20.QM의 정리(디렉)

화공과 김영훈 교수

korea1@kw.ac.kr

하이젠베르크 vs. 슈뢰딩거

- 두 식은 서로 다른 현상을 설명하는가?
  - 하이젠베르그(심상이 없다)

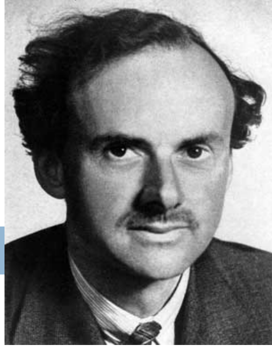
$$H(P,Q)\Theta = W\Theta \qquad H = \frac{1}{2m}P^2 + \frac{k}{2}Q^2$$

  - 슈뢰딩거(심상이 있다)

$$-\frac{\mathrm{h}^2}{2m}\nabla^2\Psi + V\Psi = E\Psi$$

- Max Born & Paul Dirac
  - 두 식은 서로 대응한다(일치한다)

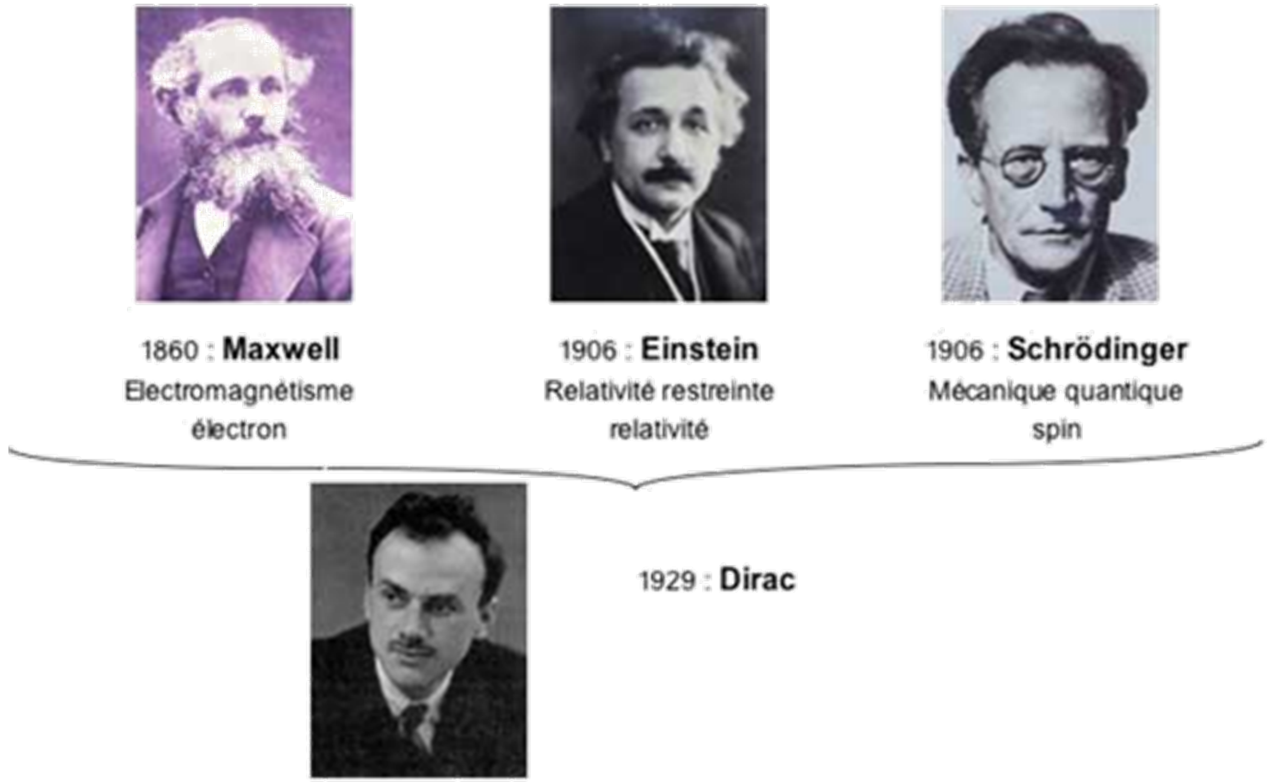
Dirac’s theory



- Removed the paradox of particle-wave duality
  - If a particle was probed in a way that was meant to demonstrate its particle like properties - it would appear to be a particle
  - If it was probed in a way that was meant to demonstrate its wave like properties - it would appear to be a wave
  - It seems that it is our own inability to conjure up an appropriate or adequate mental picture of photons, atoms, electrons and other quantum particles that is at the heart of the particle-wave duality paradox

양자역학의 집대성

- Famous for
  - 변환원리: QM 공식의 이론적 완성
  - 전자에 상대성 원리 적용
  - 반물질 예언



Dirac의 3번째 양자역학 공식

- Relativistic quantum mechanics
  - Schrodinger's wave equation + Einstein's special relativity

$$(i\gamma^{\mu}\partial_{\mu} - m)\Psi = 0$$

  - 슈뢰딩거 식
    - 파동의 성질은 잘 표현하지만, 입자성질 표현에 미약
    - 수학적으로 하이젠베르그식과 일치함을 확인
  - 아인슈타인의 특수상대성 이론 접목
    - 광자가 아닌 전자에 관하여 해석 시도
    - 정확한 magnetic moment 해석
    - 전자(입자)는 본래부터 ½ 스핀을 지닌다(ab initio)

슈뢰딩거: 비상대적인 질량만 고려

- 기본 슈뢰딩거 식

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\Psi(\mathbf{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)$$

$p^2/2m$ a potential energy operator

TOTAL ENERGY

