MODES OF MASS TRANSFER

CHAPTER OBJECTIVES

– *After you have studied this chapter, you should be able to:*

1. Explain the process of molecular diffusion and its dependence on molecular mobility.

2. Explain the process of capillary diffusion

3. Explain the process of dispersion in a fluid or in a porous solid.

4. Understand the process of convective mass transfer as due to bulk flow added to diffusion or dispersion.

5. Explain saturated flow and unsaturated capillary flow in a porous solid

6. Have an idea of the relative rates of the different modes of mass transfer.

7. Explain osmotic flow.

KEY TERMS

- diffusion, diffusivity, and. diffusion coefficient
- dispersion and dispersion coefficient
- hydraulic conductivity
- capillarity
- osmotic flow
- mass and molar flux
- Fick's law
- Darcy's law

1. A Primer on Porous Media Flow

$$n^v = -K\frac{\partial H}{\partial s}$$

$$H = h + z$$

$$v_{average} = \frac{n^v}{\Phi}$$

Physical Interpretation of Hydraulic Conductivity K and Permeability k
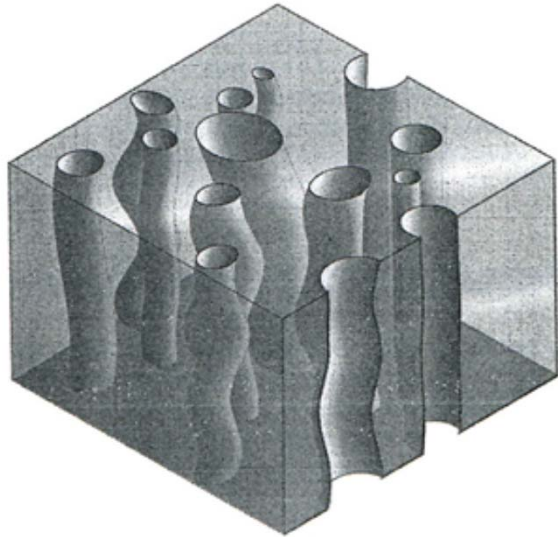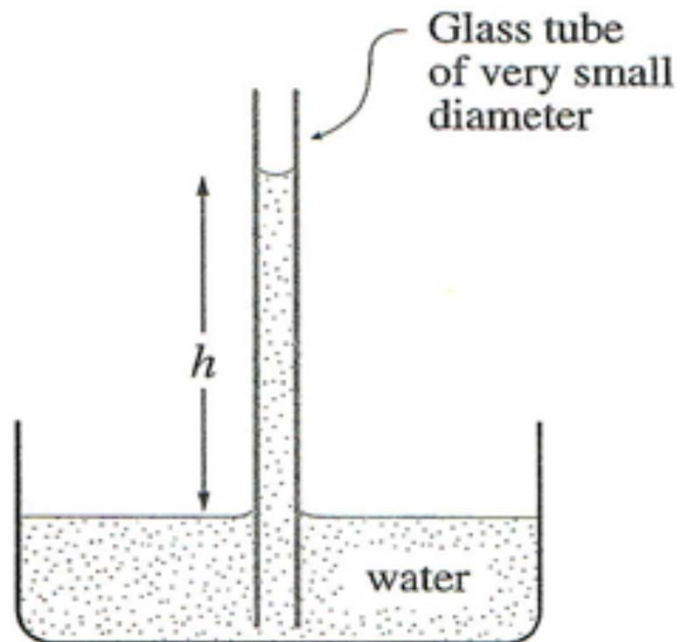

Figure 1. Idealization of a porous media as bundle of tubes of varying diameter and tortuosity.

$$Q_i = - \frac{\pi r_i^4 \rho g}{8\mu} \frac{\partial h}{\partial s}$$

$$n^v = - \frac{\rho g}{8\mu\tau} \frac{\Sigma_i \pi \omega_i r_i^4}{A} \frac{\partial h}{\partial s}$$

$$K = \underbrace{\frac{\rho g}{\mu}}_{\substack{\text{fluid} \\ \text{property}}} \underbrace{\frac{1}{8\tau} \sum_i \triangle \beta_i r_i^2}_{\substack{\text{matrix} \\ \text{property}}}$$

Capillarity and Unsaturated Flow in a Porous Media

Glass tube
of very small
diameter

h

water

$$h = \frac{2\gamma}{\rho.g\gamma}$$

Figure 2. Capillary attraction between the tube walls and the fluid causes the fluid to rise.

