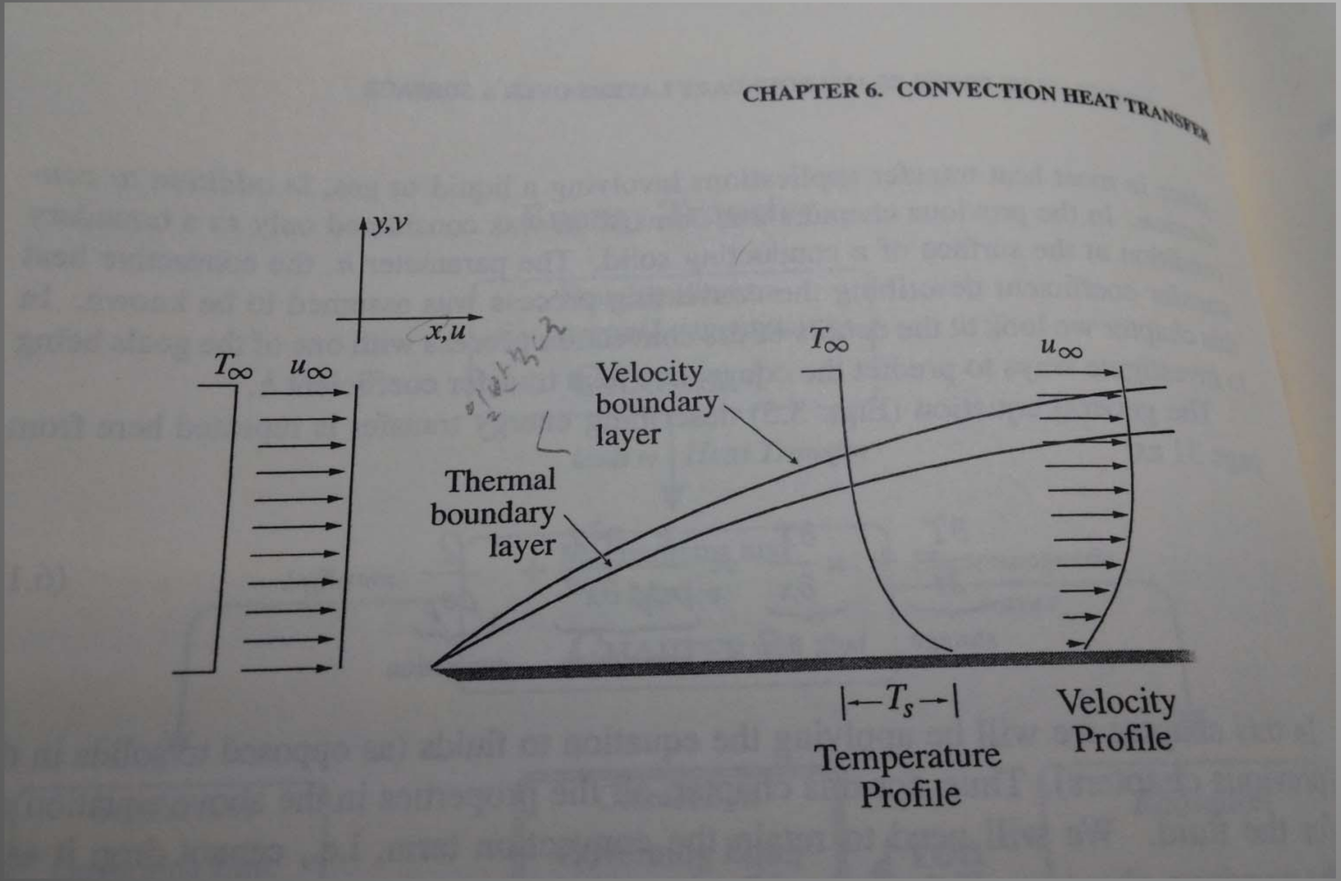
Convection **Heat Transfer**

Governing Equation for convection

$$\underbrace{\frac{\partial T}{\partial t}}_{storage} + \underbrace{u\frac{\partial T}{\partial x}}_{bulk\,flow} = \underbrace{\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial x^2}}_{conduction} + \underbrace{\frac{Q}{\rho c_p}}_{generation}$$

Temperature profiles and Boundary Layers Over a Surface



Temperature profiles and Boundary Layers Over a Surface

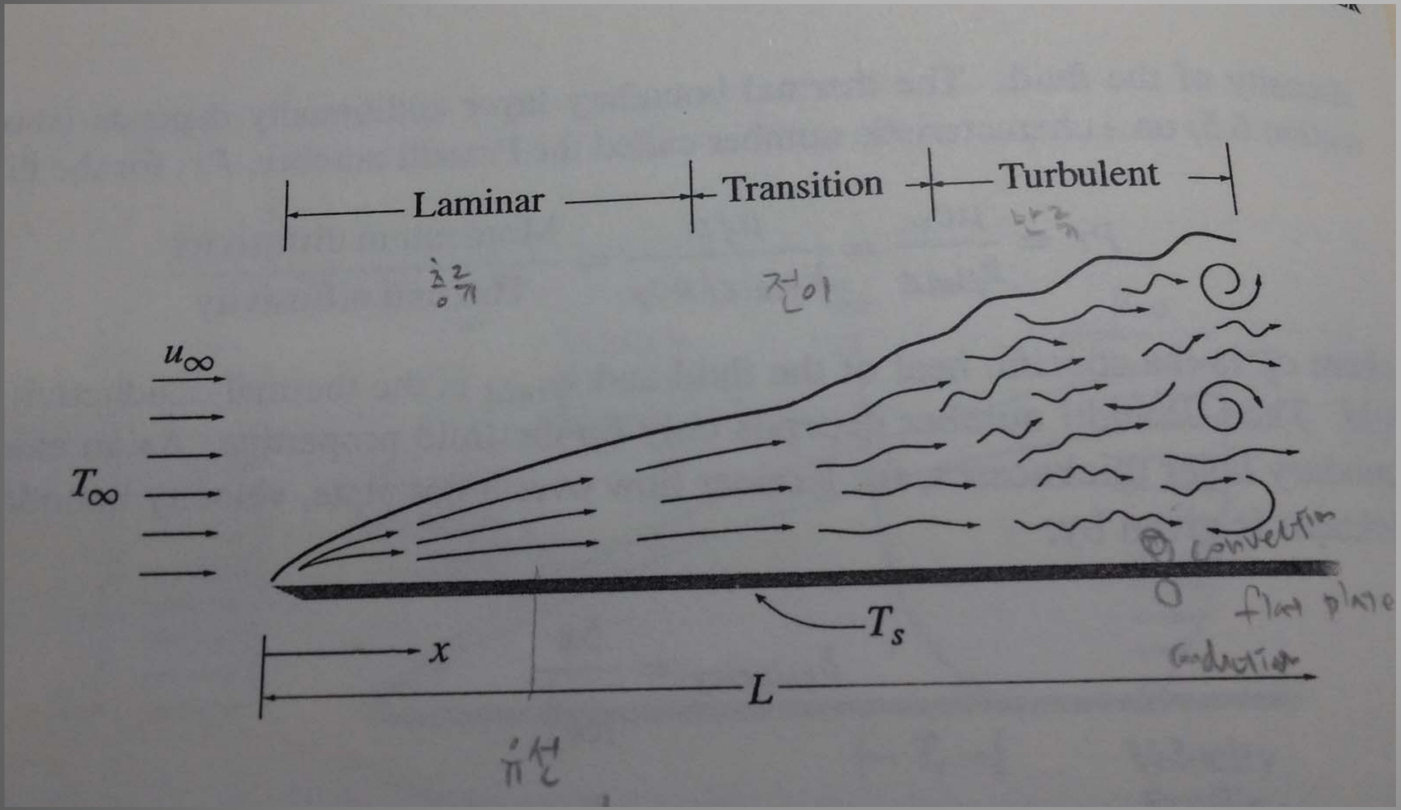
$$\frac{T_s - T_{\delta_{thermal}}}{T_s - T_\infty} = 0.99$$