- V=0일 때의 식: $\frac{1}{2m}\nabla^2\Psi(\mathbf{r},t) + i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = 0$

this equation is a "non-relativistic" approximation !

- Relativistic correction
  - 전자 속도=약 0.01c

classical energy : $E = mv^2/2 = p^2/2m$

relativistic: $E^2 = m^2c^4 + p^2c^2$

Dirac equation

$$(i\gamma^{\mu}\partial_{\mu} - m)\Psi = 0$$

this is a 4-dimensional derivative: space and time get same treatment

this is mc^2

$$\left(i\hbar\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} - mc\right)\psi = 0$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$ double-spinor

$$E = \pm\sqrt{m^2c^4 + p^2c^2}$$

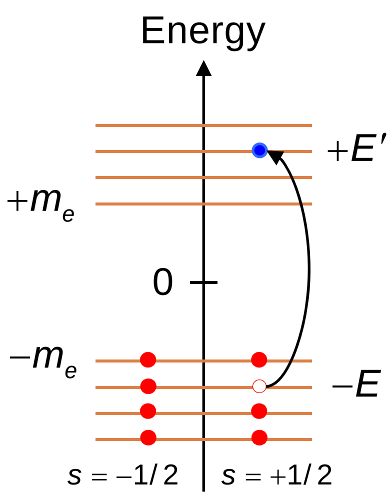
	Spin-up	*Spin-down*
Particle	$\psi^{(1)} = \psi_{E=+mc^2,+\hbar/2} = \frac{1}{\sqrt{V}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} e^{-imc^2t/\hbar}$	$\psi^{(2)} = \psi_{E=+mc^2,-\hbar/2} = \frac{1}{\sqrt{V}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} e^{-imc^2t/\hbar}$
Anti-Particle	$\psi^{(3)} = \psi_{E=-mc^2,+\hbar/2} = \frac{1}{\sqrt{V}}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} e^{+imc^2t/\hbar}$	$\psi^{(4)} = \psi_{E=-mc^2,-\hbar/2} = \frac{1}{\sqrt{V}}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} e^{+imc^2t/\hbar}$

Dirac 식에서 음의 에너지 해석

- Combining quantum mechanics with special relativity, and the wish to linearize $\partial/\partial t$, leads Dirac to the equation

$$i\gamma^{\mu}\partial_{\mu}\psi(x,t) - m\psi(x,t) = 0$$

for which solutions with negative energy appear



This picture fails for bosons !

