Chapter 7. Flow Past Immersed Objects

Drag and Drag Coefficients (항력 및 항력계수)

*** Drag**

: The force in the direction of flow exerted by the fluid on the solid
(흐름방향에서 유체가 고체에 미치는 힘)

. wall drag: drag from wall shear (벽항력)

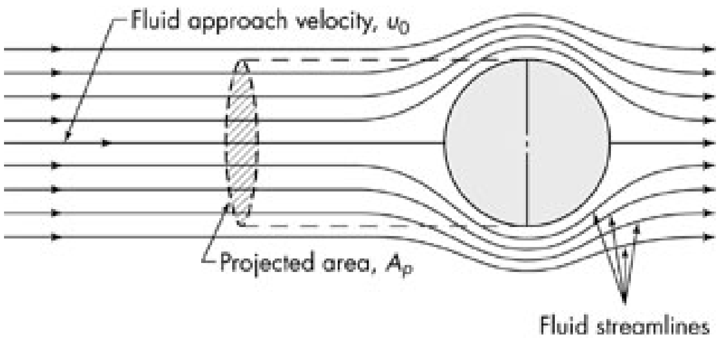
. form drag: “ pressure (형태항력)

*** Drag coefficient, C_D**

↑—— for immersed solids

cf.) friction factor, f

↑—— for flow through conduits



$$\boxed{C_D \equiv \frac{F_D / A_p}{\rho u_0^{\,2} / 2}}$$

F_D: total drag
A_p: projected area

--- Eq. (7.1)

approaching velocity ——↑

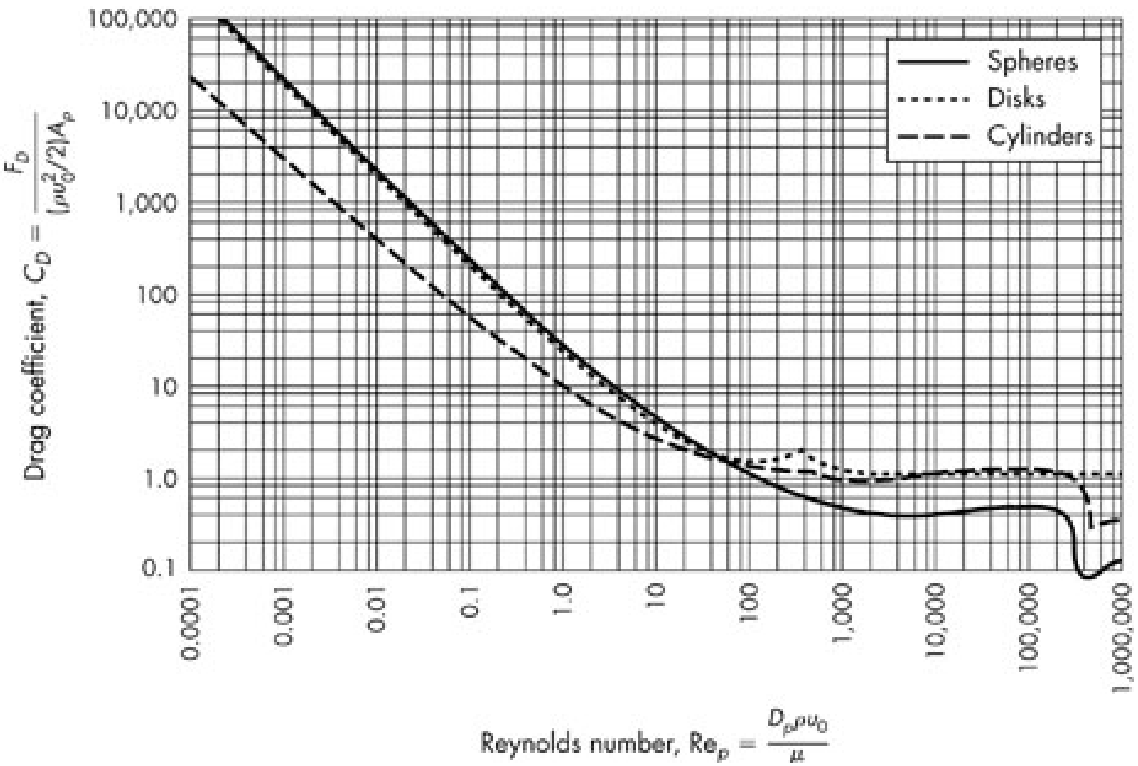
cf.) $f \equiv \dfrac{\tau_w}{\rho \bar{V}^2 / 2}$

