Chapter 10. Heat Transfer by Conduction

Three heat flow mechanisms:

- Conduction (전도)
- Convection (대류)
  - natural convection (자연대류)
  - forced convection (강제대류)
- Radiation (복사)

Basic Law of Conduction

*** Fourier's law**

$$\frac{dq}{dA} = -k\frac{dT}{dx}$$ --- Eq. (10.1)

q : rate of heat flow ***A*** : surface area ***T*** : temperature
x : distance normal to surface ***k*** : thermal conductivity

General expressions of Fourier's law in all three directions:

$$\frac{dq}{dA} = -k\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z}\right) = -k\nabla T$$

* Thermal conductivity (열전도도) *k*

Fourier's law에서의 비례상수

← Newton's law에서의 점도에 해당

$$\frac{dq}{dA} = -k\frac{dT}{dx} \longleftrightarrow \tau = -\mu\frac{du}{dy}$$

Rate of heat flow (열흐름속도) q 의 단위: W or Btu/h

dT/dx 의 단위: $^oC/m$ or $^oF/ft$

∴ 열전도도 k 의 단위: W/m·^oC or Btu/ft·h·^oF

For small ranges of T, k = constant

For larger T ranges, $k = a + bT$

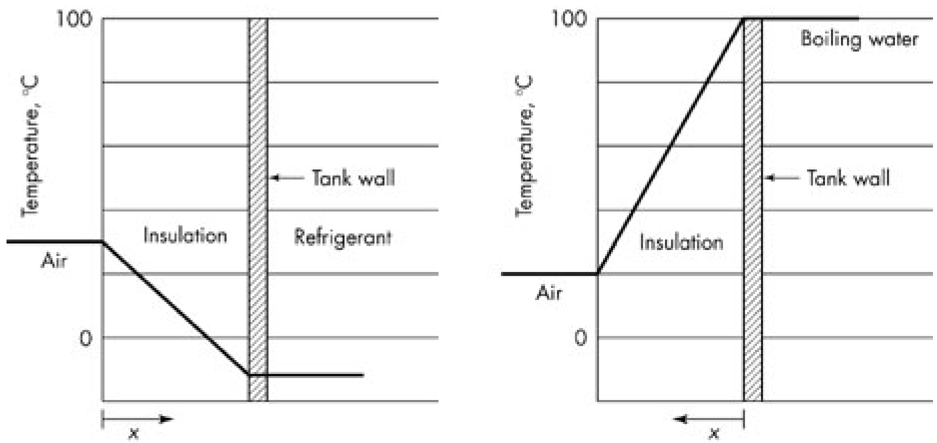
k values:
- solids: 17 W/m·^oC for stainless steel
- 415 W/m·^oC for silver
- 0.35 W/m·^oC for glass
- liquids: 0.5 W/m·^oC for water
- gases: 0.024 W/m·^oC for air

Solid having low k → "insulators" ex) polystyrene foam

Steady-State Conduction (정상상태 열전도)

← neither accumulation nor depletion of heat within the slab

q is constant along the path of heat flow



Heat flow into the tank   Heat flow from the tank

$$\frac{q}{A} = -k\frac{dT}{dx} = k\frac{T_2 - T_1}{x_2 - x_1} = k\frac{\Delta T}{B}$$

--- Eq. (10.7)