$$\boldsymbol{Re_x} = \frac{u_\infty x \rho}{\mu}$$

$$\boldsymbol{Pr} = \frac{\mu c_p}{k_{fluid}} = \frac{\mu/\rho}{k_{fluid}/\mu c_p} = \frac{Momentum\ diffusivity}{Thermal\ diffusivity}$$

$$\delta_{thermal} = \frac{\delta_{velocity}}{Pr^{1/3}}$$

Laminar and Turbulent Flows



Laminar region $Re_x < 2 \times 10^5$

Transition region $2 \times 10^5 < Re_x < 3 \times 10^6$

Turbulent region $3 \times 10^6 < Re_x$

Convective Heat Transfer Coefficient Defined

Conductive heat Flux in the fluid $= -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0, in\ fluid}$

Convective Heat flux $= h\left(T_{y=0, in\ fluid} - T_{\infty}\right)$

$$= h(T_s - T_{\infty})$$

$$-k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0, in\ fluid} = h(T_s - T_{\infty})$$

$$h = \frac{-k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0, in\ fluid}}{(T_s - T_{\infty})}$$

Significant Parameters in Convective Heat Transfer

$$\underbrace{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}}_{bulk\ flow}=\underbrace{\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)}_{conduction}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\rho\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\rho\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad u \gg v$$

$$\frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x},\frac{\partial v}{\partial x},\frac{\partial v}{\partial y} \qquad \frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\rho\frac{\partial p}{\partial x}+\nu\frac{\partial^2 T}{\partial y^2}$$

Significant Parameters in Convective heat transfer

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\rho\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$$

$$\downarrow$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{\alpha}{u_\infty L}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\rho\frac{\partial p^*}{\partial x^*}+\frac{\nu}{u_\infty L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\begin{aligned}
x^* &= x/L\\
y^* &= y/L\\
u^* &= u/u_\infty\\
v^* &= v/u_\infty\\
T^* &= (T-T_s)/(T_\infty-T_s)\\
p^* &= p/\rho {u^2}_\infty
\end{aligned}$$

Significant Parameters in Convective heat transfer

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{\alpha}{u_\infty L}\frac{\partial^2 T^*}{\partial {y^*}^2}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\rho\frac{\partial p^*}{\partial x^*}+\frac{\nu}{u_\infty L}\frac{\partial^2 u^*}{\partial {y^*}^2}$$

$$\downarrow$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{Re_L\cdot Pr}\frac{\partial^2 T^*}{\partial {y^*}^2}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\rho\frac{\partial p^*}{\partial x^*}+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial {y^*}^2}$$

$$Re_L=\frac{u_\infty L}{\nu}$$

$$Pr=\frac{\rho c_p}{k}$$

Significant Parameters in Convective Heat Transfer

$$T^* = \left(x^*, y^*, Re_L, Pr, \frac{\partial p^*}{\partial x^*}\right)$$

$$T^* = (x^*, y^*, Re_L, Pr\ )$$

$$h = \frac{-k_{fluid} \frac{\partial T}{\partial y}\Big|_{y=0, in\ fluid}}{(T_S - T_\infty)} = \frac{k_{fluid}}{L} \frac{\partial\left(\frac{(T-T_S)}{(T_\infty - T_S)}\right)}{\partial\left(\frac{y}{L}\right)} = \frac{k_{fluid}}{L} \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0}$$

$$\frac{hL}{k_{fluid}} = \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0}$$

$$\frac{hL}{k_{fluid}} = f\left(x^*, Re_L, Pr\right)$$

Significant Parameters in Convective Heat Transfer

$$h_L = \frac{\int_0^L hdx}{L}$$

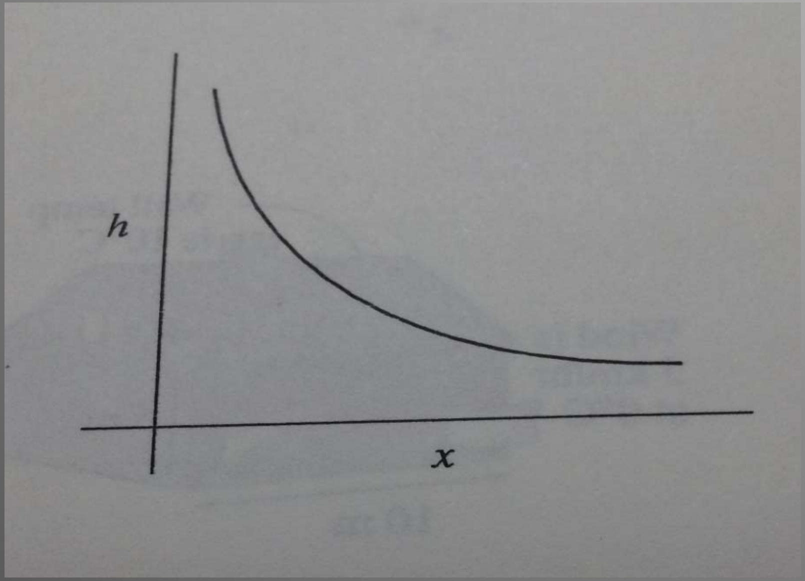
$$\frac{hL}{k_{fluid}} = f(Re_L, Pr) = Nu_L$$

