Lecture 11.
Binary Distillation (1)
[Ch. 7]

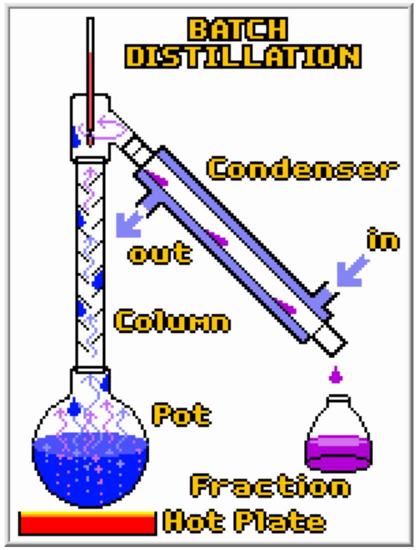
- Distillation
  - Overview & Industrial use
  - Operation
- McCabe-Thiele Method
  - Rectifying section
  - Stripping section
  - Feed-stage considerations
  - q-line
  - Optimal and nonoptimal locations of feed stage
- Limiting Conditions
  - Minimum number of equilibrium stages
  - Minimum reflux ratio
  - Perfect separation

Distillation

- Distillation (fractionation) *vs.* absorption and stripping
  : the second fluid phase is usually created by thermal means (vaporization and condensation) rather than by introduction
- 11th century, distillation was used in Italy to produce alcoholic beverages (batch process)

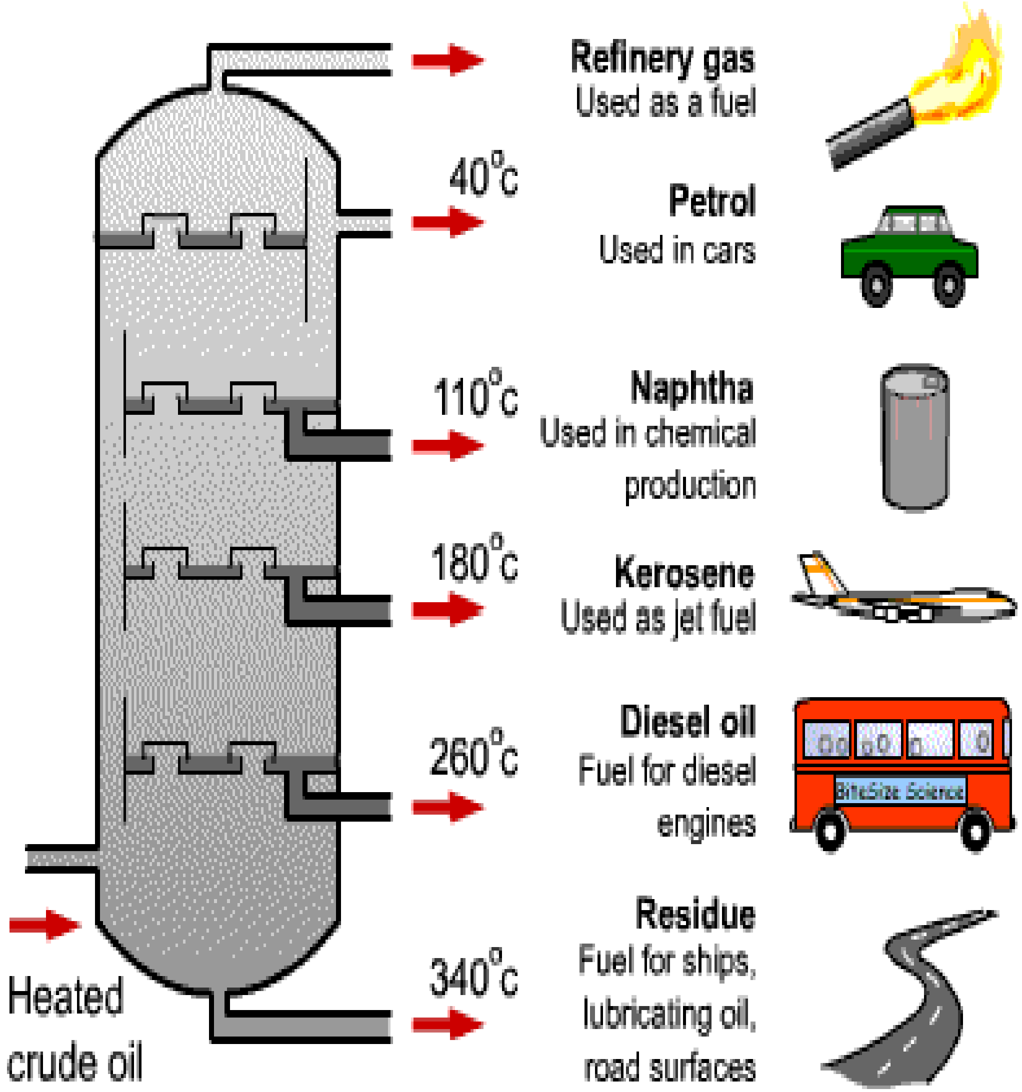
→ 16th century, it was known that separation could be improved by multiple vapor-liquid contacts (stages)

→ 20th century, multistage distillation became the most widely used industrial method for separating liquid mixtures

