Chapter 4. Basic Equations of Fluid Flow

Useful equations in fluid mechanics

: Principles of mass balance (or continuity)

momentum balance { linear / angular }

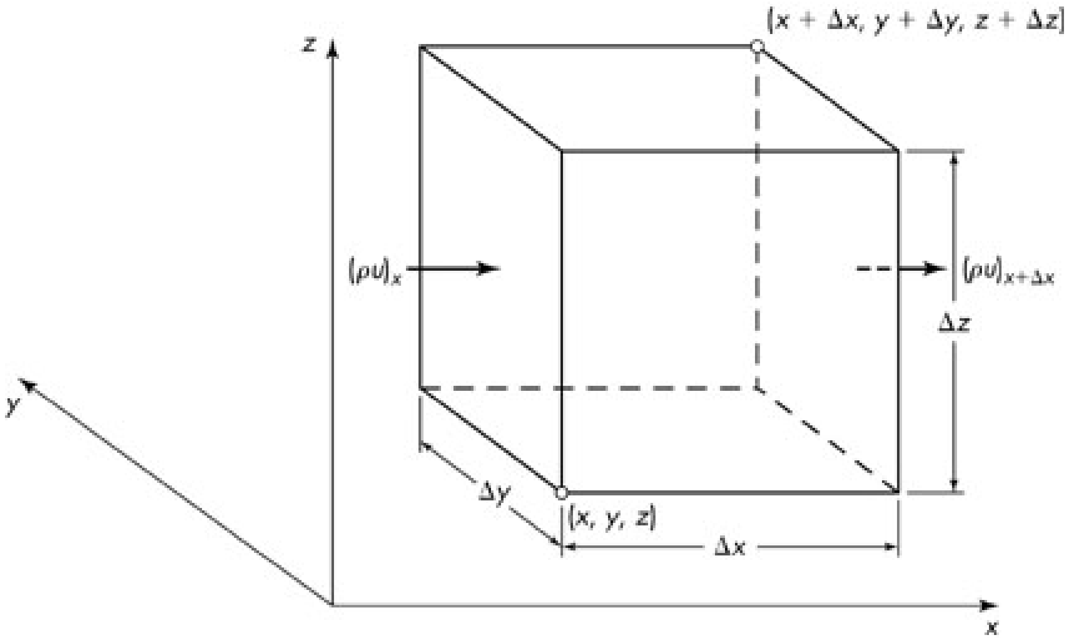
energy balance

→ Differential forms

Integrated forms

Mass Balance in a Flowing Fluid; Continuity (질량수지식, 연속식)

(Rate of mass flow in) – (Rate of mass flow out)
= (Rate of mass accumulation)



$$\mathbf{V} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\left((\rho u)_x \Delta y \Delta z + (\rho v)_y \Delta x \Delta z + (\rho w)_z \Delta x \Delta y\right)$$
$$-\left((\rho u)_{x+\Delta x} \Delta y \Delta z + (\rho v)_{y+\Delta y} \Delta x \Delta z + (\rho w)_{z+\Delta z} \Delta x \Delta y\right)$$
$$= \left(\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t}\right)$$

: 단위 시간당 질량 변화

정리하고 극한을 취하면,

$$\frac{\partial\rho}{\partial t} = -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] = -(\nabla\cdot\rho\mathbf{V}) \qquad \text{------ (4.3)}$$

: Equation of continuity (연속식, 연속방정식)

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z} = -\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -\rho(\nabla\cdot\mathbf{V})$$

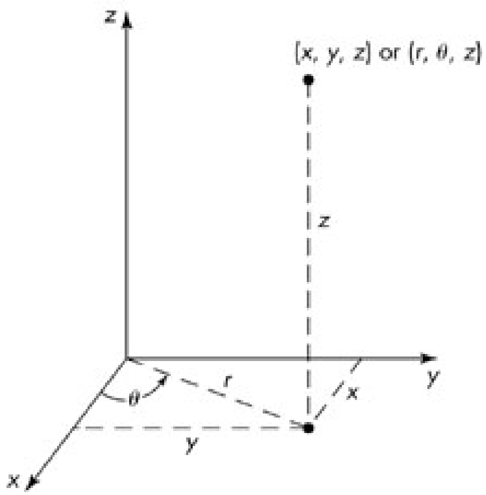
$\frac{D\rho}{Dt}$ -- substantial derivative (실질 미분, 움직임을 따르는 도함수)