Calculation and Physical Implications of Convective Heat Transfer Coefficient Values

Dimensionless	**Definition and physical significance**
Reynolds number	$Re_L = \frac{u_\infty L}{\nu} = \frac{\text{Inertia force}}{\text{Viscous force}}$
Nusselts number	$Nu = \frac{hL}{k_{fluid}} = \frac{\text{Diffusive resistance}}{\text{Convective resistance}}$
Prandtl number	$Pr = \frac{\rho c_p}{k} = \frac{\text{Viscous effect}}{\text{Thermal diffusion dffect}}$
Biot number	$Bi = \frac{hL}{k_{solid}} = \frac{\text{Internal diffusive resistance}}{\text{Surface convective resistance}}$
Grashof number	$Gr = \frac{\beta g L^3 \Delta T}{\nu^2} = \frac{\text{Buoyancy force}}{\text{Viscous force}}$
Rayleigh number	$Ra = Gr \times Pr$

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

$$h = \frac{-k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0, in\ fluid}}{(T_s - T_\infty)} \rightarrow \infty \quad at\ y = 0$$



$$T_f = \frac{T_s + T_\infty}{2}$$

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

Flat plate, Forced Convection

$$Nu_x = 0.332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}} \quad for\ laminar(Re_x < 2 \times 10^5)$$

$$Nu_L = 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} \quad for\ laminar(Re_L < 2 \times 10^5)$$

$$Nu_x = 0.0288 Re_x^{\frac{4}{5}} Pr^{\frac{1}{3}} \quad for\ turbulent(Re_x < 3 \times 10^6)$$

$$Nu_L = 0.0360 Re_L^{\frac{4}{5}} Pr^{\frac{1}{3}} \quad for\ turbulent(Re_L < 3 \times 10^6)$$

6.6 Calculation and Physical Implications of convective Heat Transfer Coefficient Values

6.6.1 Flat plate, Forced Convection

Example 6.6.1 Heat Transfer Coefficient Over a Building Wall
Wind is blowing at 5 km/hour over a building wall of size 5 m * 10 m, as shown in Figure 6.5. Air temperature is 0℃ and the wall surface temperature is 10℃.

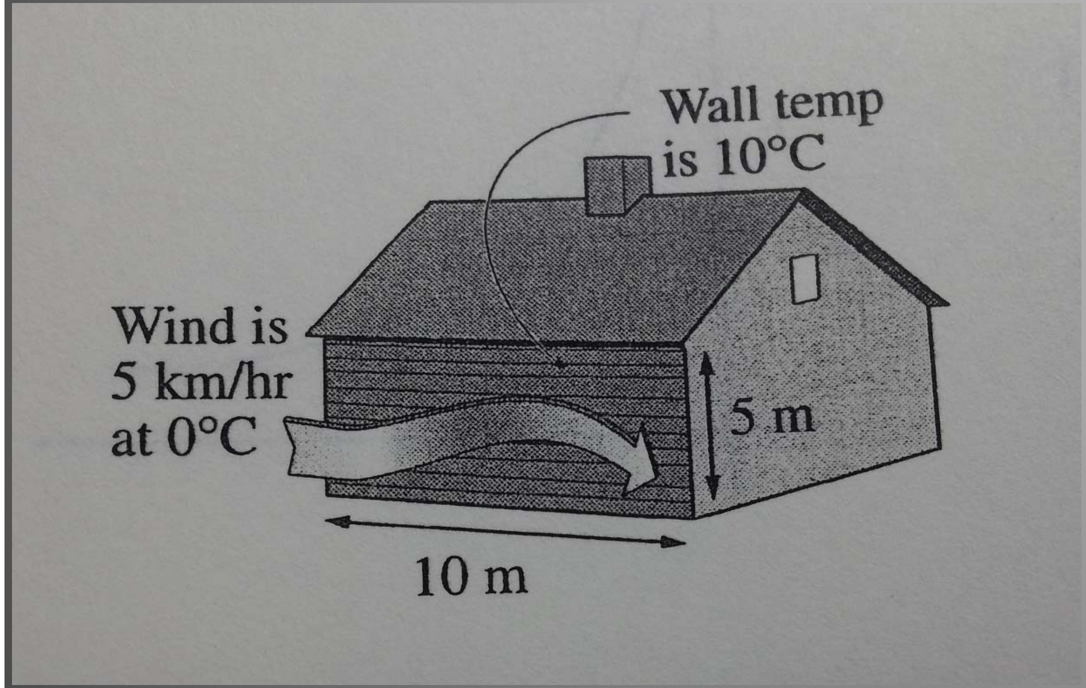
1) Calculate the average heat transfer coefficient along the 10 m width of the wall.

2) Calculate the local heat transfer coefficient at a location 10 cm from the leading edge of the wall.

Known: Speed of wind that is blowing over a building wall
Find: The average heat transfer coefficient over the wall surface and heat transfer coefficient at a given location
Schematic and Given Data: Schematic of this problem is shown in Figure 6.5.

The given data are
The wind speed is 5 km/hour
The wall dimensions are 5m * 10 m
the air temperature is 0℃ and the wall surface temperature is 10℃

Calculation and Physical Implications of convective Heat Transfer Coefficient Values



Assumptions:
Heat transfer coefficient is assumed to vary only along the direction of the air flow
The wall can be treated as a perfect flat plate

Analysis:
This heat transfer situation is treated as a flat plate under forced convection. We have to then determine if the flow is laminar or turbulent. for this, we need to find the Reynolds number $Re = u_\infty L\rho/\mu$ as follows:

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

$$u_\infty = \frac{5 \times 1000}{3600} = \frac{1.39\mathrm{m}}{\mathrm{s}}$$

$$\rho = \text{density of air at the film temperature of } \frac{(0+10)}{2} = 5℃$$

$$= 1.2708 kg/m^3$$

$$L = \text{characteristic length along the flow} = 10\mathrm{m}$$

$$\mu = \text{viscosity of air at the film temperature of } 5℃$$

$$= 1.7404 \times 10^5 [kg/m \cdot s]$$

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

$$\begin{aligned} Re_L &= \frac{u_\infty L \rho}{\mu} \\ &= \frac{1.39[m/s]10[m]1.2708[kg/m^3]}{1.7404 \times 10^{-5}[kg/m \cdot s]} \\ &= 1.015 \times 10^6 \end{aligned}$$

$$\begin{aligned} Nu_L &= 0.036 Re_L^{\frac{4}{5}} Pr^{\frac{1}{3}} \\ \frac{h[W/m^2 \cdot K]10[m]}{0.0245[W/m \cdot K]} &= 0.036(1.015 \times 10^6)^{4/5}(0.714)^{1/3} \\ h &= 5.04 W/m^2 \cdot K \end{aligned}$$