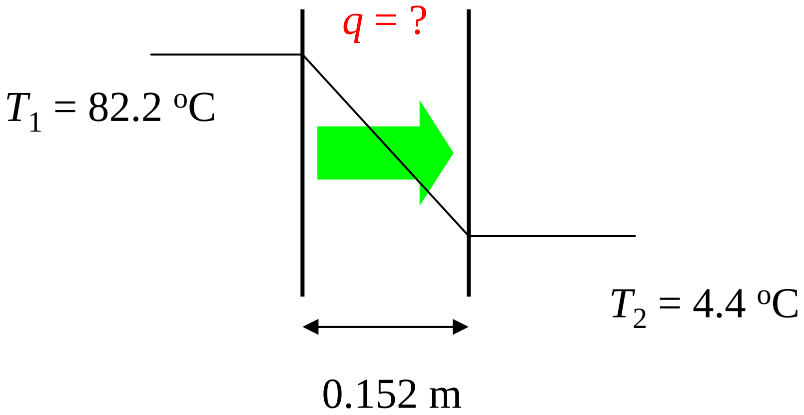
x: distance from hot side

B: thickness of slab

cf.) $\frac{q}{A} = \frac{\Delta T}{R}$

thermal resistance (B/k) $1/R$: conductance (k/B)

Ex. 10.1) **A layer of pulverized cork (insulator)**



k : 0.036 W/m^{o}C at 0 ^{o}C
0.055 W/m^{o}C at 93.3 ^{o}C

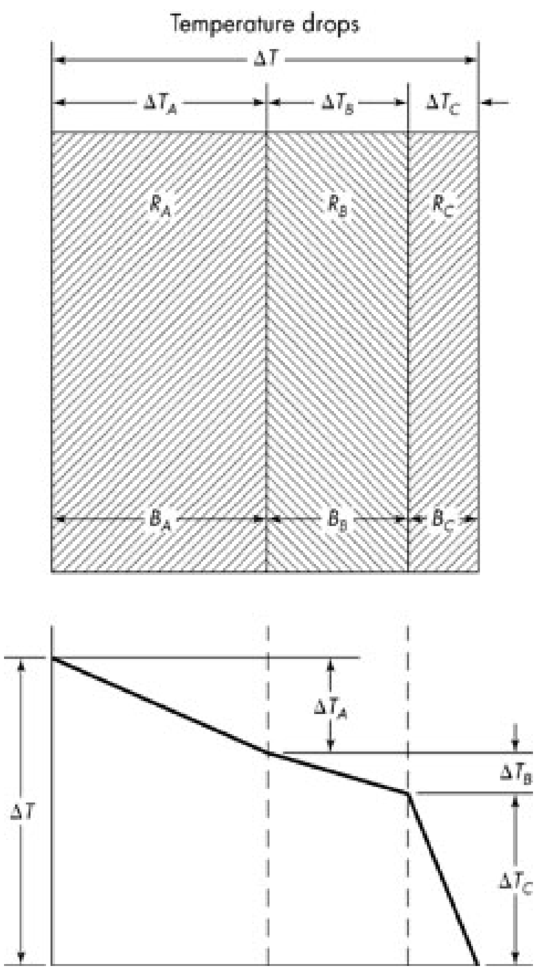
A = 2.32 m^2

q (the rate of heat flow) ?

$$q = Ak\frac{\Delta T}{B} = 2.32 \times \left(0.036 + 43.3 \times \frac{0.055 - 0.036}{93.3 - 0}\right) \times \frac{77.8}{0.152}$$

$$= 53.3\,\mathrm{W}$$

* Compound resistances in series



$$\Delta T = \Delta T_A + \Delta T_B + \Delta T_C$$

$$q_A \frac{B_A}{k_A A} \qquad q_B \frac{B_B}{k_B A} \qquad q_C \frac{B_C}{k_C A}$$