Particle Reynolds number (Re for a particle in a fluid):

$$\boxed{\mathrm{Re}_p \equiv \frac{\rho D_p u_0}{\mu}} \qquad \text{--- Eq. (7.2)}$$

D_p: characteristic length (특성길이, 보통 particle diameter)

*** Drag coefficient of typical shapes**



C_D vs. Re_p
for spheres, disks
and cylinders

. For low Re ($Re_p << 1$)

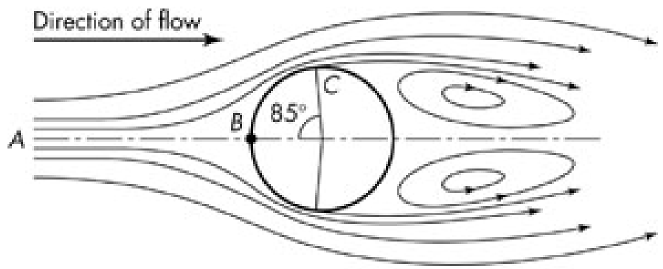
→ Creeping flow (미동흐름)

$$F_D = 3\pi \mu u_0 D_p$$ for a sphere (Stokes' law) --- Eq. (7.3)

$$\longrightarrow \boxed{C_D \equiv \frac{24}{Re_p}}$$ --- Eq. (7.4)

. For $Re_p > 20$

→ Separation occurs

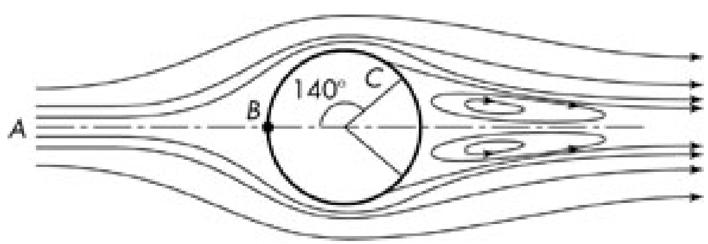


laminar flow in boundary layer ($Re_p=10^5$)

. For high Re ($10^3 < Re_p < 3\times10^5$)

→ $C_D \cong 0.40 - 0.45$

Front boundary layer is still laminar



turbulent flow in boundary layer ($Re_p=3\times10^5$)
(B: stagnation pt., C: separation pt.)

. For high Re ($Re_p > 3\times10^5$)

→ $C_D \cong 0.10$

Front boundary layer becomes turbulent

cf.) $Re_p= 3\times10^5$: *critical Reynolds number* for drag

. C_D vs. Re_p for a cylinder

→ similar to that for a sphere, but C_D is not exactly proportional to Re_p^{-1}

. C_D vs. Re_p for a disk

→ does not show C_D drop at $Re_{p,\,crit}$ ("bluff body")

C_D for a disk is nearly 1 at Re_p > 2,000.