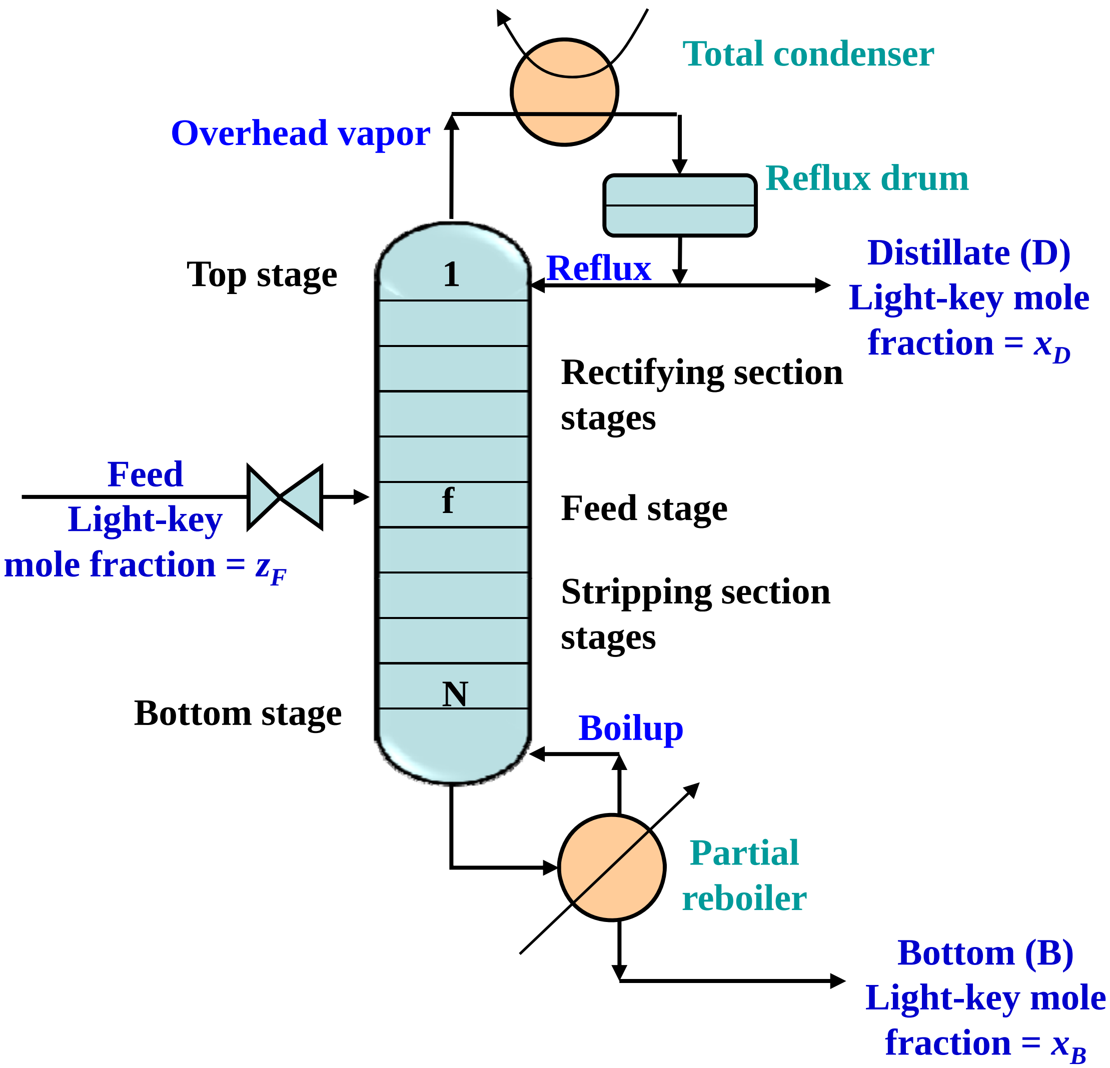


Distillation: Industrial Use

- Distillation is technically the most mature separation operation
- Distillation is very energy intensive, especially when the relative volatility, α, is low (< 1.50)
- Most significant distillation energy has been consumed by petroleum refining to separate crude oil into petroleum fractions, light hydrocarbons (C_2's to C_5's), and aromatic chemicals



Distillation Operation



- Total condenser: the overhead vapor leaving the top stage is totally condensed → liquid distillate product + liquid reflux that is returned to the top stage
- Partial reboiler: liquid from the bottom stage is partially vaporized → liquid bottoms product + vapor boilup that is returned to the bottom stage
- Multiple, countercurrent contacting stages can achieve a sharp separation unless an azeotrope is formed

Phase Equilibrium of Binary Mixture

- Goal of distillation: from the feed to produce a distillate, rich in the light key ($x_D \rightarrow 1.0$), and a bottoms product, rich in the heavy key ($x_B \rightarrow$)

- Relative volatility, α

$$\alpha_{1,2} = K_1 / K_2$$

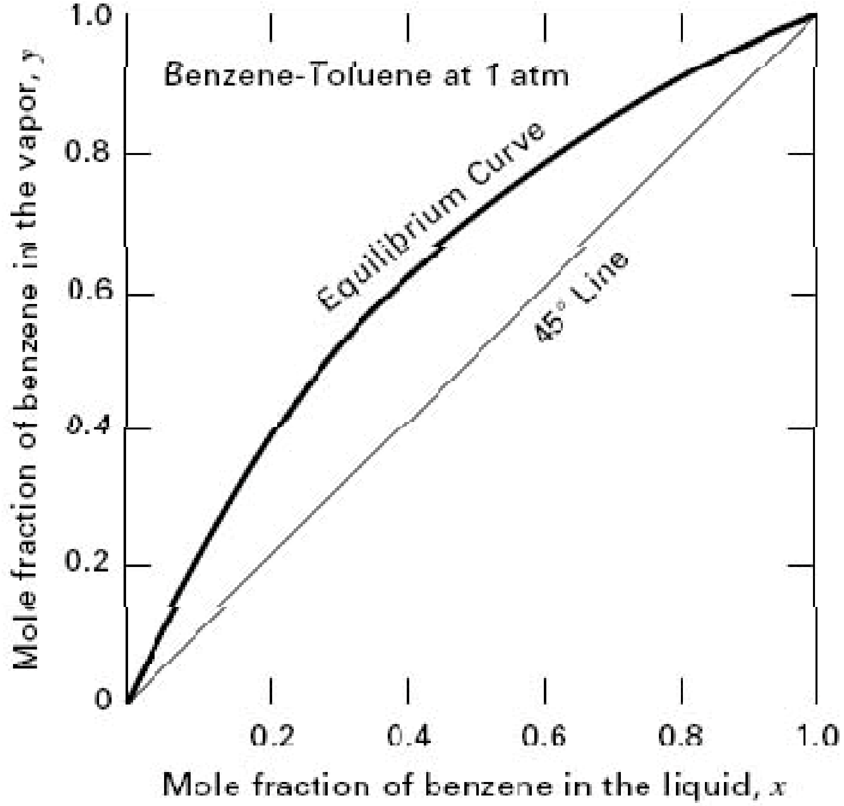
Raoult's law

$$K_1 = P_1^s / P \quad \text{and} \quad K_2 = P_2^s / P$$

$$\alpha_{1,2} = P_1^s / P_2^s$$

$$\alpha_{1,2} = \frac{y_1 / x_1}{y_2 / x_2} = \frac{y_1(1 - x_1)}{x_1(1 - y_1)}$$

