Mass transfer

Lecture 02: *Equilibrium-stage Op.*

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Learning objectives

- **Understand how cascades allow effective, continuous exchange of matter across the two streams in a qualitative manner.**

- **Become competent in analyzing and designing multi-stage distillation column, at least for 2-component systems.**

- **Comprehend how the tray shape and structure affect mass transfer between the two phases.**

Today's outline

- **Equipment for stage contacts**
  - ✓ cascades
  - ✓ distillation vs leaching
- **Principles of stage processes**
  - ✓ Terminology
  - ✓ Material balances
  - ✓ Number of ideal stages
  - ✓ Absorption factor method for calculating # of ideal stages

20.1 Introduction

- **Cascades: multi-stage system where two streams continuously move counter-currently, are mixed and then separated**

Helpful online video clips

- Khan Academy, Simple and fractional distillations:

https://www.youtube.com/watch?v=3pL2X-8-eVk&t=506s

- Andreas Pfennig, 009 TUO - distillation - distillation cascade:

https://www.youtube.com/watch?v=ITM2ibACjME

- Sneha Bharat, Distillation column:

https://www.youtube.com/watch?v=BaBMXgVBQKk

20.1 Cascades in distillation

- **Hypothetical equipment schematic**
  - ✓ rectifying vs stripping/enriching section

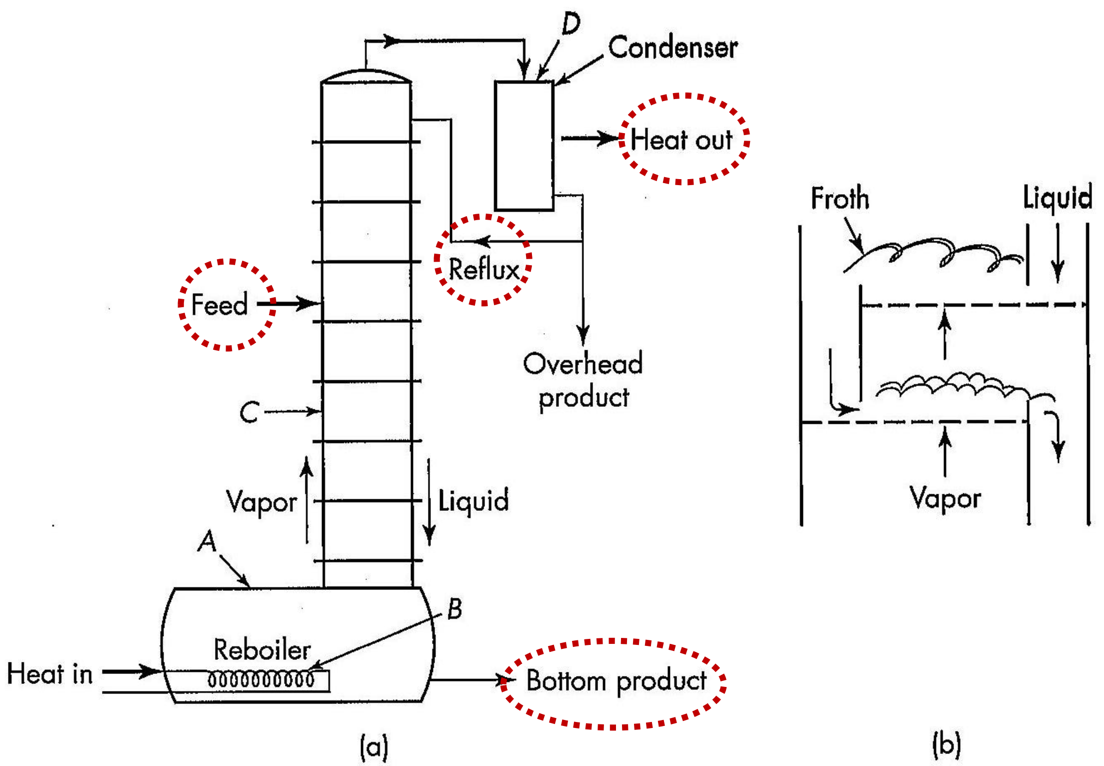
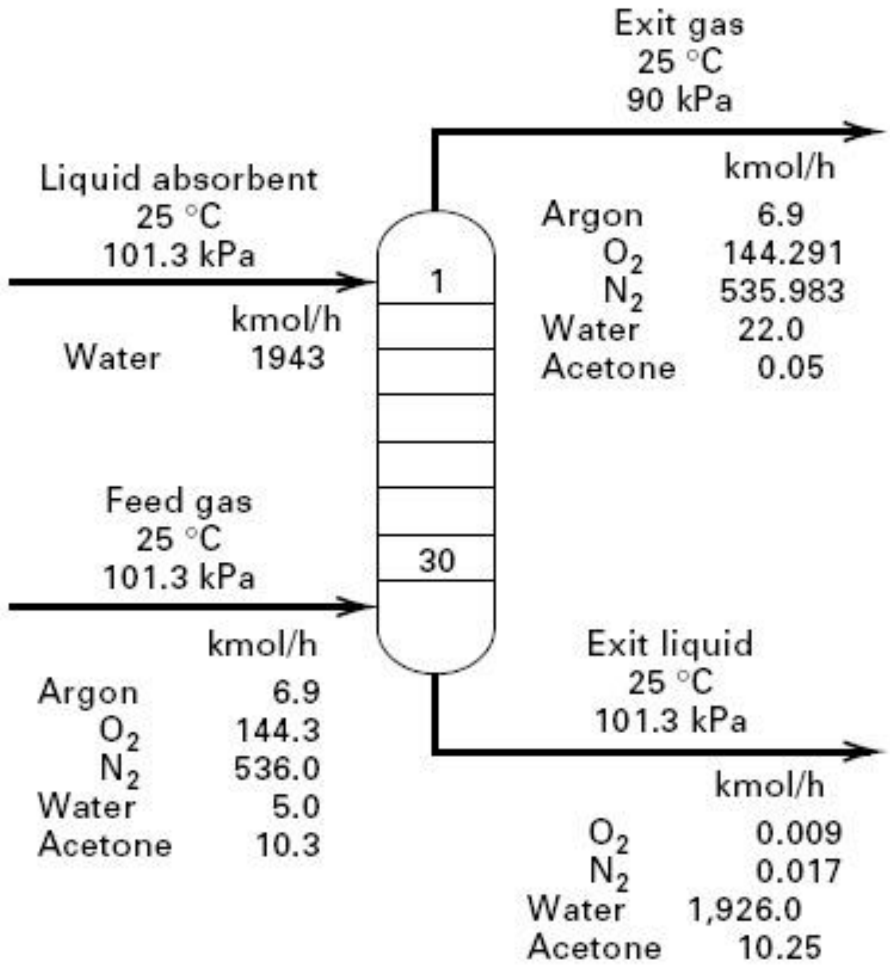