* Continuity equation for incompressible fluids

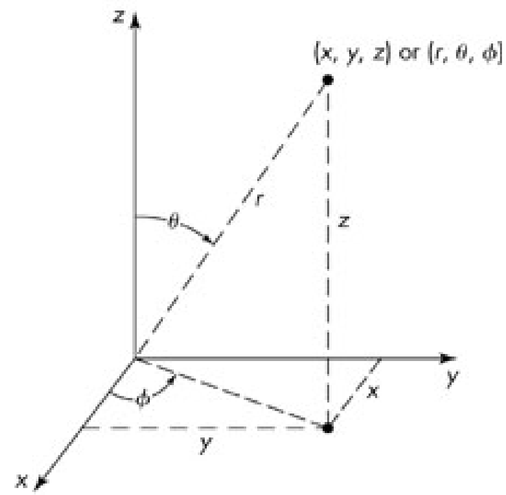
$$\therefore \quad \nabla\cdot\mathbf{V} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \qquad \text{------ (4.6)}$$

직교좌표계(rectangular coordinates, or Cartesian coordinates) 식을

원통좌표계(cylindrical coordinates)와 구면좌표계(spherical coordinates) 식으로 각각 표현하면,



원통좌표계 (r, θ, z)



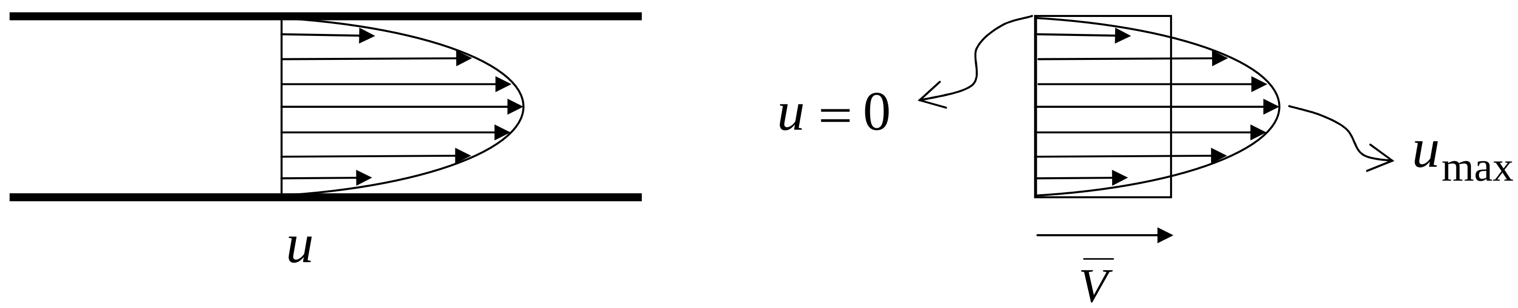
구면좌표계 (r, θ, ϕ)

$$\frac{\partial \rho}{\partial t} = -\left[\frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho r u)}{\partial z}\right] = -(\nabla \cdot \rho \mathbf{V})$$ --- (4.7: 원통좌표계)

$$\frac{\partial \rho}{\partial t} = -\left[\frac{1}{r^2}\frac{\partial(\rho r^2 u_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho u_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(\rho u_\phi)}{\partial \phi}\right] = -(\nabla \cdot \rho \mathbf{V})$$ --- (4.8: 구면좌표계)

Average velocity, $\overline{V}$

Local velocity u varies across a cross-sectional area S.



The mass flow rate through a differential area: $d\dot{m} = \rho u dS$

Total mass flow rate (질량유량) : $\dot{m} = \rho \int u dS$

$\int u dS$ → q, volumetric flow rate (체적유량, $\overline{V}S$)

Average velocity (평균속도) : $$\overline{V} = \frac{\dot{m}}{\rho S} = \frac{1}{S}\int u dS = \frac{q}{S}$$ --- volume flux

* Flow through a pipe

관의 입구와 출구를 a, b로 두면,

$$\dot{m} = \rho_a \overline{V}_a S_a = \rho_b \overline{V}_b S_b = \rho \overline{V} S$$
$$= \rho q$$

For the case through channels of circular cross section:

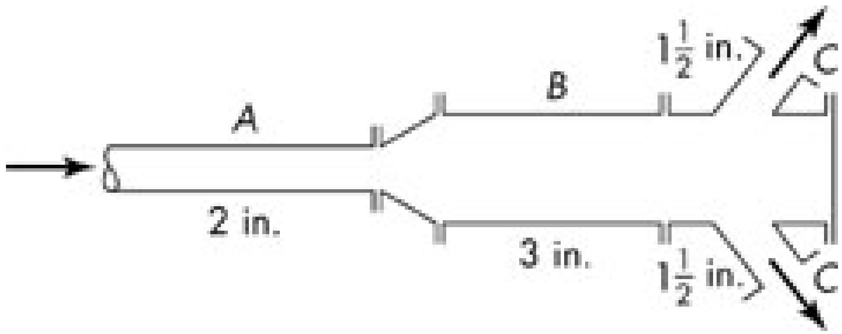
$$\frac{\rho_a \overline{V}_a}{\rho_b \overline{V}_b} = \left(\frac{D_b}{D_a}\right)^2$$

Mass velocity (질량속도), G : $$\overline{V}\rho = \frac{\dot{m}}{S} \equiv G$$ --- 단위시간, 단위면적당 질량.