$$y_1 = \frac{\alpha_{1,2} x_1}{1 + x_1(\alpha_{1,2} - 1)}$$

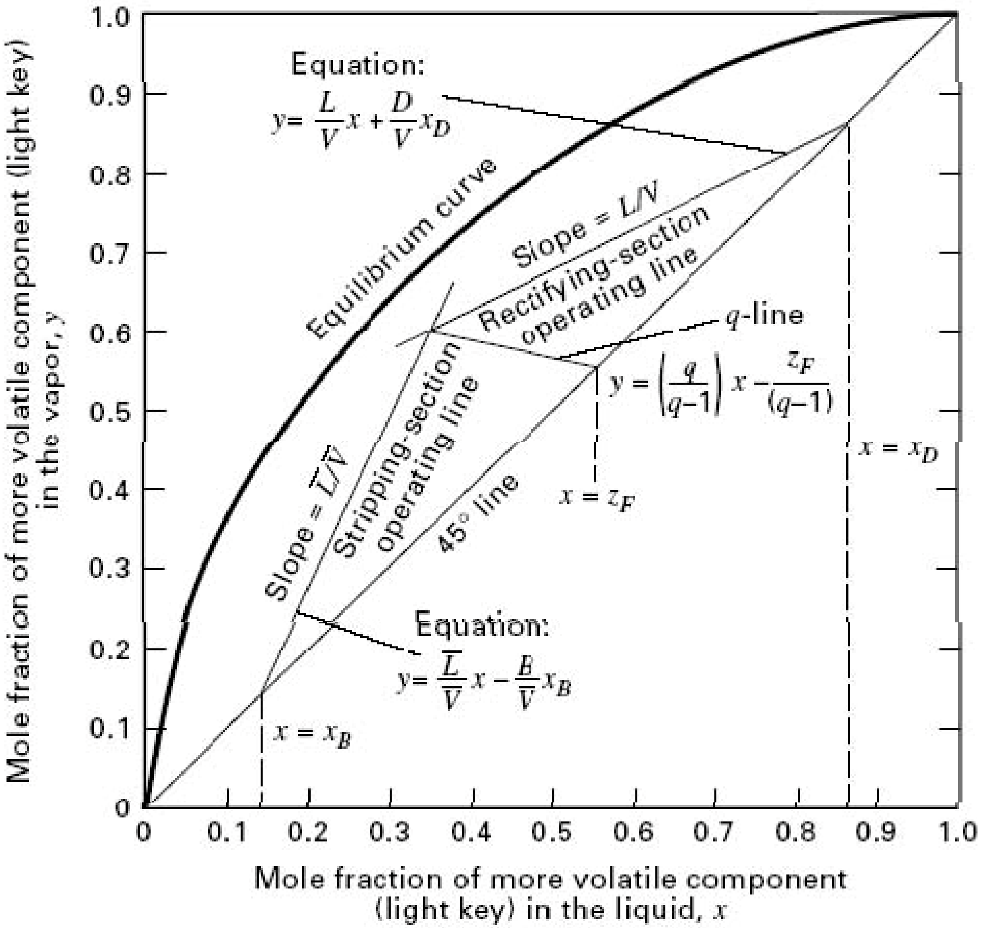


For ideal binary mixtures of components with close boiling points, T changes are small and α is almost constant

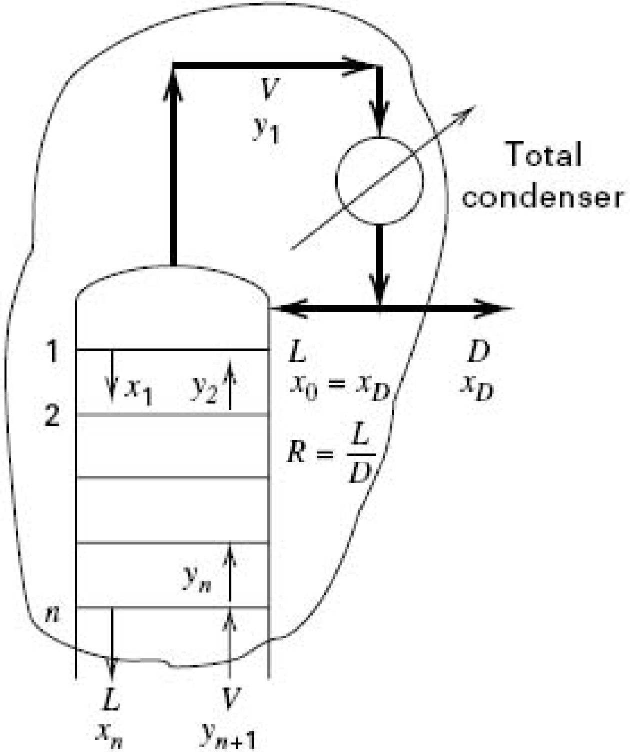
McCabe–Thiele Method

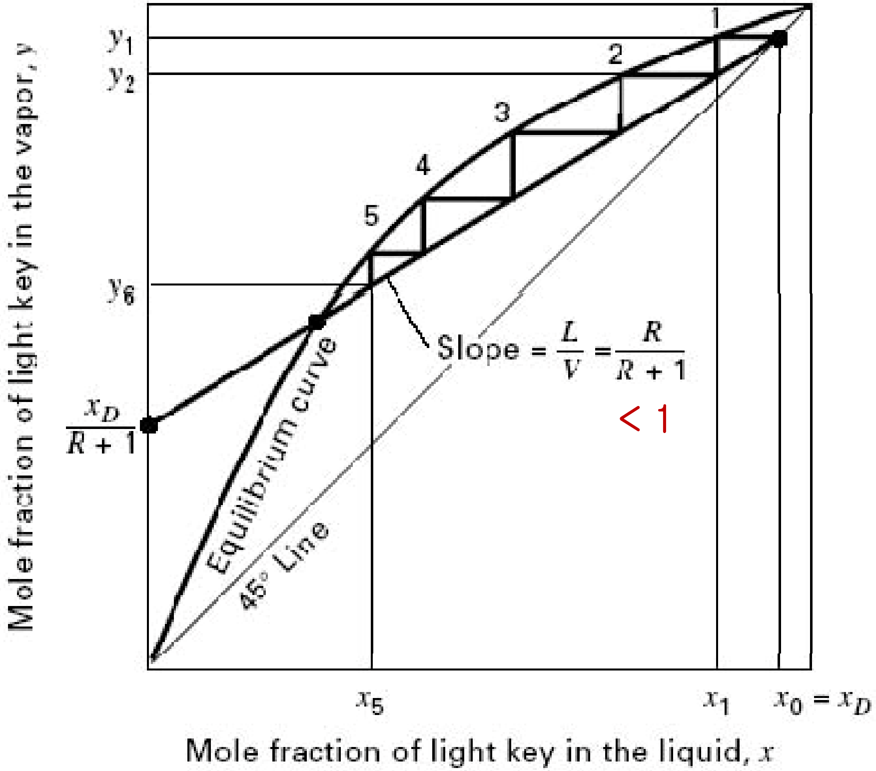
Specifications	
F	Total feed rate
z_F	Mole-fraction composition of the feed
P	Column operating pressure (assumed uniform throughout the column)
	Phase condition of the feed at column pressure
	Vapor–liquid equilibrium curve for the binary mixture at column pressure
	Type of overhead condenser (total or partial)
	Type of reboiler (usually partial)
x_D	Mole-fraction composition of the distillate
x_B	Mole-fraction composition of the bottoms
$R/R_{\min}$	Ratio of reflux to minimum reflux
Results	
D	Distillate flow rate
B	Bottoms flow rate
$N_{\min}$	Minimum number of equilibrium stages
$R_{\min}$	Minimum reflux ratio, $L_{\min}/D$
R	Reflux ratio, L/D
V_B	Boilup ratio, $\bar{V}/B$
N	Number of equilibrium stages
	Optimal feed-stage location
	Stage vapor and liquid compositions

Graphical equilibrium–stage method for trayed towers



Rectifying Section





Assuming constant molar overflow

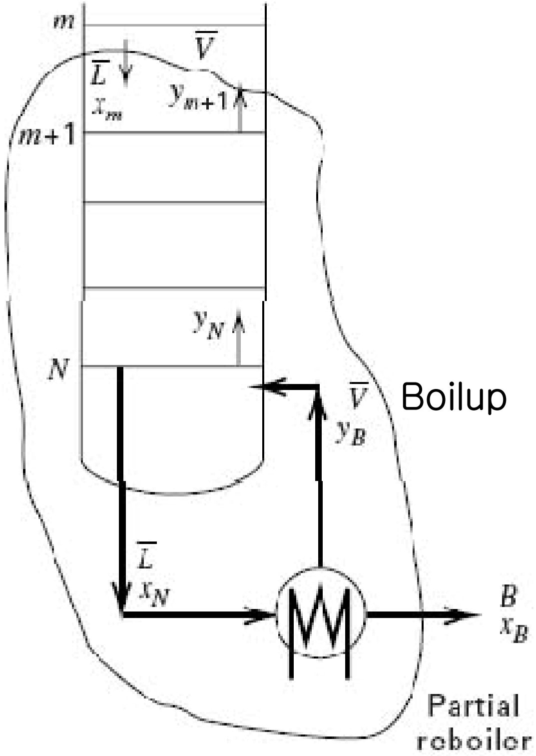
R=L/D
L/V = R/(R+1)
D/V=1/(R+1)