Osmotic Flow in a Porous Media
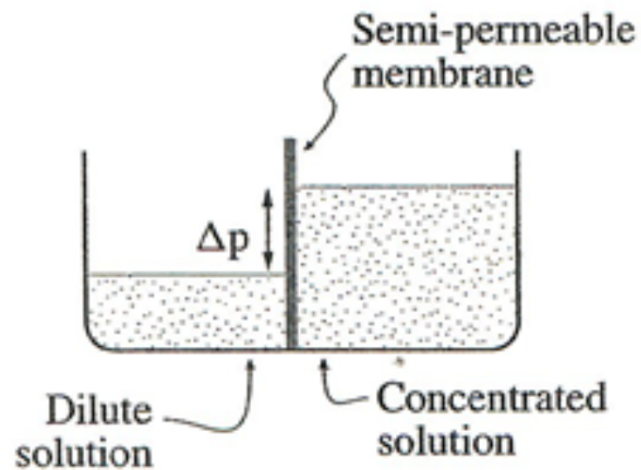
$$\Pi = cRT$$

$$n^v = -\frac{k}{\mu}\frac{\partial}{\partial s}(P - \Pi)$$

Figure 3. Osmotic flow from a dilute to a concentrated solution through a semi-permeable membrane.

2. Molecular Diffusion

• In a material with two or more mass species whose concentrations vary within the material, there is tendency for mass to move. Diffusive mass transfer is the transport of one mass component from a region of higher concentration to a region of lower concentration.

$$\underbrace{j_{A,x}}_{Diffiusive\ flux} = - \underbrace{D_{AB}}_{Diffusivity} \underbrace{\left(\frac{dc_A}{dx}\right)}_{\substack{Concentration \\ gradient}}$$
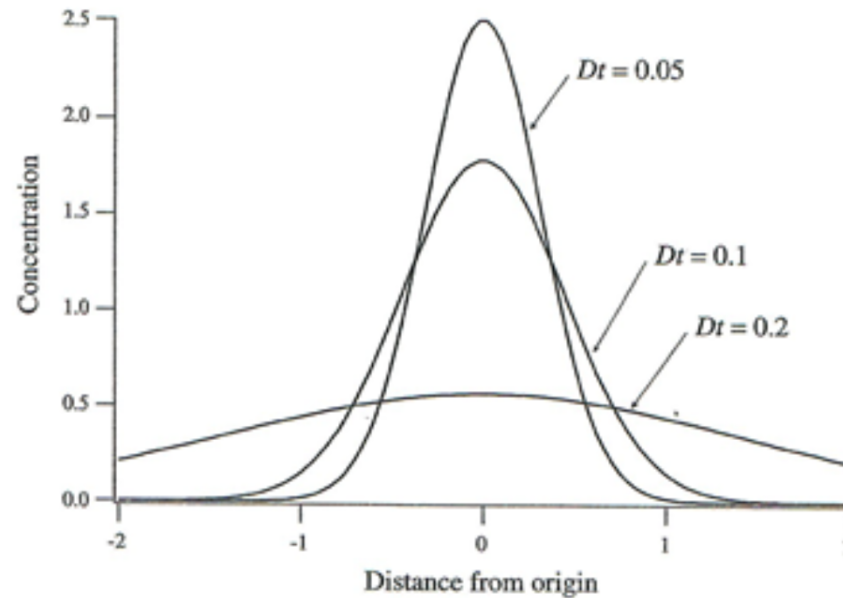
Physical interpretation of diffusivity

Figure 4. Concentration profiles at different times from an instantaneous source placed at zero distance. Here units of and are arbitrary, but related so that is dimensionless.

$$D = \frac{<x^2>}{2t} \qquad u_{diff} = \frac{(<x^2>)^{1/2}}{t} = \sqrt{\frac{2D}{t}}$$