* Form drag and streamlining

Minimizing the from drag

→ streamlined body (ex., airfoil)

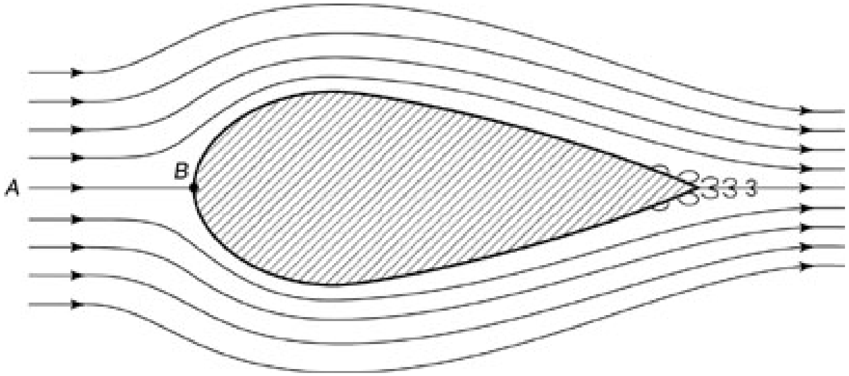
Pressure at stagnation point, p_s :

$$\frac{p_s - p_0}{\rho} = \frac{u_0^{\,2}}{2}$$

← by Bernoulli equation (from A to B)

u_0: velocity of undisturbed fluid

p_0: pressure in undisturbed fluid



B: stagnation point

Flow through Beds of Solids

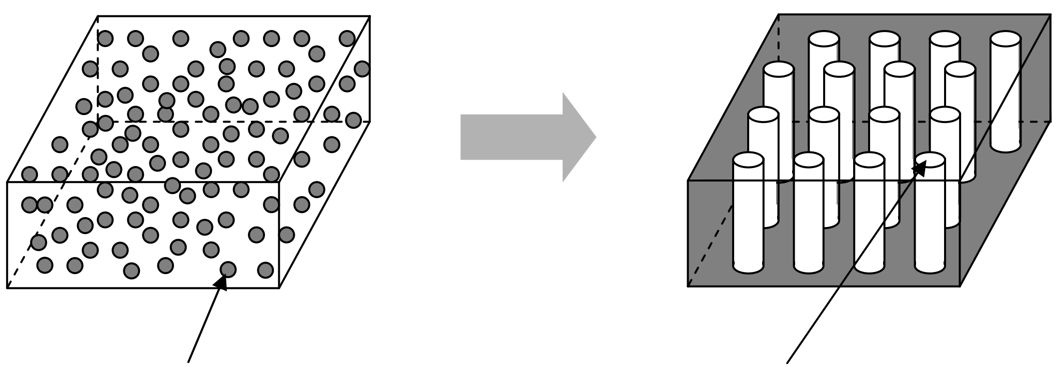
: 정지 고체 입자상을 통과하여 흐르는 단일 유체상의 흐름을 대상으로 함.

(관련 공정: filtration, flow of liquid and gas through packed towers,

ion-exchange reactor, catalytic reactor)

"actual channels" (irregular, tortuous channels) → "uniform circular channels"

S_o
cross-sectional area of the bed



n channels
channel length L

D_p
particle diameter

D_{eq}
equivalent channel diameter

Surface-volume ratio for particles

For spheres, $\dfrac{s_p}{v_p} = \dfrac{6}{D_p}$ $\leftarrow$ $\begin{cases} s_p = 4\pi R_p^{\,2} = \pi D_p^{\,2} \\ v_p = \dfrac{4}{3}\pi R_p^{\,3} = \dfrac{\pi}{6} D_p^{\,3} \end{cases}$

For other spheres, $\boxed{\dfrac{s_p}{v_p} = \dfrac{6}{\Phi_s D_p}}$ or $\Phi_s = \dfrac{6/D_p}{s_p/v_p}$

sphericity (구형도) See Table 7.1

Porosity (공극률), or void fraction: ε

$\longrightarrow$ particle volume fraction in the bed: $1-\varepsilon$

Total surface area: $n\pi D_{eq} L = \underline{S_0 L(1-\varepsilon)} \dfrac{6}{\Phi_s D_p}$