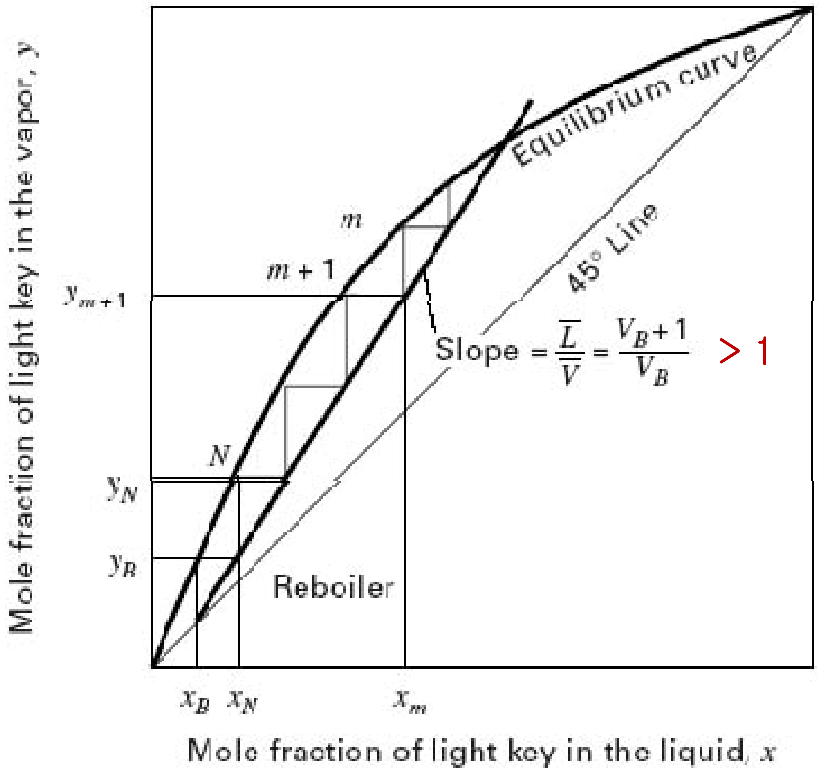
R: reflux ratio

$$V_{n+1}y_{n+1} = L_n x_n + D x_D$$

$$y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{D}{V_{n+1}} x_D \rightarrow y = \frac{L}{V} x + \frac{D}{V} x_D \rightarrow y = \frac{R}{R+1} x + \frac{1}{R+1} x_D$$

Stripping Section





$$\bar{L}_m x_m = \bar{V}_{m+1} y_{m+1} + B x_B$$

$$y_{m+1} = \frac{\bar{L}_m}{\bar{V}_{m+1}} x_m - \frac{B}{\bar{V}_{m+1}} x_B$$

Assuming constant molar overflow

$$y = \frac{\bar{L}}{\bar{V}} x - \frac{B}{\bar{V}} x_B$$

$$\boldsymbol{V_B = \bar{V} / B}$$
$$\boldsymbol{\bar{L} / \bar{V} = (V_B + 1) / V_B}$$
$$\boldsymbol{B / \bar{V} = 1 / V_B}$$

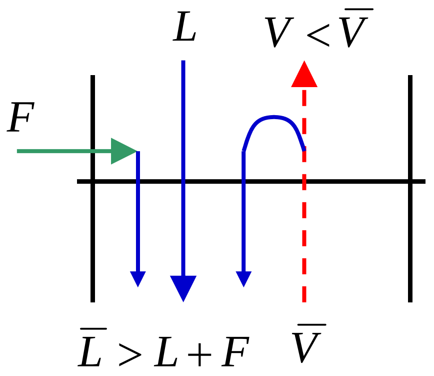
V_B: boilup ratio

$$y = \frac{V_B + 1}{V_B} x - \frac{1}{V_B} x_B$$

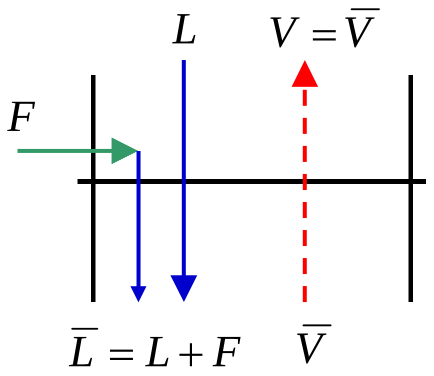
Feed-Stage Considerations

- x_D and x_B can be selected independently
- R and V_B are related by the feed phase condition

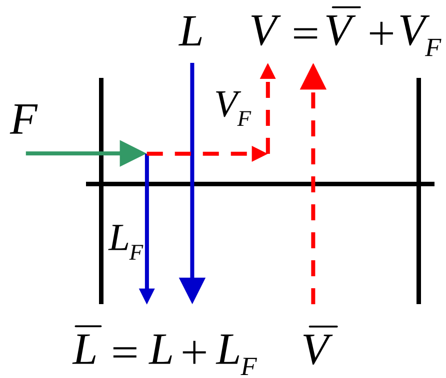
Subcooled-liquid feed



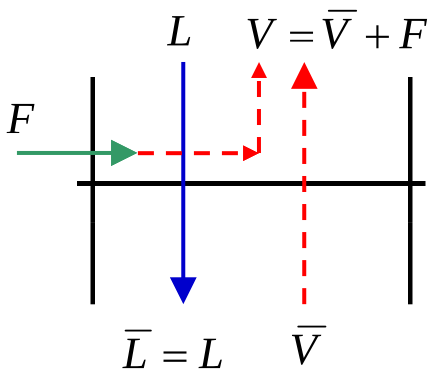
Bubble-point liquid feed



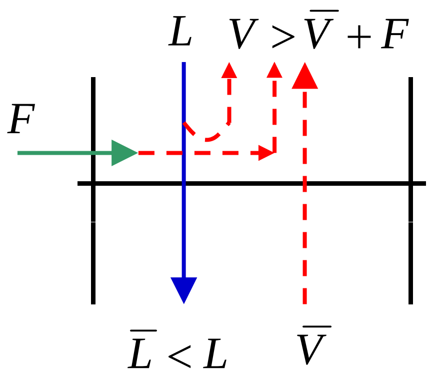
Partially vaporized feed



Dew-point vapor feed



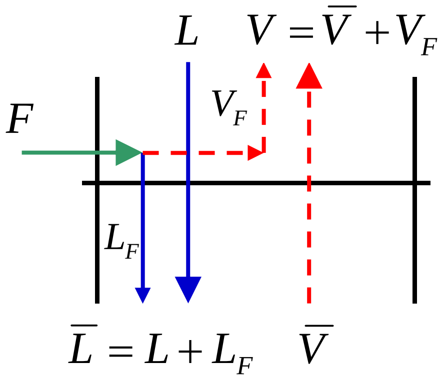
Superheated-vapor feed



Possible feed conditions

Relations for Reflux Ratio and Boilup Ratio

- Relations covering feed conditions from a saturated liquid to a saturated vapor