즉, mass flux

Ex. 4.1) 비중(60°F/60°F)=0.887인 Crude oil(원유). 관 A: 2-in. Schedule 40, 관 B: 3-in. Schedule 40, 관 C는 각각 1.5-in. Schedule 40임. 관 A의 유량(q)이 30 gal/min일 때, 각 관에서의

(a) mass flow rate $\dot{m}$, (b) average velocity $\overline{V}$, and (c) mass velocity G ?



(a) mass flow rate

$$\dot{m} = \rho \overline{V_A} S_A = \rho \overline{V_B} S_B = 2\rho \overline{V_C} S_C = \rho q$$

물의 밀도: Appendix 6 (p. 1093)에서 62.37 lb/ft^3

→ 원유의 밀도 ρ = 0.887x62.37 =55.3 lb/ft^3

$$q = (30\,\text{gal}/\text{min})(60\,\text{min}/1\text{h})(1\text{ft}^3/7.48\text{gal}) = 240.7\text{ft}^3/\text{h}$$

$$\therefore\ \dot{m} = 240.7\text{ft}^3/\text{h} \times 55.3\text{lb}/\text{ft}^3 = 13{,}300\text{lb}/\text{h}$$

$$\Rightarrow\ \dot{m}\text{ at A} = \dot{m}\text{ at B} = 13{,}300\,\text{lb/h}, \quad \dot{m}\text{ at C} = 6{,}650\,\text{lb/h}$$

(b) Average velocity

$$\overline{V}_A = \frac{q}{S_A},\ \overline{V}_B = \frac{q}{S_B},\ \overline{V}_C = \frac{q}{2S_C}$$

각 관의 단면적: Appendix 3 (p. 1090)에서

S_A=0.02330 ft^2 , S_B=0.05130 ft^2 , S_C=0.01414 ft^2

$$\overline{V}_A = \frac{240.7/3{,}600}{0.0233} = 2.87\,\text{ft/s},\ \overline{V}_B = \frac{240.7/3{,}600}{0.0513} = 1.30\,\text{ft/s},\ \overline{V}_C = \frac{240.7/3{,}600}{2\times 0.01414} = 2.36\,\text{ft/s}$$