$\rightarrow$ total particle volume

Void volume in the bed: $S_0 L\varepsilon = \dfrac{1}{4} n\pi D_{eq}^{\,2} L$

Combining the above two equations,

$$\boxed{D_{eq} = \frac{2}{3}\Phi_s D_p \frac{\varepsilon}{1-\varepsilon}}$$

ex.) $\varepsilon = 0.4, \quad D_{eq} = 0.44\Phi_s D_p \quad \therefore D_{eq} \cong \frac{1}{2} D_p$

Average velocity in the channels: $\boxed{\bar{V} = \frac{\bar{V}_0}{\varepsilon}}$ → superficial (or empty-tower) velocity

. Pressure drop at low Re_p (< 1) : $\underbrace{\frac{\Delta p}{L} = \frac{32\bar{V}\mu}{D^2}}_{\text{Hagen-Poiseuille equation}} = \frac{32\lambda_1 \bar{V}_0 \mu}{\frac{4}{9}\varepsilon \Phi_s{}^2 D_p{}^2} \frac{(1-\varepsilon)^2}{\varepsilon^2}$

λ_1 : correction factor
"channels are tortuous"
$(\lambda_1 = 2.1)$

→ $$\boxed{\frac{\Delta p}{L} = \frac{150\bar{V}_0\mu}{\Phi_s{}^2 D_p{}^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}}$$: Kozeny-Carman equation

→ Darcy's law를 따름($q \propto \frac{\Delta p}{\mu}$)

. Pressure drop at high Re_p (> 1000) :

$$\boxed{\frac{\Delta p}{L} = \frac{1.75\rho \bar{V}_0^{\,2}}{\Phi_s D_p} \frac{(1-\varepsilon)}{\varepsilon^3}}$$: Burke-Plummer equation

An equation covering the entire range of the flow rates

← Viscous losses & kinetic energy losses are additive.

$$\boxed{\frac{\Delta p}{L} = \frac{150 \bar{V}_0 \mu}{\Phi_s^{\,2} D_p^{\,2}} \frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho \bar{V}_0^{\,2}}{\Phi_s D_p} \frac{(1-\varepsilon)}{\varepsilon^3}}$$ --- Eq. (7.22)

: Ergun equation

Motion of Particles through Fluids

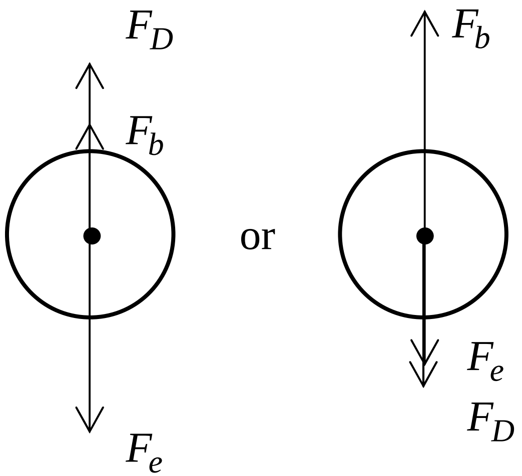
* Mechanism of particle motion

Three forces action on a particle through a fluid:

1) external force (gravitational or centrifugal), F_e

2) buoyant force, F_b

3) drag force, F_D



부력은 외력의 반대방향으로 작용

항력은 이동방향과 반대방향으로 작용

The resultant force (F) on the particle: $F_e - F_b - F_D$

The acceleration of the particle: $\dfrac{du}{dt}$

$F = m\dfrac{du}{dt}$ 에 대입하면,

$$\boxed{m\frac{du}{dt} = F_e - F_b - F_D}$$

$F_e = \boxed{ma_e}$, $F_b = \boxed{\dfrac{m\rho a_e}{\rho_p}}$, $F_D = \boxed{\dfrac{C_D u_0^{\,2} \rho A_p}{2}}$

$\dfrac{du}{dt}$: acceleration of particle

F_b: (Archimedes's principle)

$$\begin{cases} A_p = \text{projected area of particle} \\ u_0 = u \end{cases}$$

Motion from gravitational force

If the external force is gravity, a_e is g.

$$\frac{du}{dt} = g\frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} \qquad \text{--- Eq. (7.30)}$$

*** Terminal velocity (종말속도)**

: 중력 하의 유체 속에서 낙하하는 입자는 속도증가에 따라 drag이 증가

→ 가속도는 시간에 따라 감소하게 되고 0에 접근하여 일정 속도에 이르게 됨

→ 이는 maximum attainable velocity이며 이를 terminal velocity u_t 라 함.