Boilup ratio

$$\bar{V} = V - V_F = L + D - V_F$$

$$V_B = \frac{\bar{V}}{B} = \frac{L + D - V_F}{B}$$

Reflux ratio

$$L = \bar{L} - L_F = \bar{V} + B - L_F$$

$$R = \frac{L}{D} = \frac{\bar{V} + B - L_F}{D}$$

For the specification of distillation operation, R or R/R_{min} is used traditionally because the distillate product is often the more important product

- q: ratio of the increase in molar reflux rate across the feed stage to the molar feed rate

$$q = \frac{\bar{L} - L}{F} \qquad q = 1 + \frac{\bar{V} - V}{F}$$

Feed condition	Value of q
Subcooled liquid	> 1
Bubble-point liquid	1
Partially vaporized	L_F/F
Dew-point vapor	0
Superheated vapor	< 0

q–Line

- q–line: one point of which is the intersection of the rectifying and stripping operating lines

Rectifying operating line

$$y = \frac{L}{V}x + \frac{D}{V}x_D$$

Stripping operating line

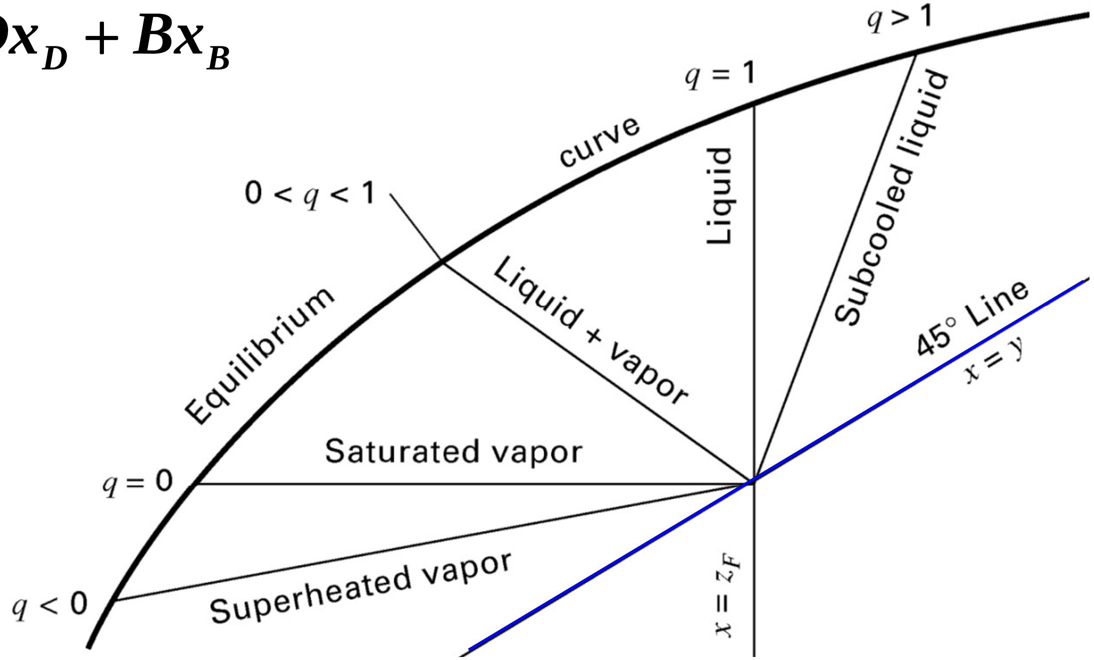
$$y = \frac{\bar{L}}{\bar{V}}x - \frac{B}{\bar{V}}x_B$$

$$y(V - \bar{V}) = (L - \bar{L})x + Dx_D + Bx_B$$

$$\begin{cases} Dx_D + Bx_B = Fz_F \\ F + \bar{V} + L = V + \bar{L} \end{cases}$$

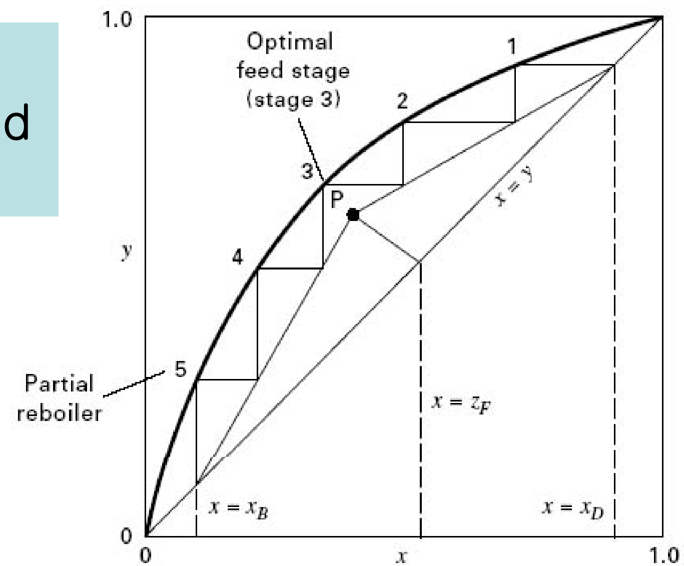
$$q = \frac{\bar{L} - L}{F}$$

$$y = \frac{q}{q-1}x - \frac{z_F}{q-1}$$



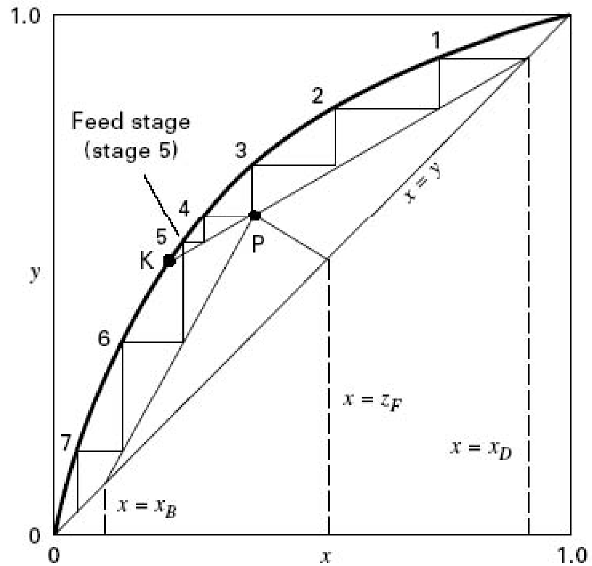
Optimal and Nonoptimal Locations of Feed Stage

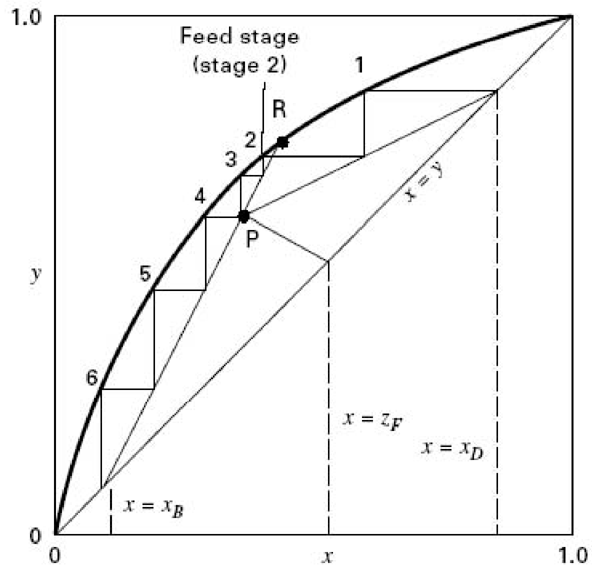
5 stages, optimal feed location



The smallest number of total stages occurs when the transfer is made at the first opportunity after a horizontal line of the staircase passes over point P