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

$$Re_x = \frac{u_\infty x \rho}{\mu}$$

$$= \frac{1.39[m/s]0.1[m]1.2708[kg/m^3]}{1.7404 \times 10^{-5}[kg/m \cdot s]}$$

$$= 1.015 \times 10^4$$

$$Nu_x = 0.332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

$$\frac{h[W/m^2 \cdot K]0.1[m]}{0.0245[W/m \cdot K]} = 0.332(1.015 \times 10^4)^{1/2}(0.714)^{1/3}$$

$$h = 7.32 W/m^2 \cdot K$$

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

Natural Convection

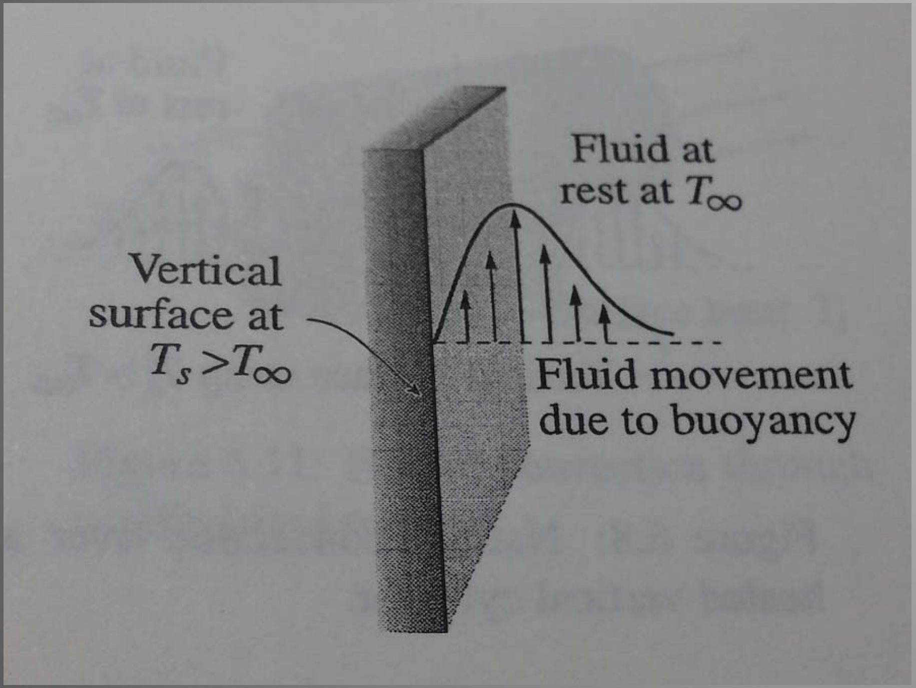
$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\bigg|_{p=constant}$$

$$\rho = P/R_g T$$

$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\bigg|_{p=constant} = \frac{1}{\rho}\frac{P}{R_g T^2} = \frac{1}{T}$$

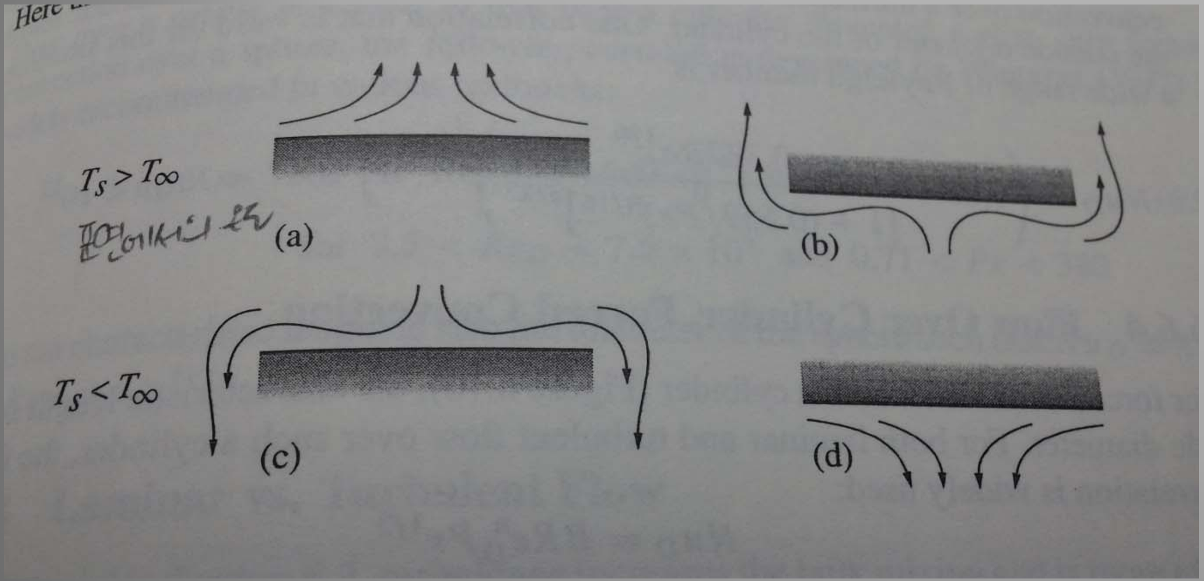
$$Gr = \frac{\beta g L^3 \Delta T}{\nu^2}$$

6.6 Calculation and Physical Implications of convective Heat Transfer Coefficient Values



$$Nu_L = \left(0.825 + \frac{0.387 {Ra_L}^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right)^2$$