식 (7.30)에서 $du/dt = 0$ 으로 두면,

$$\boxed{u_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A_p \rho_p C_D \rho}}} \qquad \text{--- Eq. (7.33)}$$

Motion of spherical particles

$$m = \frac{1}{6}\pi D_p{}^3 \rho_p \qquad A_p = \frac{1}{4}\pi D_p{}^2 \quad \text{이므로}$$

$$\therefore\ u_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D \rho}} \qquad \text{--- Eq. (7.37)}$$

At low Re_p (<< 1) ← Stokes' law range

$$C_D = \frac{24}{\text{Re}_p} \longleftarrow F_D = 3\pi\mu u_t D_p$$

$$\boxed{u_t = \frac{g D_p^{\,2}(\rho_p - \rho)}{18\mu}}$$ Stokes' law --- Eq. (7.40)

For 1,000 < Re_p < 200,000 ← Newton's law range

$$C_D = 0.44 \qquad F_D = 0.055\pi D_p^{\,2} u_t^{\,2} \rho$$

$$\boxed{u_t = 1.75\sqrt{\frac{g(\rho_p - \rho)D_p}{\rho}}}$$ Newton's law --- Eq. (7.43)

* Settling and rise of bubbles and drops

Drops of liquid or bubbles of gas

→ change their shapes

Form drag → flattens drops

Surface tension → keeps spherical shapes

Drop size ↓ → surface energy per volume ↑

Drop or bubble < 0.5 mm → nearly spherical

∴ C_D & u_t are about the same as solid sphere,

but not exactly the same.

(∵ circulation of fluid inside a drop)

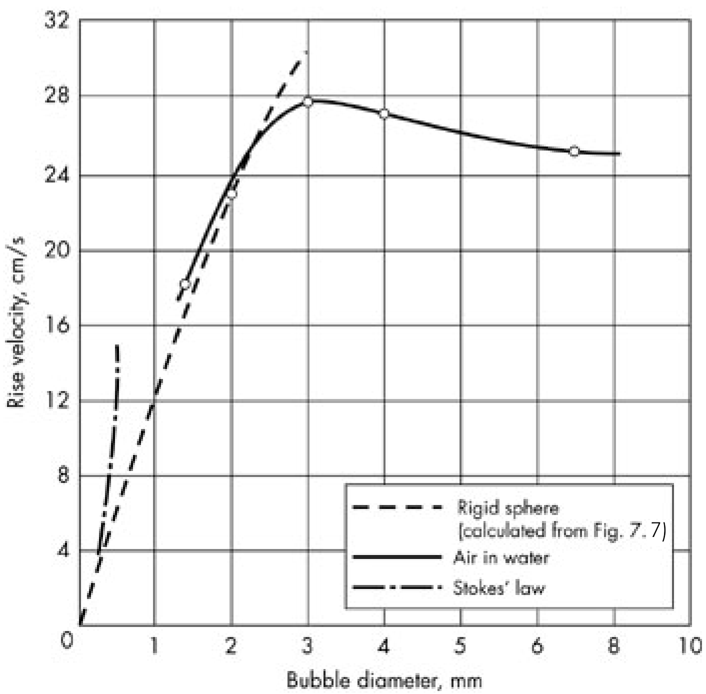
→ Total drag is somewhat less than

a rigid sphere

Large drops become flattened ellipsoids

or may oscillate from oblate to prolate form.

Drops larger than about 10 mm in diameter usually break apart.