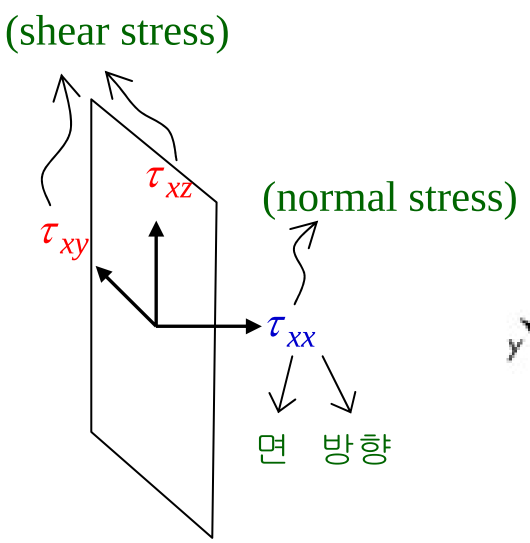
(c) Mass velocity

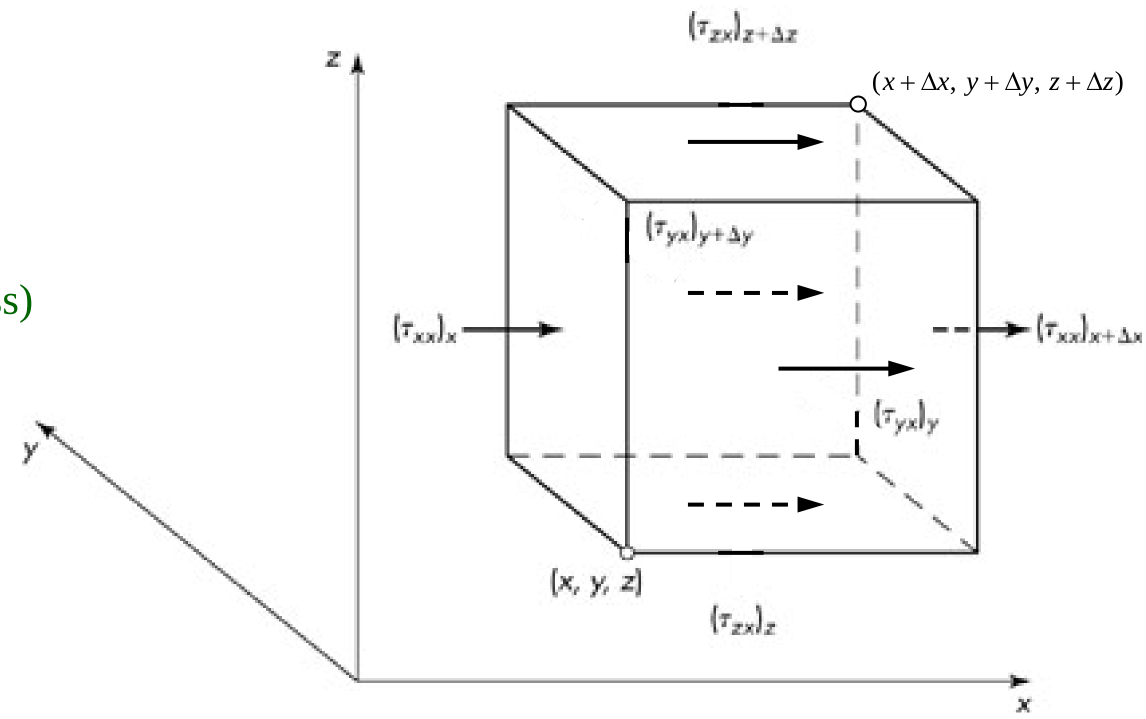
$$G_A = \frac{\dot{m}}{S_A},\ G_B = \frac{\dot{m}}{S_B},\ G_C = \frac{\dot{m}}{2S_C}$$

$$G_A = \frac{13{,}300}{0.0233} = 571{,}000\,\text{lb/ft}^2\text{h},\ G_B = \frac{13{,}300}{0.0513} = 259{,}000\,\text{lb/ft}^2\text{h},\ G_C = \frac{13{,}300}{2\times 0.01414} = 470{,}000\,\text{lb/ft}^2\text{h}$$

Differential Momentum Balance; Equation of Motion (운동방정식)

(Rate of momentum accumulation) = (Rate of momentum entering)
– (Rate of momentum leaving) + (Sum of forces acting on the system) --- (4.16)





x component of momentum

* momentum은 벡터량이므로 (*x*, *y*, *z*) 3 성분이 존재

* Rate of *x* momentum in and out { by convection of bulk flow / by viscous action }

. Convective flow of *x* momentum: Eq. (4.17)

$$\Delta y \Delta z\left[(\rho uu)_x - (\rho uu)_{x+\Delta x}\right] + \Delta x \Delta z\left[(\rho vu)_y - (\rho vu)_{y+\Delta y}\right] + \Delta x \Delta y\left[(\rho wu)_z - (\rho wu)_{z+\Delta z}\right]$$

. Flow of *x* momentum by viscous action: Eq. (4.18)

$$\Delta y \Delta z\left[(\tau_{xx})_x - (\tau_{xx})_{x+\Delta x}\right] + \Delta x \Delta z\left[(\tau_{yx})_y - (\tau_{yx})_{y+\Delta y}\right] + \Delta x \Delta y\left[(\tau_{zx})_z - (\tau_{zx})_{z+\Delta z}\right]$$

* Forces in the *x* direction acting on the system from pressure *p* & gravitational force per mass **g**:

$$\Delta y \Delta z\left(p_x - p_{x+\Delta x}\right) + \rho g_x \Delta x \Delta y \Delta z$$ --- Eq. (4.19)

* Rate of accumulation of *x* momentum:

$$\Delta x \Delta y \Delta z \frac{\partial \rho u}{\partial t}$$

각 항을 수지식 (4.16)에 대입한 후 $\Delta x \Delta y \Delta z$ 로 나누고 극한을 취하면,

$$\frac{\partial}{\partial t}\rho u = -\frac{\partial p}{\partial x} - \left(\frac{\partial}{\partial x}\rho uu + \frac{\partial}{\partial y}\rho vu + \frac{\partial}{\partial z}\rho wu \right) - \left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx} \right) + \rho g_x \quad \text{--- Eq. (4.20)}$$

→ 정리한 후 연속식(4.3)을 적용하면,

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) + \rho g_x \quad \text{--- Eq. (4.21)}$$

y 성분, z 성분에 대해서도 마찬가지로 표현 가능

→ <u>벡터 형태</u>로 표현하면,

$$\rho \frac{DV}{Dt} = -\nabla p - [\nabla \cdot \tau] + \rho \mathbf{g} \quad \text{--- Eq. (4.22)}$$