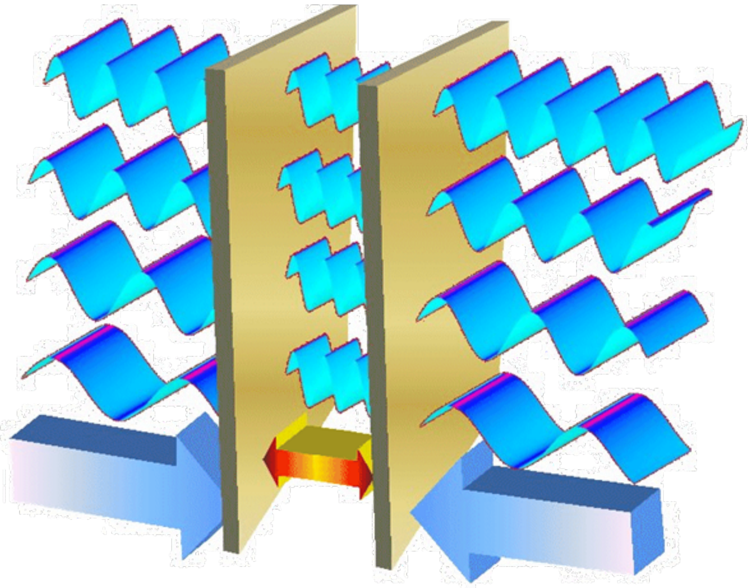
- Vacuum represents a "sea" of such negative-energy particles (fully filled according to Pauli's principle)
- Dirac identified holes in this sea as "antiparticles" with opposite charge to particles … (however, he conjectured that these holes were protons, despite their large difference in mass, because he thought "positrons" would have been discovered already)
- An electron with energy E can fill this hole, emitting an energy $2E$ and leaving the vacuum (hence, the hole has effectively the charge $+e$ and positive energy).

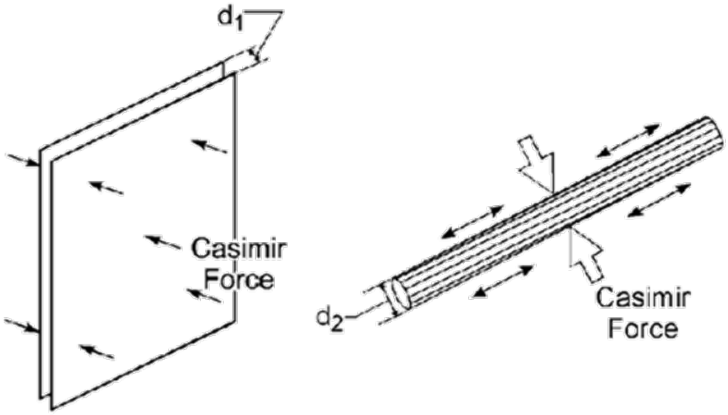
가상의 입자의 불확실성에 의한 해석

- 하이젠베르그 불확실성 원리
  - $\Delta E x \Delta t > \hbar/2$
  - 텅빈 공간이라도 국지적으로 zero energy를 갖지 않는다
  - 단, 매우 짧은 시간에 충분한 에너지를 지니며, 이 무한히 작은 시간에 matter-antimatter 쌍이 생긴다
- Empty space
  - Virtual particle 형성을 위한 질량의 quantum fluctuation 발생: Casimir effect
  - Recomination, annihilation 발생