Diffusivity for gases

$$D_{AB} = \frac{0.001858\,T^{3/2}(1/M_A + 1/M_B)^{1/2}}{p\sigma_{AB}^2\,\Omega_{D,AB}}$$
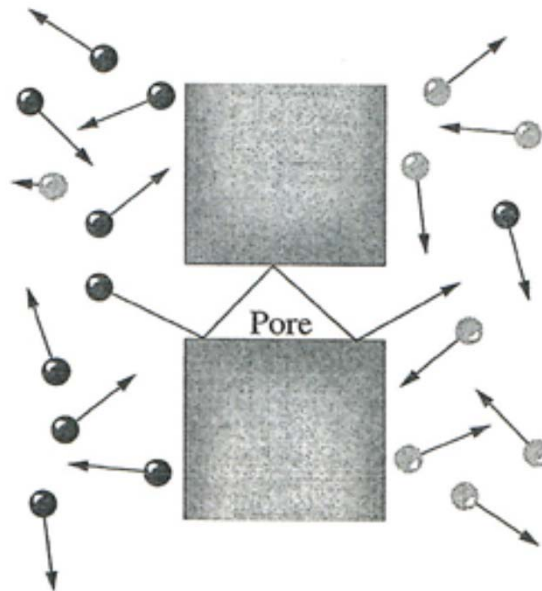

Figure 5. Schematic of Knudsen diffusion of a gas in a solid.

Diffusivity for liquids

$$f = 6\pi\mu r$$

$$D = \frac{\kappa T}{f}$$

$$D = \frac{\kappa T}{6\pi\mu r}$$

Capillary Diffusion

$$n^v = -K\frac{\partial h}{\partial s}$$
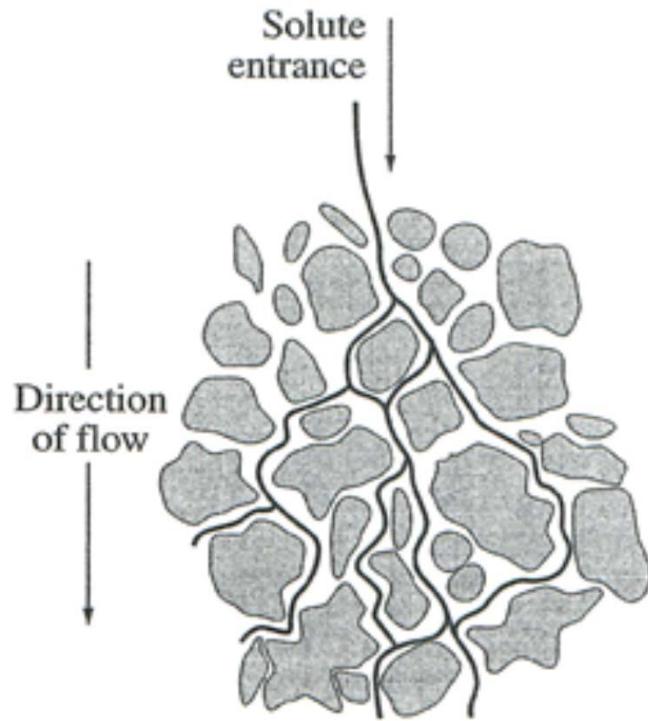
$$n = -\rho_{liquid}K\frac{\partial h}{\partial s}$$

$$c = c^*\rho_{liquid}$$

Capillary Diffusion

$$n = -\rho_{liquid} K \frac{\partial h}{\partial c} \frac{\partial c}{\partial s}$$

$$= -\frac{K}{\dfrac{\partial(c/\rho_{liquid})}{\partial h}} \frac{\partial c}{\partial s}$$

$$= -\frac{K}{\left(\dfrac{\partial c^{*}}{\partial h}\right)} \frac{\partial c}{\partial s}$$

$$= -\underbrace{D_{cap}}_{\substack{\text{capillary} \\ \text{diffusivity}}} \underbrace{\frac{\partial c}{\partial s}}_{\substack{\text{concentration} \\ \text{gradient}}}$$

$$D_{cap} = \frac{K}{\left(\dfrac{\partial c^{*}}{\partial h}\right)}$$

3. Dispersive Mass Transfer

Figure 6. Convection-dispersion of fluid in a porous media.