Rise velocity of air bubbles in water

Fluidization (유동화)

Consider a fluid (liquid or gas) passing up through a bed of solid particles.

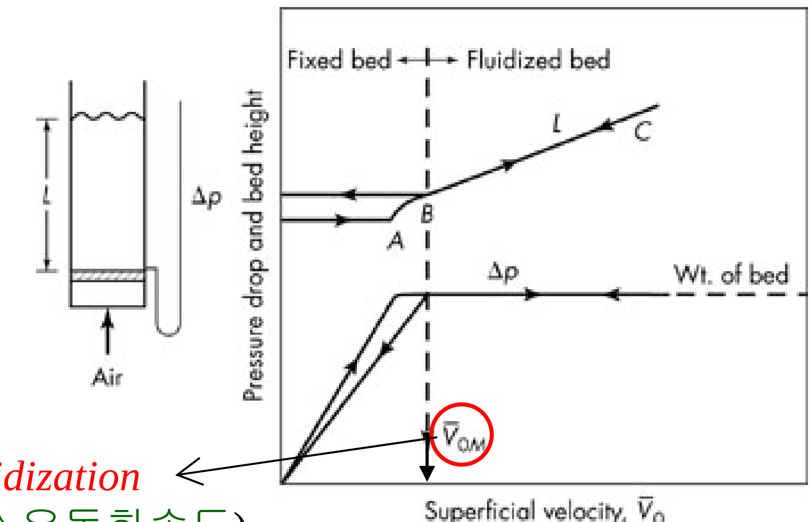
. 입자가 움직이지 않을 때의 pressure drop (Δp)

→ Ergun equation (Eq. 7.22)

. 유체의 속도가 증가함에 따라 Δp 와

개별 입자의 drag force가 증가

→ Particles become suspended in the fluid.



Fully suspended particles in fluid

minimum fluidization velocity (최소유동화속도)

Fluidized Bed (유동층)

Fig. 7.11. Pressure drop & bed height vs. superficial velocity

A vertical tube partly filled with a fine granular material.

open at the top, porous at the bottom, air flow from below

(Fig. 7.11)

*** Minimum fluidization velocity (최소유동화속도), $\bar{V}_{0M}$**

. Net upward force: $\Delta p\, A$

. Net downward force: $\underline{AL(1-\varepsilon)}(\rho_p - \rho)\, g$

net gravitational & buoyant force volume of solid particles

At incipient fluidization (초기유동화):

$$\boxed{\therefore\ \frac{\Delta p}{L} = g\,(1-\varepsilon_M)(\rho_p - \rho)}$$

← Two forces are equal

Ergun equation (Eq. 7.22)

minimum porosity(최소공극률)

즉, 초기유동화가 일어나는 지점에서는 다음의 식으로 구성:

$$\frac{150\mu \bar{V}_{0M}}{\Phi_s{}^2 D_p{}^2}\frac{(1-\varepsilon_M)}{\varepsilon_M{}^3} + \frac{1.75\rho \bar{V}_{0M}{}^2}{\Phi_s D_p}\frac{1}{\varepsilon_M{}^3} = g(\rho_p - \rho)$$

→ 최소유동화속도 $\bar{V}_{0M}$ 를 구하면,

$$\begin{cases} \text{With } Re_p < 1, \ \bar{V}_{0M} \approx \dfrac{g(\rho_p - \rho)}{150\mu} \dfrac{\varepsilon_M{}^3}{1-\varepsilon_M} \Phi_s{}^2 D_p{}^2 \\ \text{With } Re_p > 10^3, \ \bar{V}_{0M} \approx \left[\dfrac{\Phi_s D_p \, g(\rho_p - \rho) \varepsilon_M{}^3}{1.75\rho} \right]^{1/2} \end{cases}$$

← ε_M : 0.40~0.45
for roughly spherical particles

Related problems:

(Probs.) 7.1, 7.5, 7.6, 7.11 and 7.17