6.4 stages, feed-stage location below optimal stage

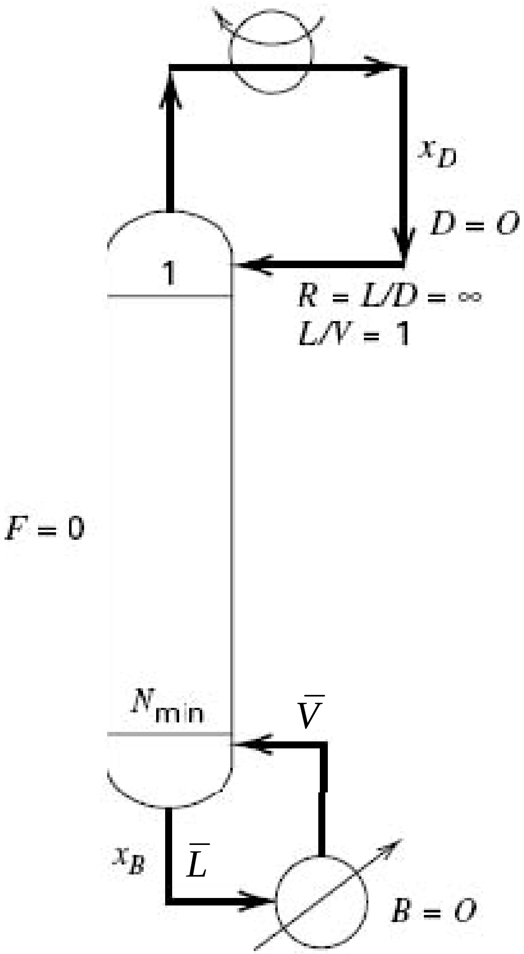




5.9 stages, feed-stage location above optimal stage

Minimum Number of Equilibrium Stages

- Increasing reflux → L/V increases to limiting value 1
- Increasing boilup ratio → $\bar{L}/\bar{V}$ decreases to limiting value 1

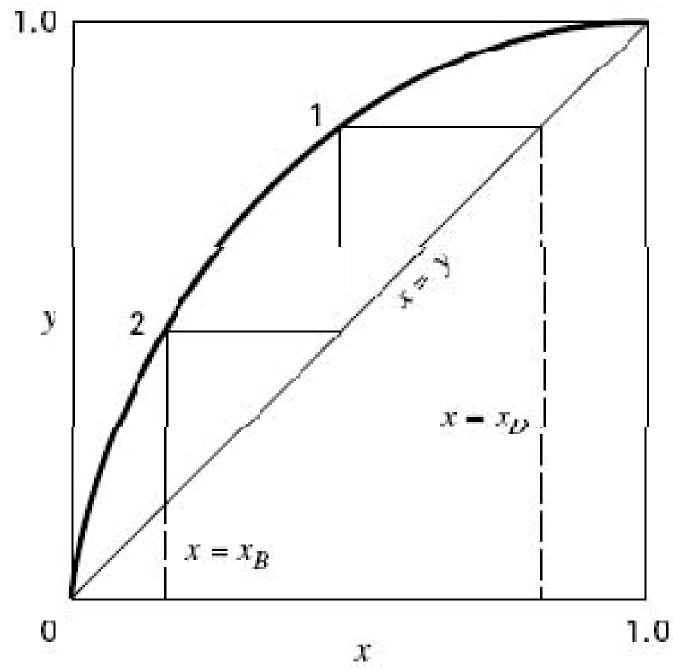


Rectifying operating line

$$y = \frac{L}{V}x + \frac{D}{V}x_D$$

Stripping operating line

$$y = \frac{\bar{L}}{\bar{V}}x - \frac{B}{\bar{V}}x_B$$

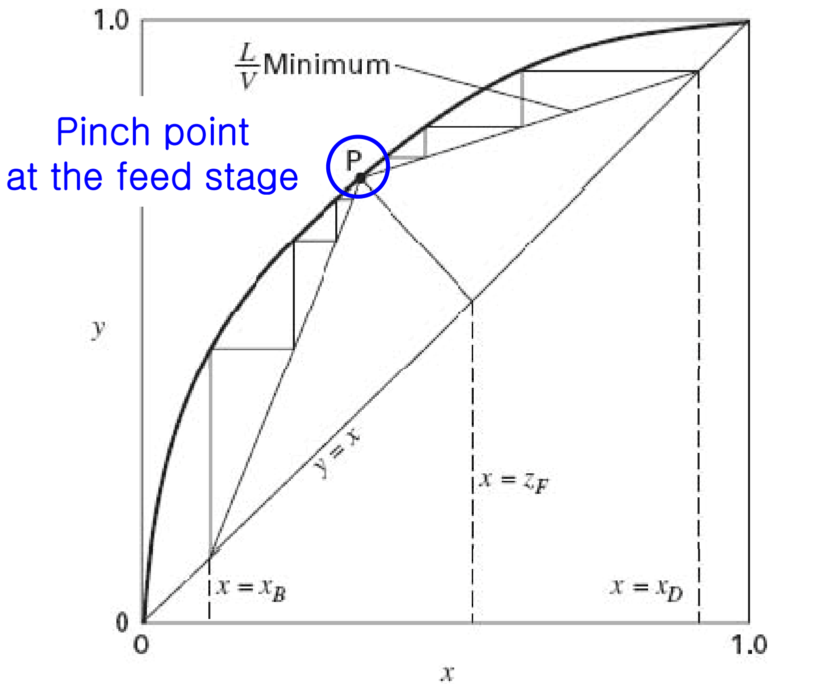


Minimum number of stages
L= V : total reflux
B=D=0 : no product

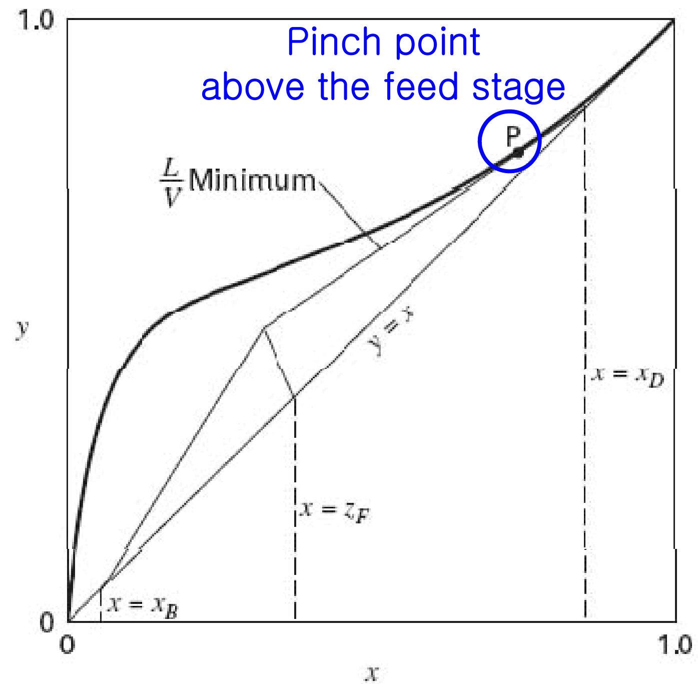
Minimum Reflux Ratio

- The number of equilibrium stages increases when operating line moves closer to equilibrium curve