6.6 Calculation and Physical Implications of convective Heat Transfer Coefficient Values



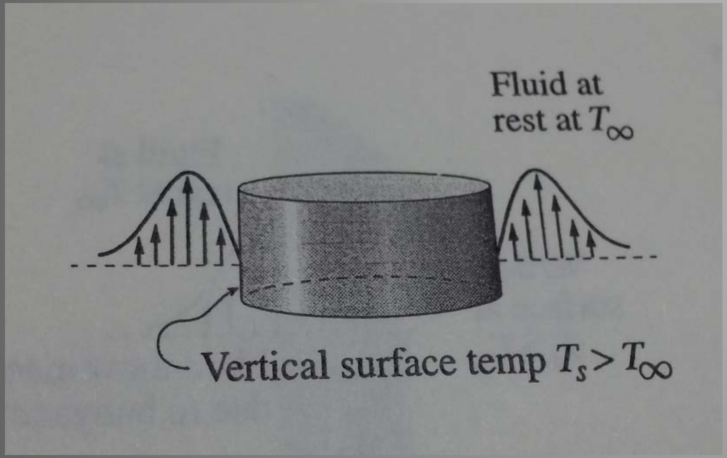
$$Nu_L = 0.54 Ra_L^{\frac{1}{4}} \; (10^5 < Re_L < 2 \times 10^7)$$

$$Nu_L = 0.14 Ra_L^{\frac{1}{4}} \; (2 \times 10^7 < Re_L < 3 \times 10^{10})$$

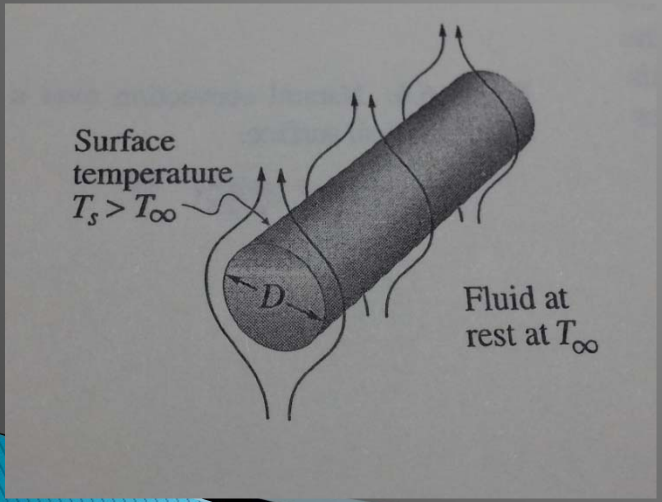
$$Nu_L = 0.27 Ra_L^{\frac{1}{4}} \; (3 \times 10^{10} < Re_L < 10^{10})$$

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

Flow Over Cylinder, Natural Convection



$$\frac{D}{L} \geq \frac{35}{Gr_L^{\frac{1}{4}}}$$

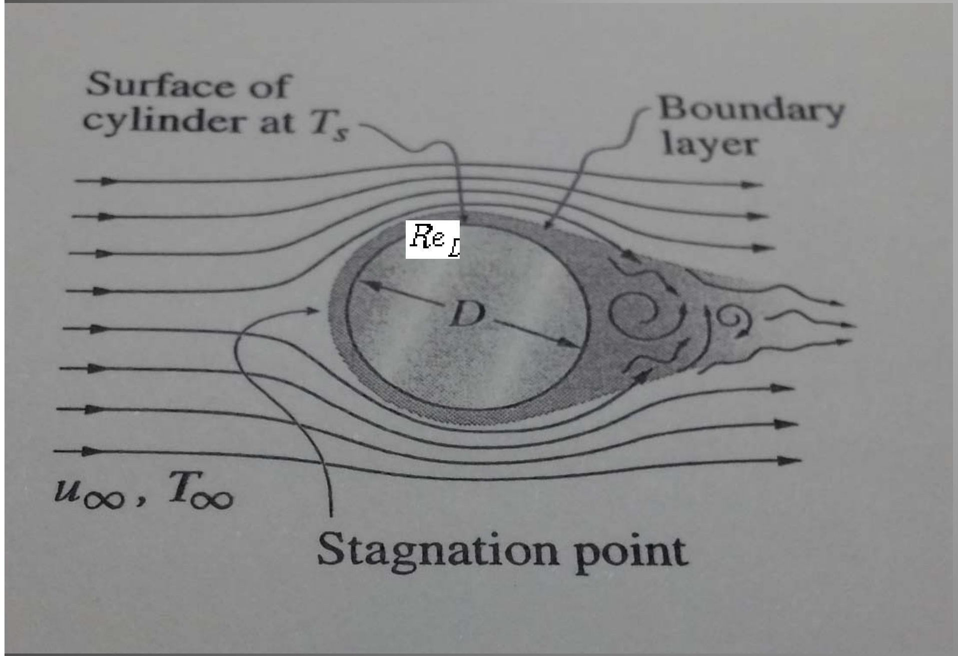


$$Nu_L = \left(0.60 + \frac{0.387 Ra_D{}^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right)^2$$

$$for \quad 10^{-5} < Ra_D < 10^{12}$$

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

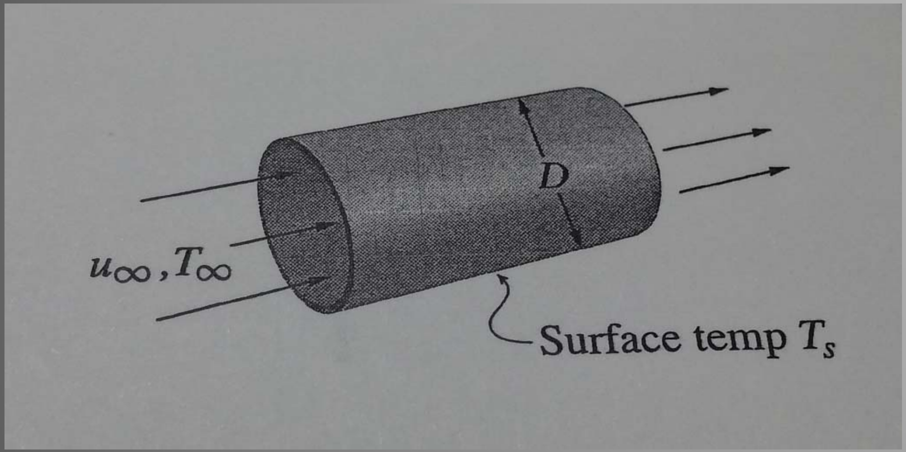
Flow Over Cylinder, Forced Convection



$$Nu_D = B{Re_D}^n Pr^{\frac{1}{3}}$$

	B	n
0.4~4	0.989	0.330
4~40	0.911	0.385
40~4000	0.683	0.366
4000~40,000	0.193	0.618
40,000~400,000	0.027	0.805

Calculation and Physical Implications of convective Heat Transfer Coefficient Values



$$Nu_D = 3.66 \text{ for } Re_D \leq 2300$$

$$Nu_D = 0.023 {Re_D}^{0.8} Pr^n \text{ for } Re_D \leq 10{,}000 \; and \frac{L}{D} \geq 10$$