Navier-Stokes equations

: equation of motion for a fluid of constant density and viscosity

이 경우 stress와 velocity gradient와의 관계는 다음과 같으므로,

$$\tau_{xx} = -2\mu\frac{\partial u}{\partial x}, \quad \tau_{yy} = -2\mu\frac{\partial v}{\partial y}, \quad \tau_{zz} = -2\mu\frac{\partial w}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = -\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad \tau_{yz} = \tau_{zy} = -\mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \quad \tau_{xz} = \tau_{zx} = -\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

→ 운동방정식(4. 21)에 대입하여 정리하면 x 성분은

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x \quad \text{--- Eq. (4.29)}$$

y 성분, z 성분에 대해서도 같은 방법으로 표현하면

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g_y \quad \text{--- Eq. (4.30)}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho g_z \quad \text{--- Eq. (4.31)}$$

Navier-Stokes 방정식을 벡터 형태로 표현하면,

$$\rho \frac{DV}{Dt} = -\nabla p + \mu \nabla^2 V + \rho \mathbf{g}$$ --- Eq. (4.32)

Navier-Stokes equations in cylindrical coordinates:

Eqs. (4.33) – (4.35)

Navier-Stokes equations in spherical coordinates:

Eqs. (4. 36) – (4.38)

Euler equation

: equation of motion for a fluid of constant density and zero viscosity

(즉, potential flow에 적용할 수 있는 운동방정식)

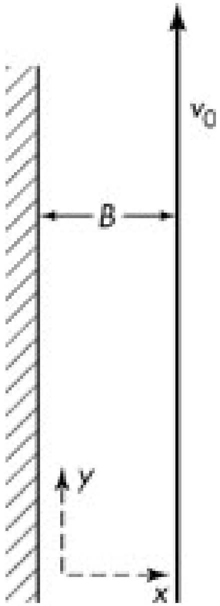
$$\rho \frac{DV}{Dt} = -\nabla p + \rho \mathbf{g}$$ --- Eq. (4.40)

Ex. 4.3) A Newtonian fluid confined between two parallel vertical plates

left plate – stationary

right plate – moving upward at v_0

Laminar flow일 때 steady-state velocity profile에 대한 식은 ?



(풀이) steady state → $\dfrac{\partial v}{\partial t} = 0$

y 방향의 흐름만 존재 → $u = w = 0$

$$v = f(x) \quad \rightarrow \quad \frac{\partial v}{\partial y} = \frac{\partial v}{\partial z} = 0$$

연속식(continuity equation)으로부터,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \rightarrow \quad \frac{\partial v}{\partial y} = 0$$

Navier-Stokes equation으로부터,

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g_y \qquad \text{--- Eq. (4.30)}$$

$\rho g_y \rightarrow -\rho g$

$$\therefore \ \mu\frac{d^2 v}{dx^2} - \frac{dp}{dy} - \rho g = 0 \qquad \text{--- Eq. (4.41)}$$

Eq. (4.29)와 Eq. (4.31)로부터 $\frac{\partial p}{\partial x}=0,\ \frac{\partial p}{\partial z}=0$

⟶ $\therefore$ p는 x, z의 함수 아님.

$$\mu\frac{d^2v}{dx^2}=\frac{dp}{dy}+\rho g \quad (=\text{const.})$$

v는 x만의 함수 / p는 y만의 함수 / 상수

Eq. (4.41)을 적분하면, $\frac{dv}{dx}-\frac{x}{\mu}\left(\frac{dp}{dy}+\rho g\right)=C_1$

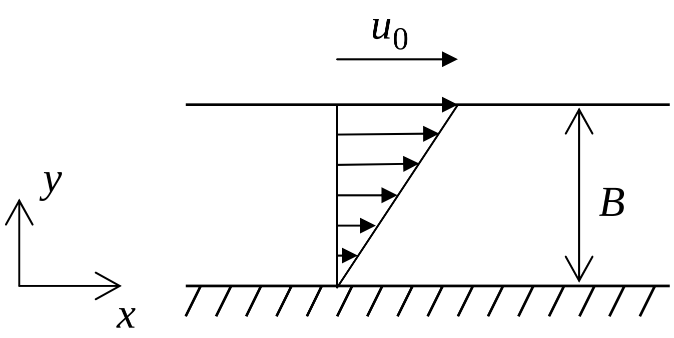
한번 더 적분하면, $v-\frac{x^2}{2\mu}\left(\frac{dp}{dy}+\rho g\right)=C_1x+C_2$

경계조건(boundary conditions) $v=0$ at $x=0$ 을 적용하면

$v=v_0$ at $x=B$

$$\therefore\ v=-\frac{1}{2\mu}\left(\frac{dp}{dy}+\rho g\right)\left(Bx-x^2\right)+v_0\frac{x}{B}$$