In steady heat flow,

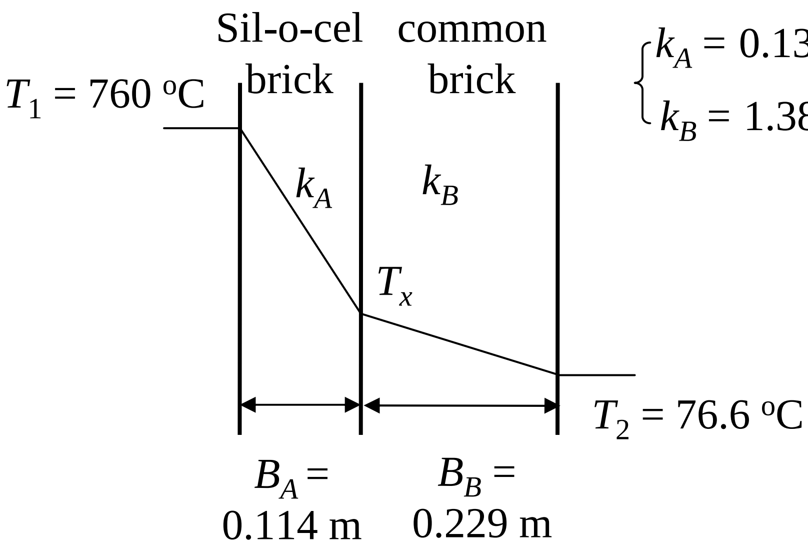
$$q_A = q_B = q_C$$

$$\therefore \frac{q}{A} = \frac{\Delta T}{B_A / k_A + B_B / k_B + B_C / k_C} = \frac{\Delta T}{R_A + R_B + R_C} = \frac{\Delta T}{R}$$

&

$$\frac{q_A}{A} = \frac{q_B}{A} = \frac{q_C}{A} = \frac{q}{A} \quad \Rightarrow \quad \frac{\Delta T}{R} = \frac{\Delta T_A}{R_A} = \frac{\Delta T_B}{R_B} = \frac{\Delta T_C}{R_C}$$

Ex. 10.2) A flat furnace wall constructed of a layer of Sil-o-cel brick backed by a common brick



$$\begin{cases} k_A = 0.138 \text{ W/m °C} \\ k_B = 1.38 \text{ W/m °C} \end{cases}$$

(a) Heat loss through the wall, q = ?

Assume $A = 1 \text{ m}^2$

$$R_A = \frac{B_A}{k_A} = 0.826 \qquad R_B = \frac{B_B}{k_B} = 0.159$$

$$R = R_A + R_B = 0.985 \text{m}^2 \, {}^{\text{o}}\text{C/W} \qquad \Delta T = 683.4 \, {}^{\text{o}}\text{C}$$

$$\therefore \; q/A = 683.4/0.985 = 693.81 \text{W/m}^2 \qquad \underline{q = 693.81\text{W}}$$

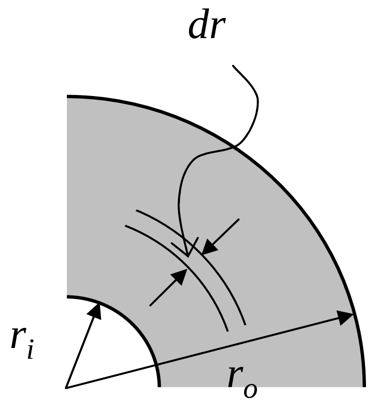
(b) Temperature of the interface between the two bricks

$$\Delta T / R = \Delta T_A / R_A \qquad 683.4/0.985 = \Delta T_A / 0.826 \qquad \Delta T_A = 573.08 \, {}^{\text{o}}\text{C} \qquad \therefore \; \underline{T_x = T_1 - \Delta T_A = 186.9 \, {}^{\text{o}}\text{C}}$$

(c) In case that the contact between the two bricks is poor and the contact resistance is 0.088 m^2 °C/W, the heat loss q = ?

$$R = 0.985 + 0.088 = 1.073 \text{m}^2 \, {}^{\text{o}}\text{C/W} \qquad \therefore \; \underline{q = \Delta T / R = 636.9 \text{ W}}$$