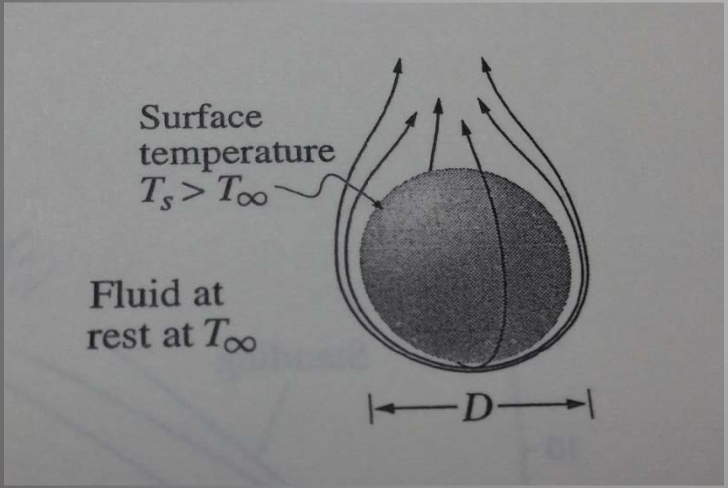
$$= 0.6 \;\leq \mathrm{Pr} \leq\; 160$$

$n = 0.3$ for fluid being cooled and

$n = 0.4$ for fluid being heated

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

Flow Over Sphere, Natural Convection



$$Nu_D = 2 + 0.43 Ra_D^{\frac{1}{4}} \quad for\ 1 < Ra_D < 10^5, \qquad Pr \cong 1$$

$$Nu_D = hD/k \qquad\qquad Gr_D = \frac{\beta g D^3 \Delta T}{\nu^2}$$

Calculation and Physical Implications of convective Heat Transfer Coefficient Values

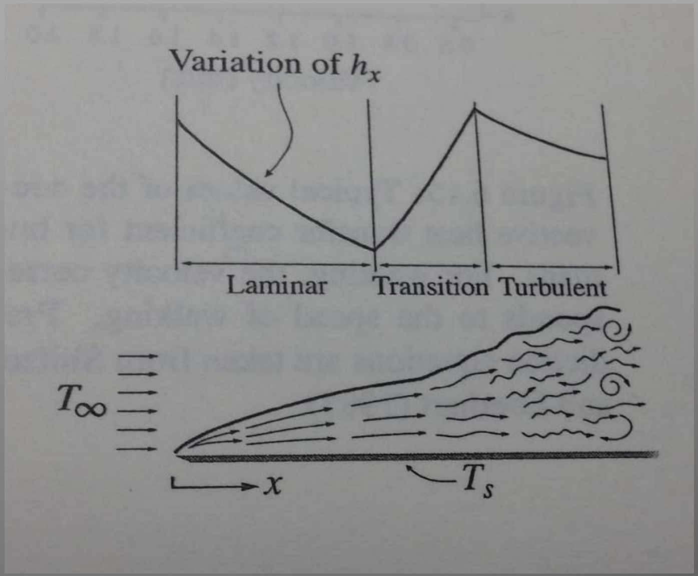
Flow Over Sphere, Forced Convection

$$Nu_D = 2 + \left(0.4{Re_D}^{1/2} + 0.06{Re_D}^{2/3}\right)Pr^{0.4}$$

$$\text{for} \quad 0.5 \quad < Re_D < 7.6 \times 10^4$$

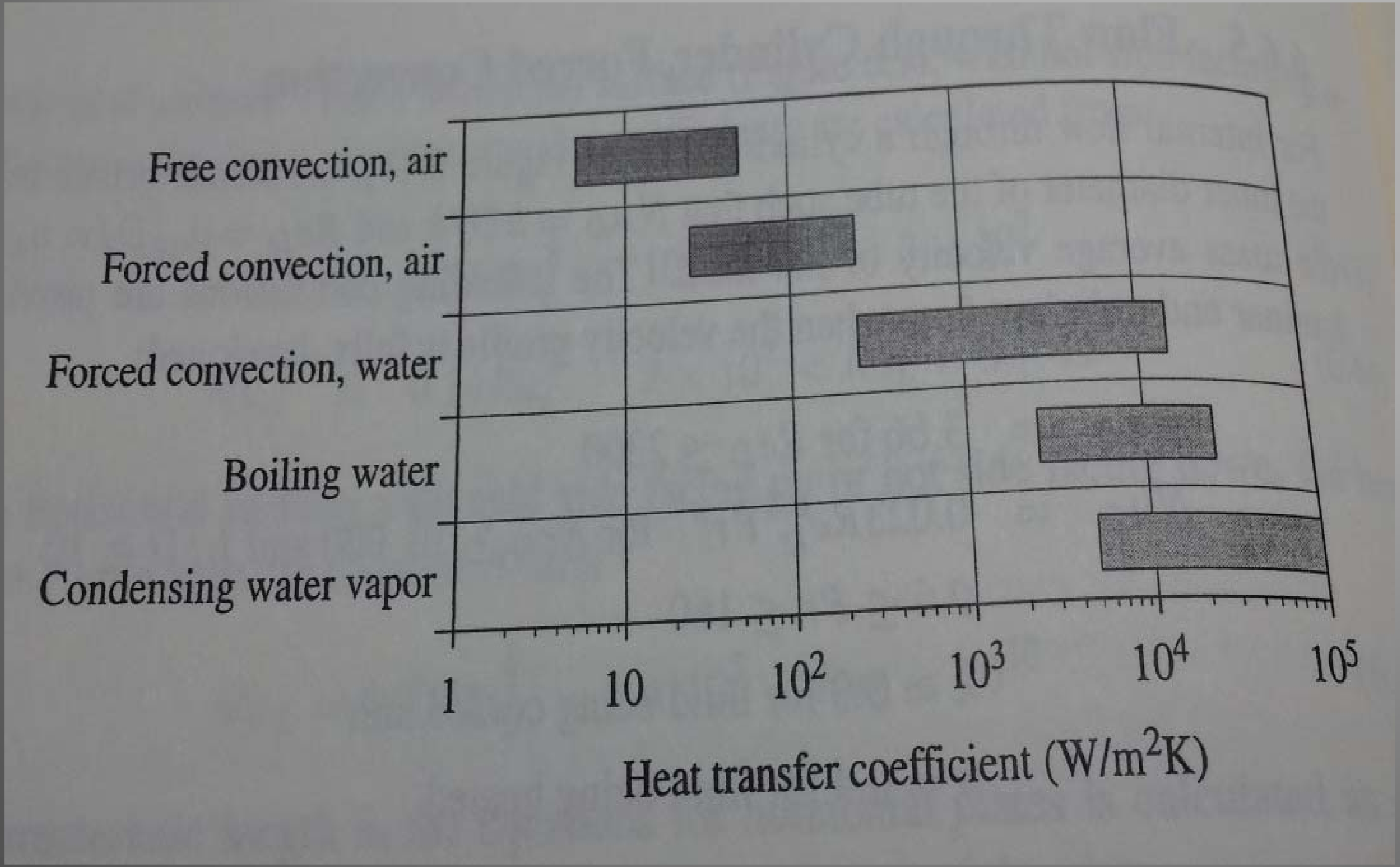
$$and\ 0.71 \ \leq \ \ \mathrm{Pr} \ \leq \ 380$$