Typical ideal or near-ideal system



Typical nonideal system

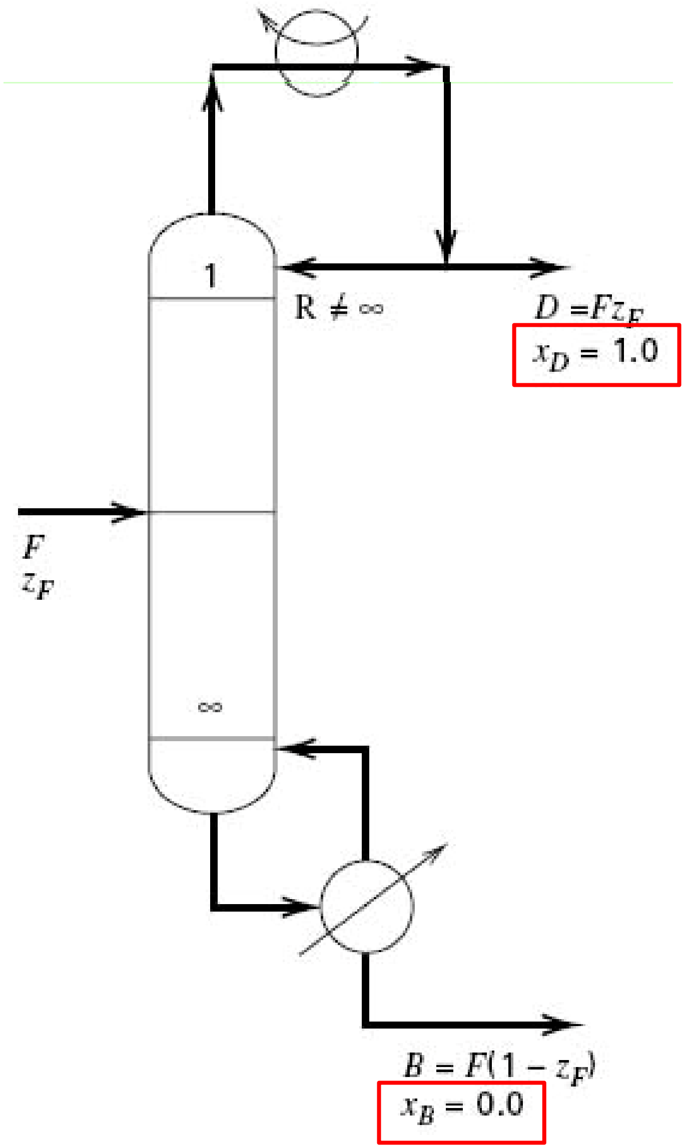


$$(L/V)_{\min} = R_{\min}/(R_{\min}+1) \longrightarrow R_{\min} = (L/V)_{\min}/[1-(L/V)_{\min}]$$

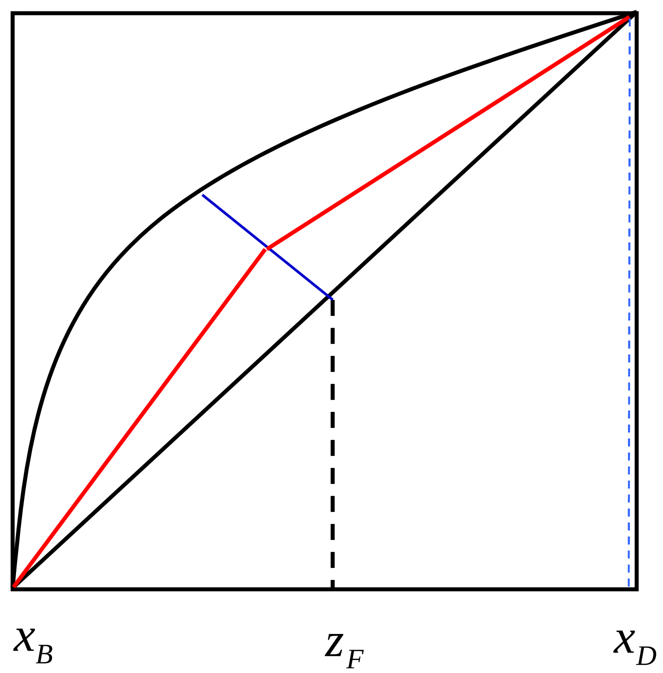
Minimum boilup ratio

$$(V_B)_{\min} = 1/[(\bar{L}/\bar{V})_{\max} - 1]$$

Perfect Separation



- Perfect separation
  $x_D = 1,\ x_B = 0$
- Number of stages : infinite
- Reflux ratio : finite value
- Slope of operating line : finite value



[Example] Distillation of a Binary Mixture of Benzene and Toluene

A trayed tower is to be designed to continuously distill 450 lbmol/hr of a binary mixture of 60 mol% benzene and 40 mol% toluene. A liquid distillate and bottom product of 95 mol% and 5 mol% benzene are to be produced. The feed is preheated so that it enters the column with a molar percent vaporization equal to the distillate-to-feed ratio. Use the McCabe-Thiele method to compute following, assuming a uniform pressure of 1 atm throughout the column.

(a) N_{min}, (b) R_{min}, and (c) N for R/R_{min}=1.3 and the optimal location of feed stage