* Heat flow through a cylinder



cylinder length: L

$$q = -k\frac{dT}{dr}(2\pi rL)$$

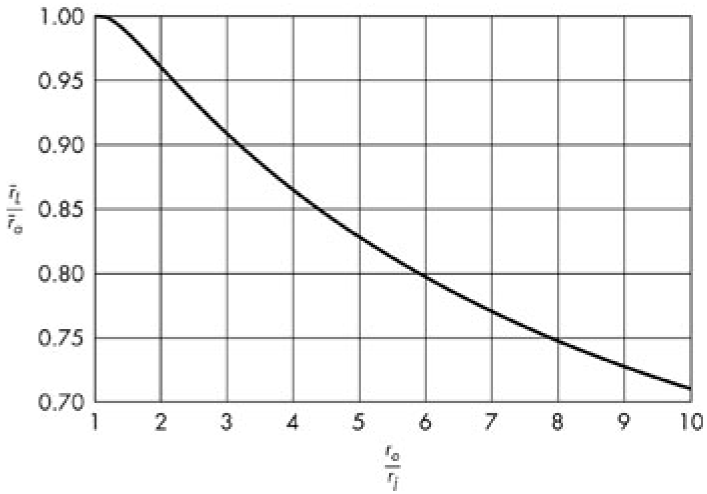
$$\int_{r_i}^{r_o}\frac{dr}{r} = \frac{-2\pi Lk}{q}\int_{T_i}^{T_o} dT \quad \Rightarrow \quad q = \frac{k(2\pi L)(T_i - T_o)}{\ln(r_o / r_i)}$$

$$= \frac{k\bar{A}_L(T_i - T_o)}{r_o - r_i}$$

$$\bar{A}_L = 2\pi \bar{r}_L L$$

$$\bar{r}_L = \frac{r_o - r_i}{\ln(r_o / r_i)}$$

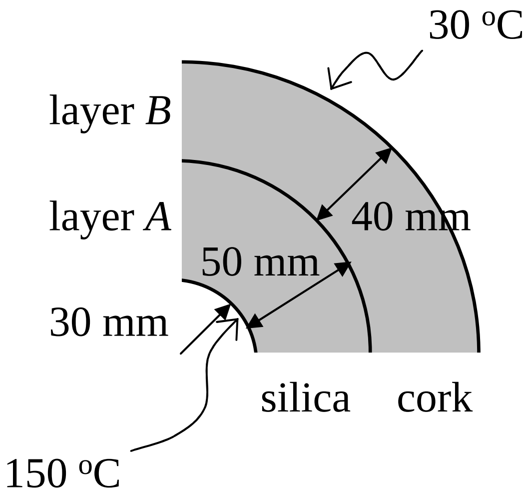
: logarithmic mean radius
(로그평균반지름)



Logarithmic mean radius $\bar{r}_L$ vs. arithmetic mean radius $\bar{r}_a$

Ex. 10.3) A tube of 60 mm OD insulated with a 50 mm silica foam layer and a 40 mm cork layer

Calculate the heat loss q of pipe in W/m ?



$$\begin{cases} k_A = 0.055 \text{ W/m °C} \\ k_B = 0.05 \text{ W/m °C} \end{cases}$$

$$\bar{r}_L = \frac{r_o - r_i}{\ln(r_o / r_i)}$$

For silica layer, $\bar{r}_L = \dfrac{80-30}{\ln(80/30)}$

For cork layer, $\bar{r}_L = \dfrac{120-80}{\ln(120/80)}$

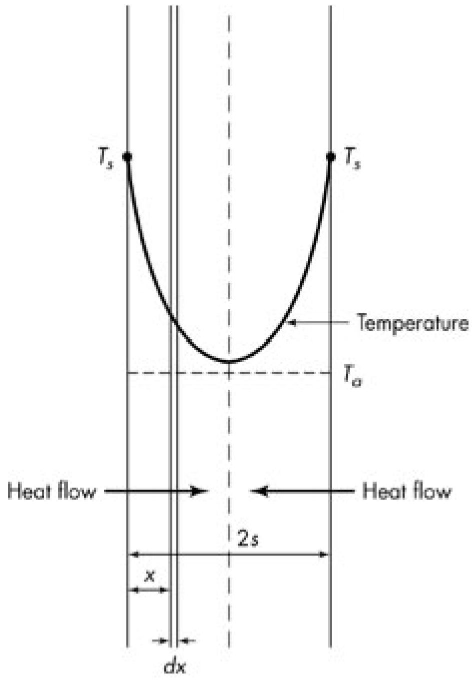
$$q_A = \frac{k_A \bar{A}_A (T_i - T_x)}{x_A} \qquad q_B = \frac{k_B \bar{A}_B (T_x - T_o)}{x_B} \qquad \bar{A}_A = 2\pi \bar{r}_L L \qquad \bar{A}_B = 2\pi \bar{r}_L L$$