Laminar vs. Turbulent Flow

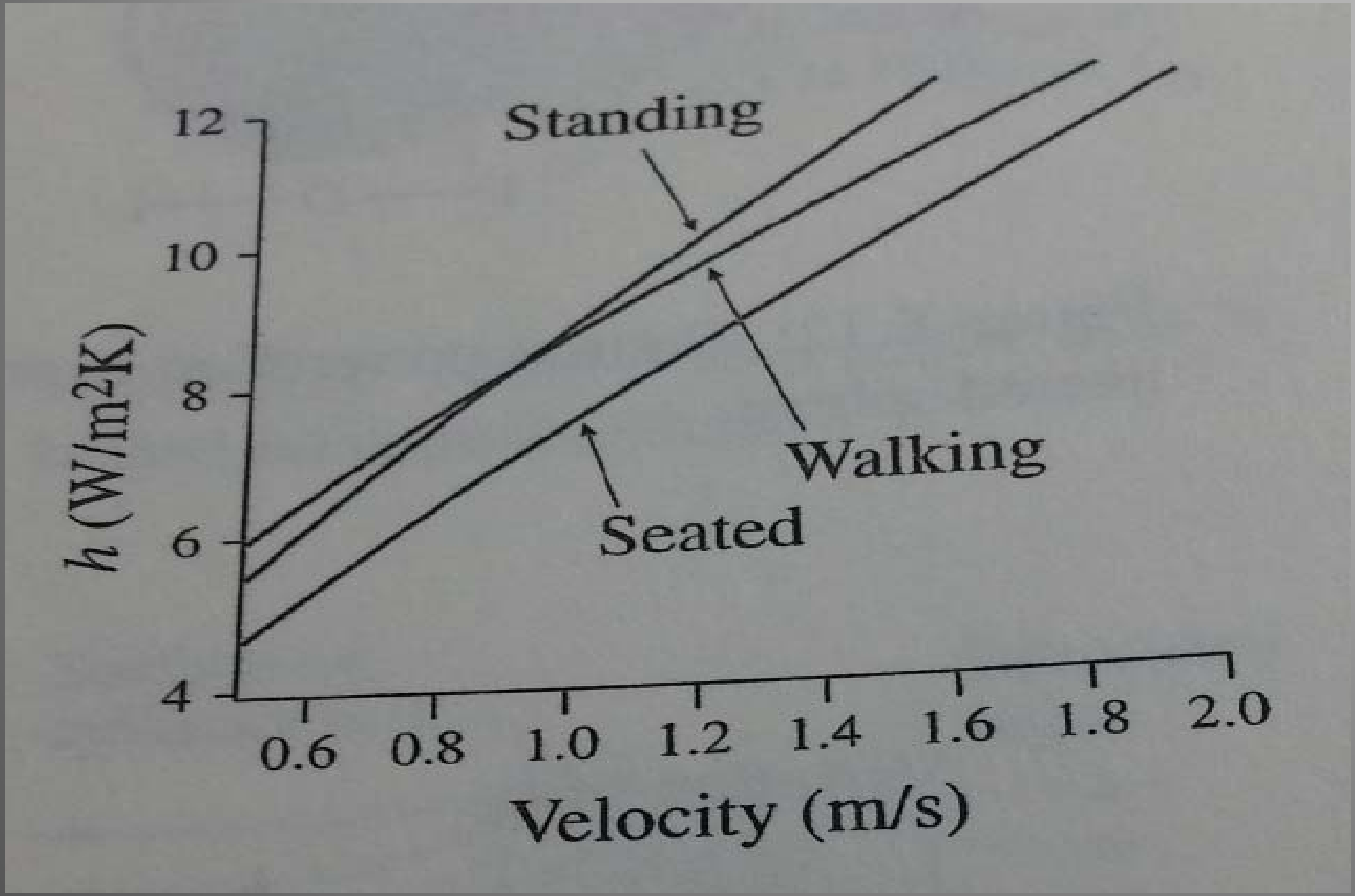


$$\frac{Nu_{l,\mathrm{turbulent}}}{Nu_{l,\mathrm{laminar}}} = 0.0542{Re_L}^{0.3}$$