* **Couette flow** (Plane Couette flow)



When the plates are horizontal
(gravitational force -- negligible)

$$g = 0$$

$$\frac{dp}{dx} = 0 \quad (\because \text{ 수평면에서 압력차이가 있으면 흐름이 생기므로})$$

N-S equation의 x 성분 식 (4.29)에서

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$

연속식에서 $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \therefore \frac{\partial u}{\partial x} = 0$

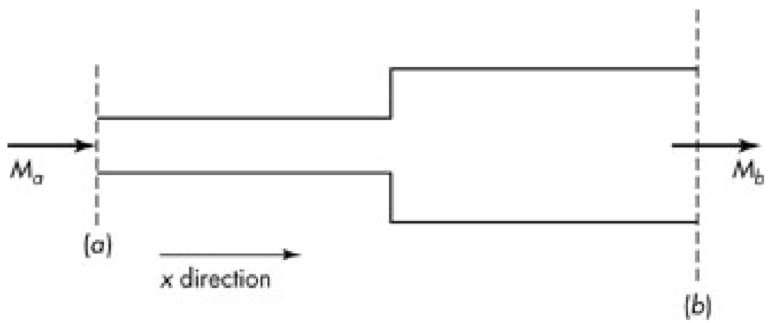
$$\therefore \frac{\partial^2 u}{\partial y^2} = 0 \rightarrow u = C_1 y + C_2 \rightarrow u = u_0 \frac{y}{B}$$

경계조건 대입

$$\therefore \mu = \frac{\tau_s}{du/dy} = \frac{F_s}{A}\frac{B}{u_0} \qquad \text{--- Eq. (4.45)}$$

Macroscopic Momentum Balances (거시적 운동량 수지)

(Rate of momentum accumulation) = (Rate of momentum entering) – (Rate of momentum leaving) + (Sum of forces acting on the system) --- (4.16)



$\dot{M}$: momentum flow rate $(=\dot{m}u)$

Steady, unidirectional flow → accumulation=0

$$\therefore\ 0=\dot{M}_a-\dot{M}_b+\sum F$$ --- Eq. (4.46) ← from Eq. (4.16)

*** Momentum correction factor** (운동량 보정인자), β

For an area dS, $\dfrac{d\dot{M}}{dS}=\dfrac{d(\dot{m}u)}{dS}=(\rho u)u=\rho u^2$ ← $d\dot{m}=\rho u dS$

$\rightarrow\ \dot{M}=\int_S \rho u^2 dS=\rho\int_S u^2 dS$ for all const. ρ

$\therefore$ momentum flux of whole stream: $\dfrac{\dot{M}}{S}=\dfrac{\rho\int_S u^2 dS}{S}$

Momentum correction factor β is defined by

$$\beta \equiv \frac{\dot{M}/S}{\rho \overline{V}^2} \longleftarrow \frac{\text{momentum flux of whole stream}}{\text{momentum flux calculated by average velocity}}$$

$$\therefore\ \beta = \frac{1}{S}\int_S \left(\frac{u}{\overline{V}}\right)^2 dS \quad \text{or} \quad \beta = \frac{\int u^2 dS}{\overline{V}^2 S} \qquad \text{--- (4.50)}$$

따라서 Eq. (4.46)을 바꿔 표현하면 $\sum F = \dot{m}\left(\beta_b \overline{V}_b - \beta_a \overline{V}_a\right)$

All force components acting on the fluid

1) pressure change in the direction of flow
2) shear stress at the boundary or external force on wall
3) gravitational force

$$\therefore\ \sum F = p_a S_a - p_b S_b + F_w - F_g$$

$\sum F = \dot{M}_a - \dot{M}_b$

F_w: net force of channel wall on fluid

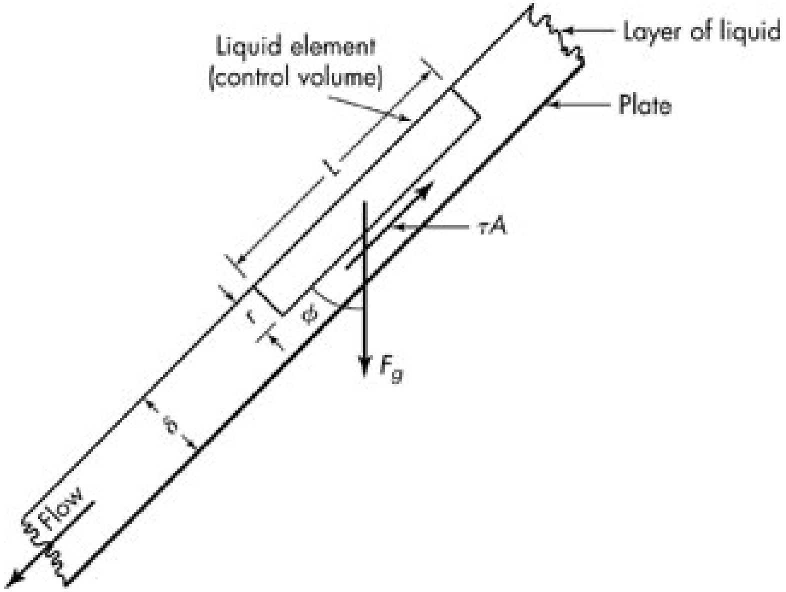
F_g: gravitational force (+ for upward direction)