$$\Rightarrow \quad q_A = 0.3522 L (T_i - T_x)$$

$$q_B = 0.7748 L (T_x - T_o)$$

At steady state, $q = q_A = q_B$ $\qquad \therefore \ q/L = 29.1\text{W}$

Unsteady-State Conduction (비정상상태 열전도)



Unsteady-state conduction in solid slab

Heat input – Heat output = Accumulation of heat

$$\Rightarrow \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

thermal diffusivity (열확산계수) $= \dfrac{k}{\rho C_p}$

단위: [m^2/s]

cf.) D : mass diffusivity

ν : kinematic viscosity

Solutions are available for certain simple shapes, such as infinite slab (Eq. 10.20), infinitely long cylinder (Eq. 10.21) and sphere (Eq. 10.22).

→ Fig. 10.5

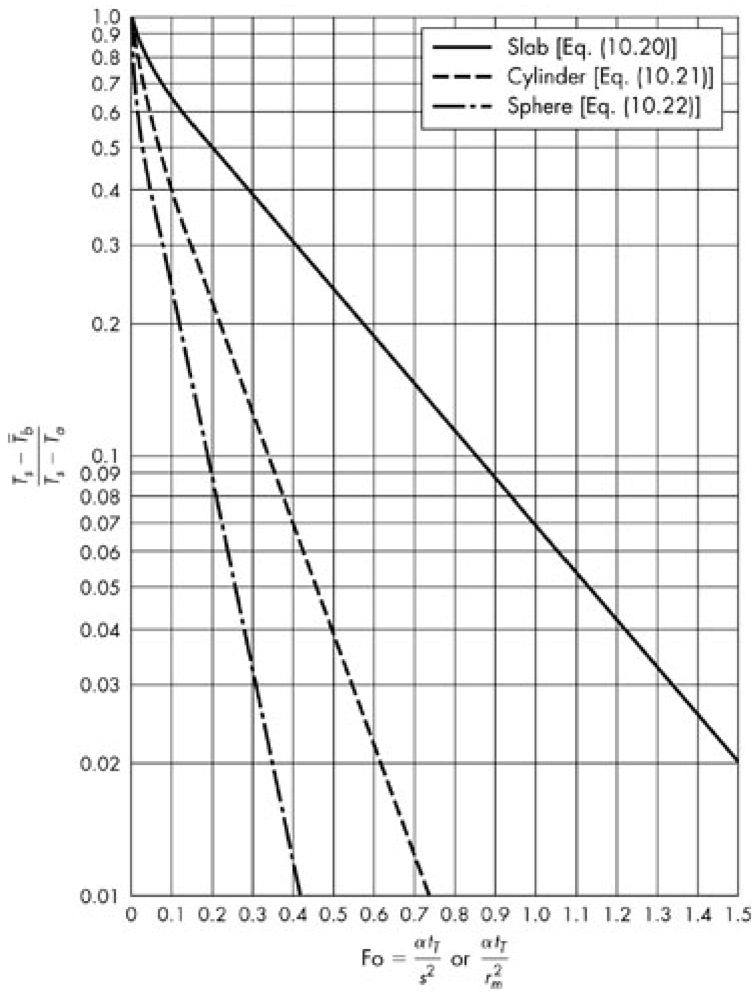


Fig. 10.5. Average temperature during unsteady-state hearing or cooling of a large slab, an infinitely long cylinder, or a sphere.

Related problems: (Probs.) 10.1, 10.2, 10.3, 10.9 and 10.12