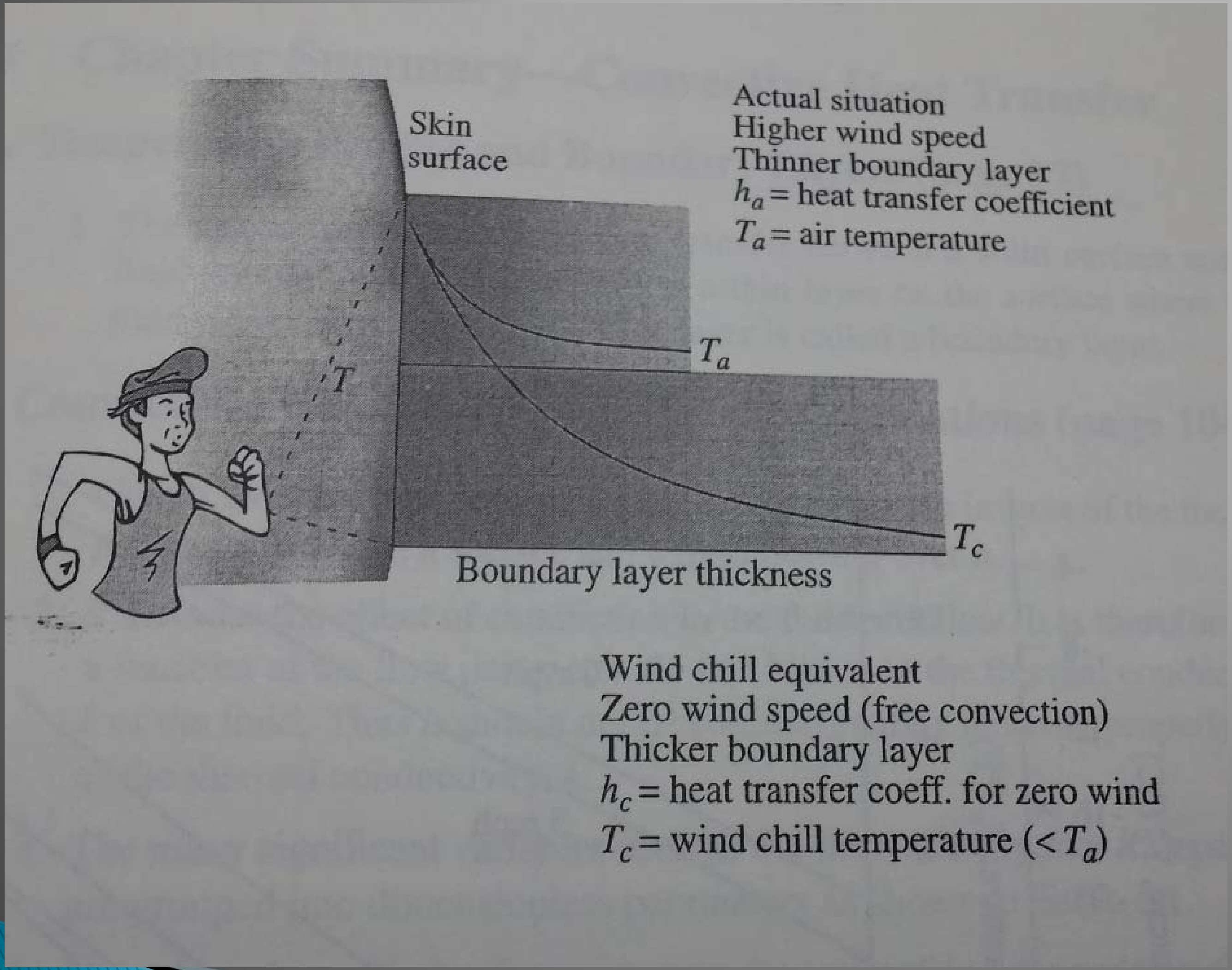
Orders of Magnitude for Heat Transfer Coefficient Values



Coefficients for Air Flow Over Human Subject

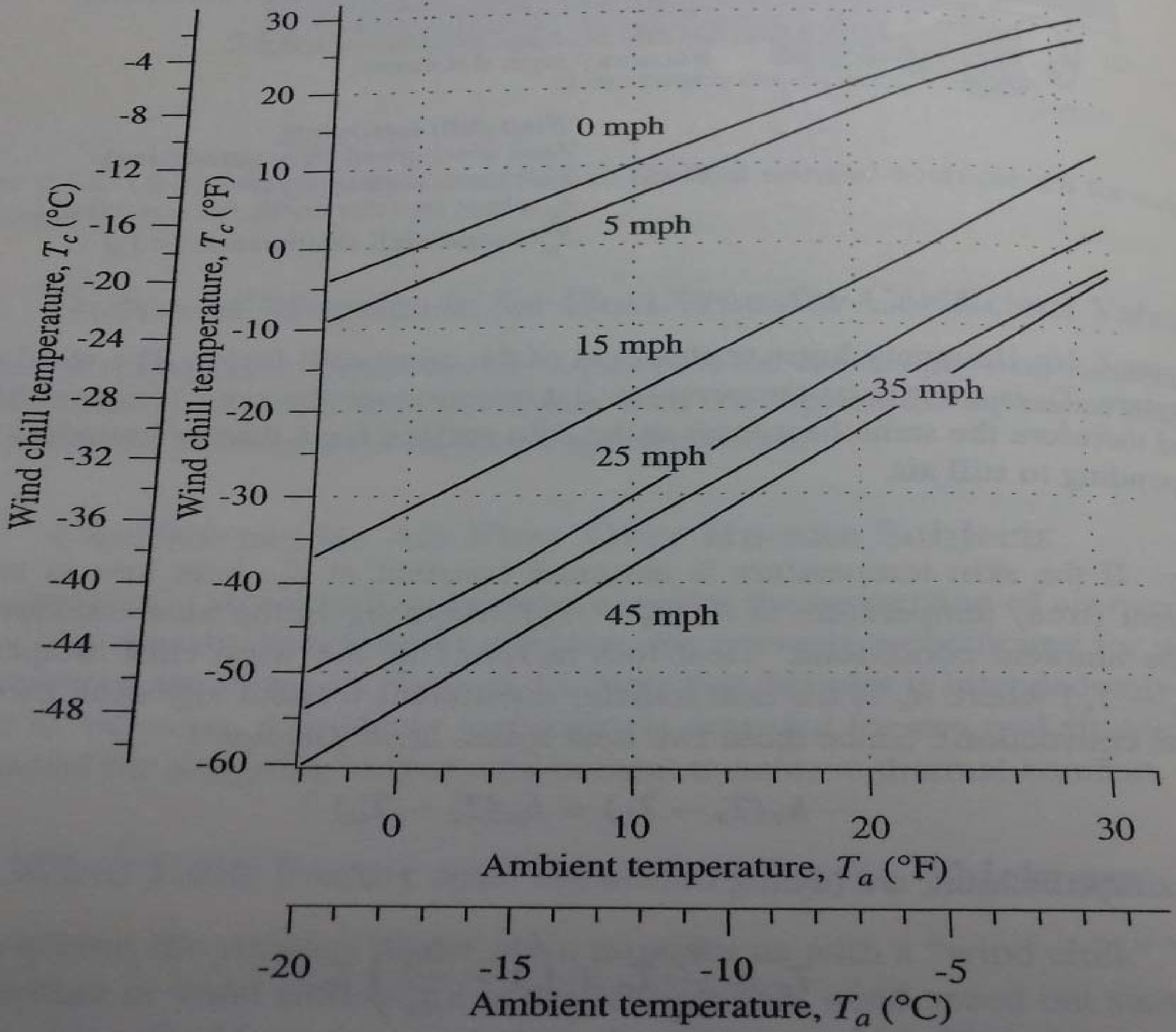


Wind Chill Factor and Boundary layer Thickness



$$h_c(T_s - T_c) = h_a(T_s - T_a)$$

$$T_c = \frac{h_a}{h_c} T_a + (1 - \frac{h_a}{h_c}) T_s$$


Wind chill temperature, T_c (°C)
-4
-8
-12
-16
-20
-24
-28
-32
-36
-40
-44
-48
Wind chill temperature, T_c (°F)
30
20
10
0
-10
-20
-30
-40
-50
-60
0 mph
5 mph
15 mph
25 mph
35 mph
45 mph
0
10
20
30
Ambient temperature, T_a (°F)
-20
-15
-10
-5
Ambient temperature, T_a (°C)