$$\underbrace{j_{A,x}}_{Dispersive\,flux} = - \underbrace{E_x}_{\substack{Dispersion \\ coefficient}} \underbrace{\left(\frac{dc_A}{dx}\right)}_{\substack{Concentration \\ gradient}}$$

4. Convective Mass Transfer

- Convective mass transfer is studied the same way as convective heat transfer. It is the added effect of bulk flow on diffusion or dispersion. Convective mass transfer is the movement of mass through a medium as a result of the net motion of a material in the medium
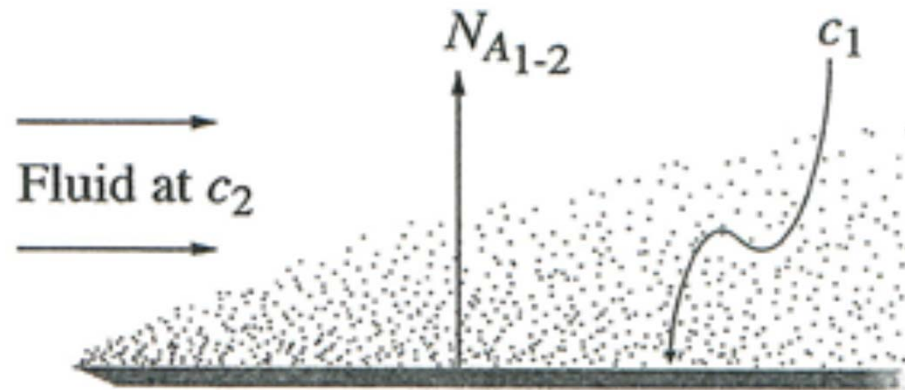
Convection-Diffusion Mass Transfer Over a Surface

Figure 7. Schematic of convection-diffusion over a surface.

$$\underbrace{N_{A_{1-2}}}_{\text{Mass flow rate}} = \underbrace{h_m}_{\substack{\text{Convective} \\ \text{coefficient}}} \underbrace{A}_{\text{Area}} \underbrace{(c_1 - c_2)}_{\substack{\text{Concentration} \\ \text{difference}}}$$

5. Comparison of the Modes of Mass Transfer

- Molecular diffusion

- Capillary "diffusion"

- Dispersion

- Convection

- Advection

Chapter Summary-Modes of Mass Transfer

- **(Bulk) Flow Through Porous Media** 1. It is the bulk movement of a fluid in a porous media due to hydraulic forces (pressure, matric attractive forces, and gravity).

2. It is described by Darcy's law (Eqn. 10.1) which states that the volumetric flux of a fluid is proportional to the gradient of the hydraulic potential. The proportionality constant is termed the hydraulic conductivity.

3. Hydraulic conductivity depends on the fluid property called permeability (that depends on density and viscosity) and matrix property such as the pore size distribution, shape of pores, porosity, and tortuosity.

4. When the porous media is unsaturatesf, the flow is driven primarily by the matric attractive (capillary) forces. Flow in such a media can be written in terms of Pick's law and an apparent (capillary) diffusivity.

Chapter Summary-Modes of Mass Transfer

- **Molecular Diffusion**

1. It is the movement of mass from a higher to a lower concentration due to random molecular motion.

2. It is described by Fick's law (Eqn. 10.18) which states that the flux of a mass species is proportional to the concentration gradient of that species. The proportionality constant is called the diffusivity.

3. Diffusivity measures speed of movement- it is one half of the mean square displacement of a number of molecules per unit time through a given medium.

4. Diffusivity for gases are the highest due to their highest mobility. They are lower for liquids and the lowest for solids.

5. Diffusivity of a gas in porous solid depends on the porous structure.

6. Diffusivity of a liquid in a porous media is really a measure of capillary flow .

Chapter Summary-Modes of Mass Transfer

- **Dispersion**

1. Dispersion is analogous to diffusion and is described by Eqn. 10.31. However, dispersion is due to completely separate mechanisms- turbulent flow in a fluid or flow through uneven and tortuous paths in a porous media.

2. Dispersion coefficients are generally much higher than diffusion coefficients.

- **Convection**

1. It is the effect of adding bulk flow to diffusion or dispersion. Convection-diffusion mass transfer over a surface is described by Eqn. 10.33 in complete analogy to heat transfer.