* Layer flow with free surface

경사면을 흐르는 흐름에서의 층 두께 해석

(가정)

. Newtonian liquid
. Laminar flow
. Steady state
. Fully developed flow
. Film thickness is constant (no ripples)



The breadth of the layer: *b*

Forces acting on the control volume:

- the pressure forces on the ends
- the shear forces on the upper and lower faces
- the force of gravity

$$\sum F_i = 0 \quad \Rightarrow \quad F_g \cos\phi - \tau A = 0 \qquad \text{--- Eq. (4.53)}$$

$$F_g \rightarrow \rho r b L g, \quad A \rightarrow bL$$

Newtonian liquid의 laminar flow 이므로 $\tau = -\mu \dfrac{du}{dr}$

따라서 Eq. (4.53)은 $-\mu \dfrac{du}{dr} = g\rho r \cos\phi$

정리하고 적분하면

$$\int_0^u du = -\frac{g\rho\cos\phi}{\mu}\int_\delta^r r\,dr$$

$$u = \frac{\rho g\cos\phi}{2\mu}\left(\delta^2 - r^2\right) \qquad \text{--- parabolic}$$

Total mass flow rate:

$$\dot{m} = \int_0^\delta \rho u b\,dr = \frac{\delta^3\rho^2 b\,g\cos\phi}{3\mu}$$

Therefore, thickness of the layer:

$$\therefore \quad \delta = \left(\frac{3\mu\dot{m}}{b\rho^2 g\cos\phi}\right)^{1/3}$$

Mechanical Energy Equation (Bernoulli equation)

← derived by forming the scalar product of **V** with equation of motion

Consider the unidirectional flows of const. ρ & $\mu = 0$ (→ Euler equation)

* Energy equation for potential flow (Bernoulli equation w/o friction)

x component of Euler equation (4.40):

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x$$

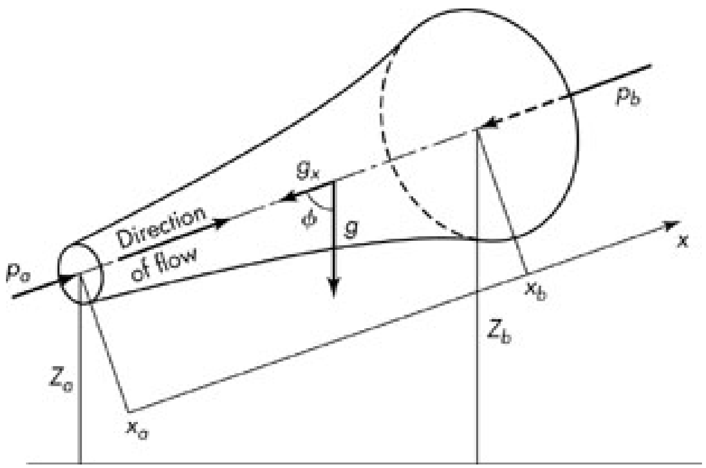
By multiplying the velocity u

$$\rho u\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -u\frac{\partial p}{\partial x} + \rho u g_x$$

or $$\rho\left(\frac{\partial (u^2/2)}{\partial t} + u\frac{\partial (u^2/2)}{\partial x}\right) = -u\frac{\partial p}{\partial x} + \rho u g_x$$ --- Eq. (4.64)

steady flow

* Consider a volume element of a stream tube



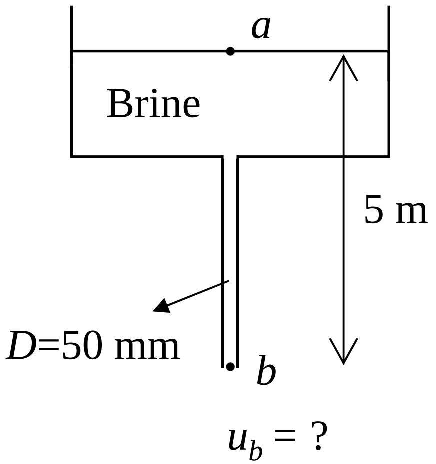
$$g_x = -g\cos\phi,$$
$$Z = Z_a + x\cos\phi,$$
$$dZ = \cos\phi\, dx$$
$$\rightarrow \cos\phi = \frac{dZ}{dx}$$

$$\therefore \frac{d(u^2/2)}{dx} + \frac{1}{\rho}\frac{dp}{dx} + g\frac{dZ}{dx} = 0$$ ← from Eq. (4.64)