- Overall material balance on benzene & total balance

$$z_F F = x_D D + x_B B \qquad F = B + D$$

$\longrightarrow$

$$0.60(450) = 0.95(D) + 0.05(B) \qquad 450 = B + D$$

$\longrightarrow$

$$D = 275 \text{ lbmol/h}$$
$$B = 175 \text{ lbmol/h}$$
$$D/F = 0.611$$

[Example] (a) Minimum Number of Theoretical Stages, N_{min}

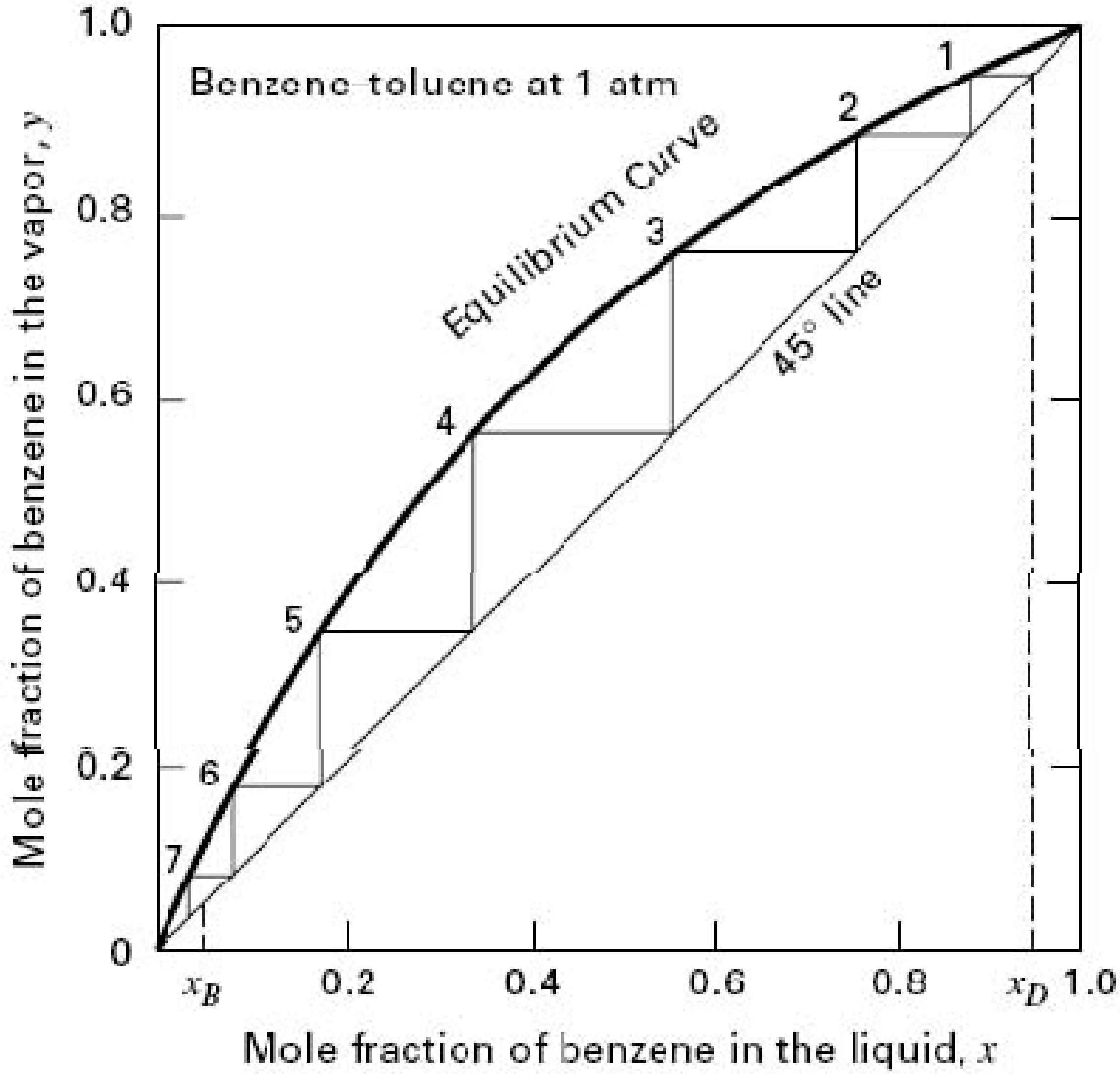
$$\frac{L}{V} \rightarrow 1 \quad \text{and} \quad \frac{\bar{L}}{\bar{\bar{V}}} \rightarrow 1$$

x and y : benzene, more-volatile component

$x_D = 0.95$

$x_B = 0.05$

$\Rightarrow N_{min} = 6.7$



[Example] (b) Minimum Reflux Ratio, R_{min}

$$V_F / F = D / F = 0.611$$

$$q = \frac{L_F}{F} = \frac{(F - V_F)}{F} = 0.389$$

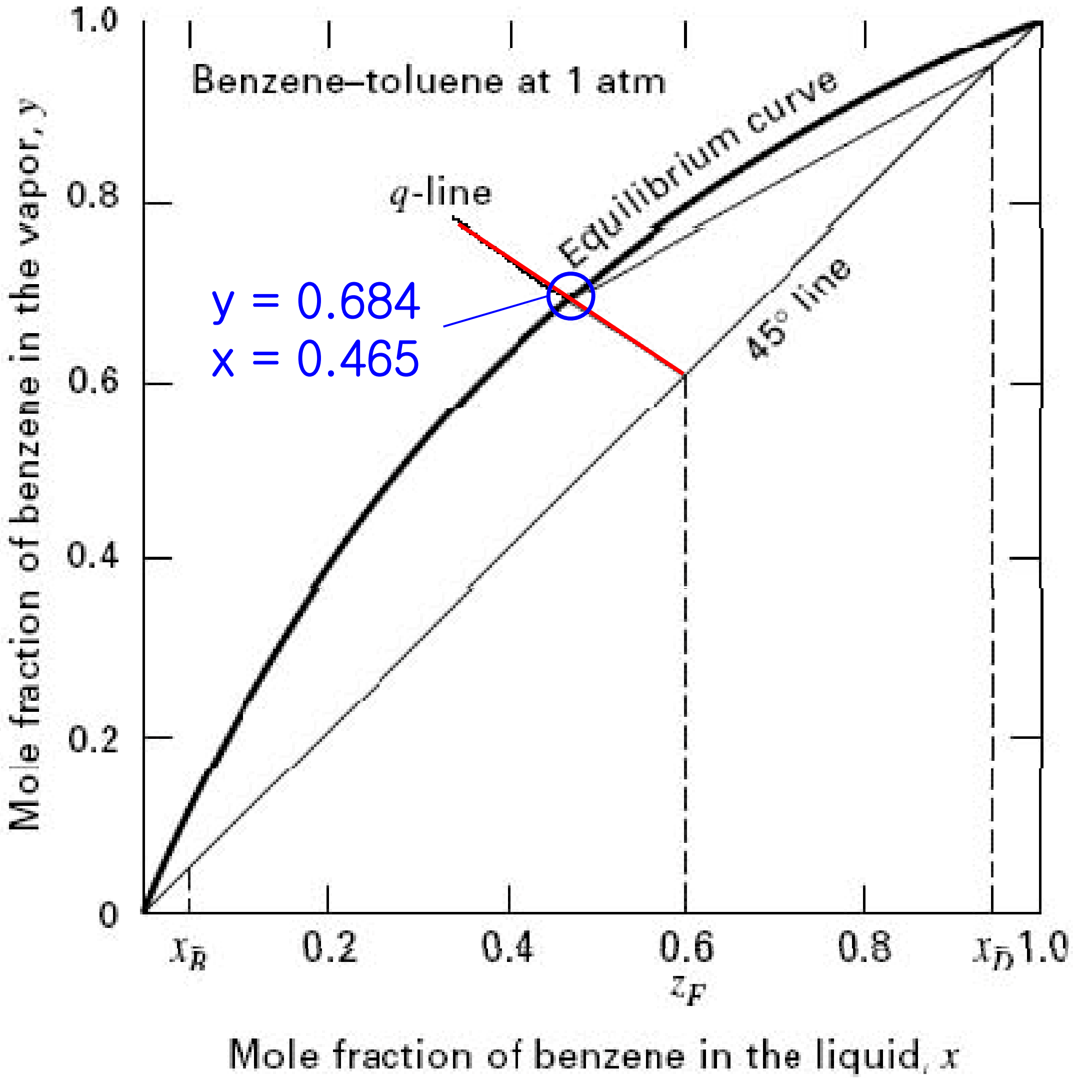
Slope of q–line

$$\frac{q}{q-1} = \frac{0.389}{0.389-1} = -0.637$$

Slope of operating line

$$\frac{0.95-0.684}{0.95-0.465} = 0.55 = \frac{R}{R+1}$$

$$\Rightarrow R_{\min} = 1.22$$



[Example] (c) Number of Equilibrium Stages, N

$$R = 1.3R_{\min}$$
$$= 1.3(1.22) = 1.59$$

Slope of operating line for rectifying section

$$\frac{R}{R+1} = \frac{1.59}{1.59+1} = 0.614$$

$$\Rightarrow N = 13.2$$

Optimal location of feed stage: 7