Casimir effect

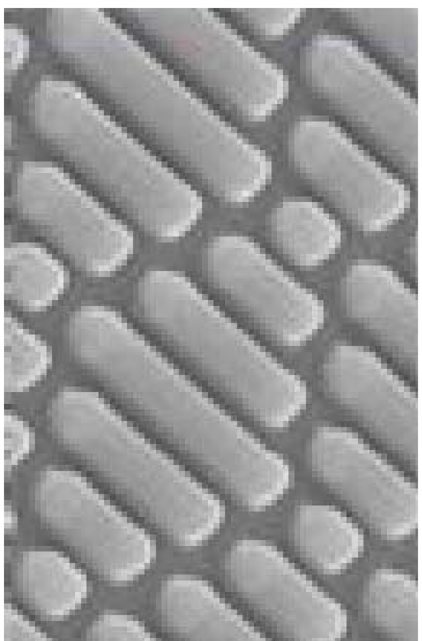
- Hendrick Casimir
  - 진공에서 전하를 지닌 두 판 사이가 충분히 가깝게 설치
  - 매우 큰 전자기적 quantum fluctuation이 발생하고, 두판 사이의 안쪽면보다 바깥쪽의 면에서 인력이 작용
  - 두판이 좁혀지려고 하지만 일정 거리 유지
  - 필요간격
    - 판형 d1: 0.6-6 μm
    - 원동형 d2: ~15 nm





응용분야

- 공중부양
  - Hoverboard (reverse Casimir force)
  - 현재는 nanomachine의 friction 감소에 사용



Pair production

- 전하량 보존 법칙
  - 광자가 전자를 생산하면 charge conservation에 의해 양전하를 지닌 물질 생산해야 함

$$\gamma \rightarrow e^{+} + e^{-}$$

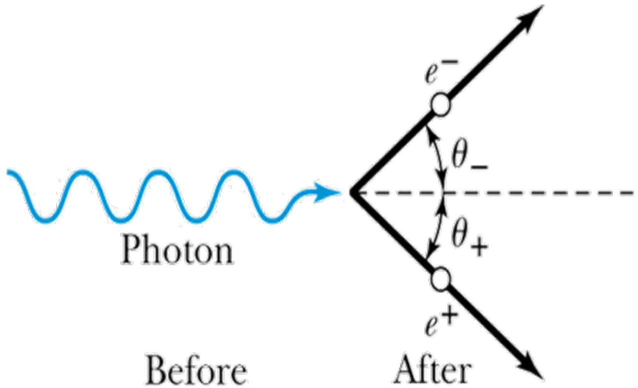
  - Anderson
    - 우주선(cosmic radiation)에서 양전하를 지닌 전자 발견
    - Positron으로 명명

Pair production in empty space

- 진공에서 에너지(moment) 보존
  - Empty space에서의 paper production에 대한 에너지 보존

$$h\nu = E_+ + E_-$$

  - 입자에 대한 전체 에너지 $E_\pm^2 = p_\pm^2 c^2 + m^2 c^4$ $E_\pm > p_\pm c$
    - 최하위값: $h\nu > p_- c + p_+ c$
    - 보존값: $h\nu = p_- c \cos(\theta_-) + p_+ c \cos(\theta_+)$
    - 최상위값: $h\nu < p_- c + p_+ c$
  - 최상위와 최하위가 서로 상충