Integrating between points *a* & *b*

$$\frac{p_a}{\rho} + gZ_a + \frac{u_a^2}{2} = \frac{p_b}{\rho} + gZ_b + \frac{u_b^2}{2}$$ --- Eq. (4.67)

: Bernoulli equation w/o friction (단위: 단위질량당 에너지)



Ex. 4.4) Brine (염수) in large open tank

비중 (60 °F/°F)=1.15

배출관의 직경: 50 mm

관 출구와 brine 표면과의 거리: 5 m

Friction along streamline: negligible

배출관 출구에서의 속도는 ?

(풀이) 염수 표면을 *a*, 관 출구를 *b*로 두고 마찰이 없는 경우의 Bernoulli 식(4.67)을 사용:

$$\frac{p_a}{\rho} + gZ_a + \frac{u_a^2}{2} = \frac{p_b}{\rho} + gZ_b + \frac{u_b^2}{2}$$

이 식에 조건 $p_a = p_b,\quad u_a = 0,\quad Z_a - Z_b = 5\text{m}$ 을 적용하면

$$5g = \frac{1}{2}u_b^2 \qquad \therefore u_b = \sqrt{5\times 2\times 9.80} = 9.90\,\text{m/s}$$

← 이 경우 속도는 밀도 및 관의 직경과 무관 (potential flow로 가정했으므로)

* **Two modifications** for practical situations

i) correction of kinetic energy term

ii) correction of fluid friction

i) Correction of kinetic energy term

. Kinetic energy flow rate (운동에너지 유량), $\dot{E}_k$

$$d\dot{E}_k = (\rho u dS)\frac{u^2}{2} = \frac{\rho u^3 dS}{2}$$

$\rho u dS \rightarrow d\dot{m}$

적분하면 $\dot{E}_k = \frac{\rho}{2}\int u^3 dS$ $\leftarrow$ assume $\rho =$const.

$$\frac{\dot{E}_k}{\dot{m}} = \frac{\rho\frac{1}{2}\int u^3 dS}{\rho\int u dS} = \frac{\frac{1}{2}\int u^3 dS}{\bar{V} S}$$

$\frac{\dot{E}_k}{\dot{m}} \rightarrow \frac{u^2}{2}$

$\bar{V} \rightarrow \bar{V} = \frac{1}{S}\int u dS$

. Kinetic energy correction factor (운동에너지 보정인자), α

$$\frac{u^2}{2} \equiv \frac{\alpha \bar{V}^2}{2} = \frac{\dot{E}_k}{\dot{m}} = \frac{\int u^3 dS}{2\bar{V}S}$$

$$\alpha = \frac{\int u^3 dS}{\bar{V}^3 S} \quad \text{or} \quad \alpha = \frac{1}{S}\int \left(\frac{u}{\bar{V}}\right)^3 dS$$

→ 만일 α를 알면 국부속도 u 대신 평균속도 $\bar{V}$를 사용하여 운동에너지 계산 가능

(즉, $\frac{u^2}{2}$ 대신 $\frac{\alpha \bar{V}^2}{2}$를 사용)

일반적으로 $\alpha = 2.0$ for laminar flow

$\cong 1.05$ for highly turbulent flow

ii) Correction of fluid friction

. Fluid friction (유체마찰), h_f (≥ 0 always)

: all the friction generated between a and b

skin friction (표면마찰): generated in boundary layer (Fig. 3.9a)

form friction (형태마찰): generated in wakes (Fig. 3.9b)

. Pump work (펌프 일)

W_p : work done by the pump per unit mass of fluid

h_{fp} : total friction in the pump " "

Net work to the fluid $= W_p - h_{fp}$

$\equiv \eta W_p \quad (\eta < 1)$

η → pump efficiency

or $\eta = \dfrac{W_p - h_{fp}}{W_p}$

*** Final form of Bernoulli equation**

$$\frac{p_a}{\rho} + gZ_a + \frac{\alpha_a \bar{V}_a^{\,2}}{2} + \eta W_p = \frac{p_b}{\rho} + gZ_b + \frac{\alpha_b \bar{V}_b^{\,2}}{2} + h_f$$ --- Eq. (4.74)

(Ex. 4.5a) and (Ex. 4.6): 책의 자세한 풀이과정대로 꼭 풀어 보기 바람.

Related problems:

(Probs.) 4.1, 4.3, 4.4 and 4.8