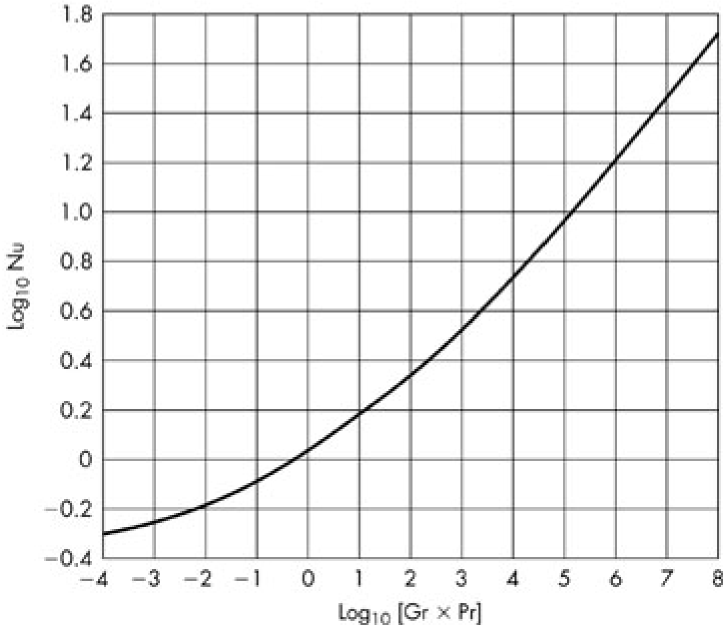
**Grashof number**, $\mathrm{Gr} \equiv \dfrac{D_o^{\,2} \rho^2 \beta g \Delta T_o}{\mu^2}$ ~ 중력의 영향

: the ratio of the buoyant force to the frictional force

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$ $v$: specific volume

For liquids, $\beta = \dfrac{\rho_1 - \rho_2}{\bar{\rho}_a (T_2 - T_1)}$ ← $\bar{\rho}_a = \dfrac{\rho_1 + \rho_2}{2}$

For ideal gases, $\beta = \dfrac{1}{T}$




Heat transfer from a single horizontal cylinders to liquids or gases in natural convection.

Empirical equation:

$$Nu = 0.53(Gr\,Pr)_f^{0.25} \quad \text{for } Gr\,Pr > 10^4$$

*** Natural convection to air from vertical shapes & horizontal planes**

$$Nu_f = b(Gr\,Pr)_f^{\,n}$$

← Constants *b* & *n* are given in Table 12.4.

*f* means that the properties are taken at the mean film between wall and bulk *T*.

Related problems: (Probs.) 12.1, 12.8, 12.17 and 12.18.