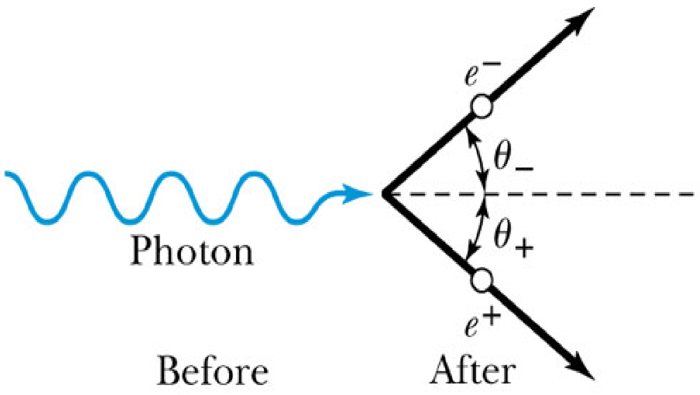


Pair production in matter

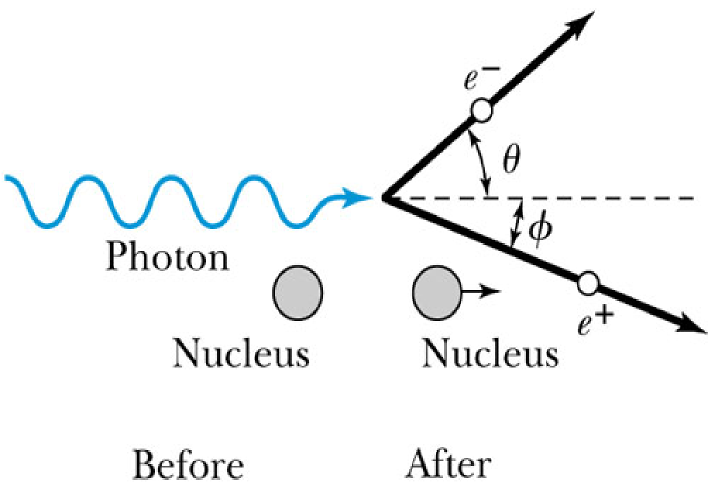
- In the presence of matter, the nucleus absorbs some energy and momentum
- The photon energy required for pair production in the presence of matter is

$$h\nu = E_+ + E_- + K.E.(nucleus)$$

$$h\nu > 2m_e c^2 = 1.022\ MeV$$



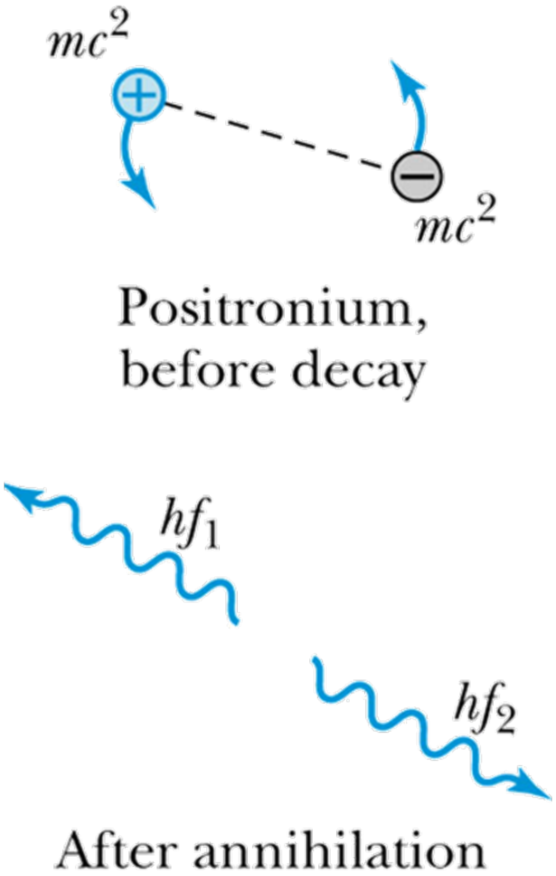
(a) Free space (**cannot occur**)



(b) Beside nucleus

Pair annihilation

- Pair의 붕괴 전후 과정
  - A positron passing through matter will likely annihilate with an electron
  - The electron and positron can form an atom-like configuration first, called positronium
  - Pair annihilation in empty space produces two photons to conserve momentum
  - Annihilation near a nucleus can result in a single photon



Pair annihilation의 수학적 해석

- Conservation of energy: $2m_e c^2 \approx h\nu_1 + h\nu_2$
- Conservation of momentum: $\frac{h\nu_1}{c} - \frac{h\nu_2}{c} = 0$

So the two photons will have the same frequency: $\nu_1 = \nu_2 = \nu$

The two photons from positronium annihilation will move in opposite directions with an energy:

$$h\nu = m_e c^2 = 0.511\,\mathrm{MeV}$$

Positron의 응용분야

- PET(Positron Emission Tomography)
  - 뇌 단층촬영(MRI 보다 선명), 감마선 이용
    - Radioactive nuclei in a fluid are injected into the subject.
    - The radioactive nuclei then emit positrons at low velocities and these then annihilate with nearby electrons.
    - The positrons and electrons are moving slowly and don't have the energy required to create a new pair of particle and antiparticle. Instead, 2 gamma rays are emitted and these are used to actively scan the brain.