FIGURE 20.1
(*a*) Reboiler with fractionating column: *A*, reboiler; *B*, heating element; *C*, column; *D*, condenser. (*b*) Detail of sieve plate.

20.1 Cascades in distillation

- **Example process flowsheet**



20.2 Cascades in leaching

- **Hypothetical equipment schematic**
  - ✓ Extracts soluble material from the solid mixture using liquid solvent

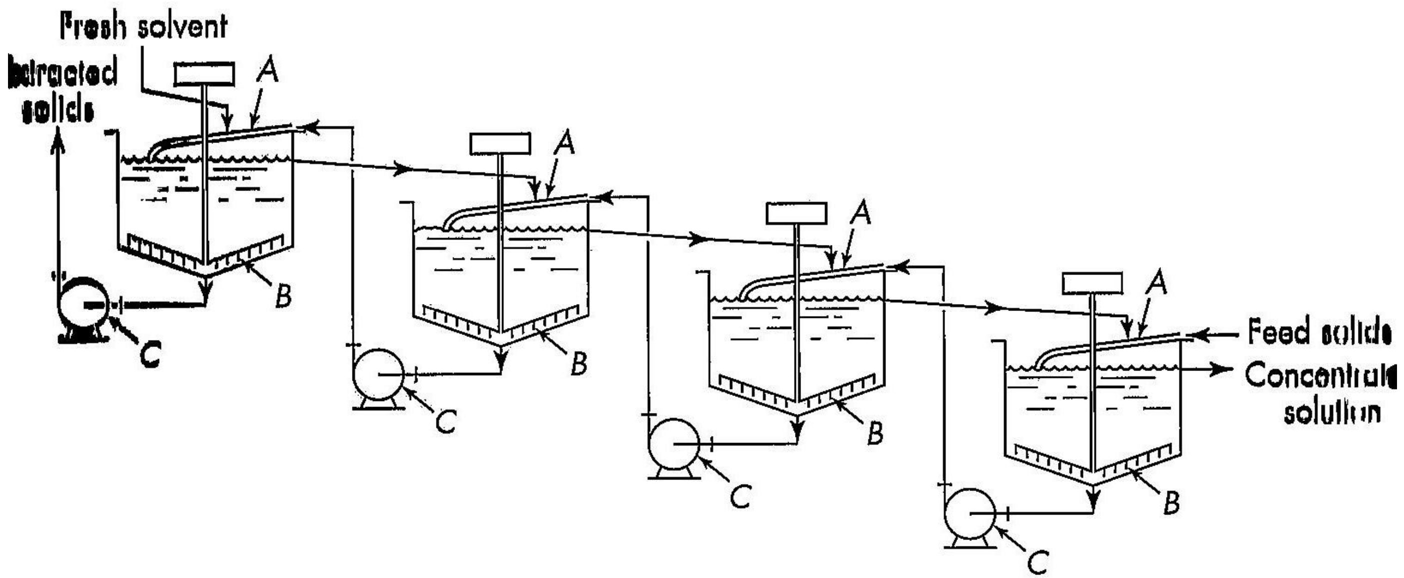


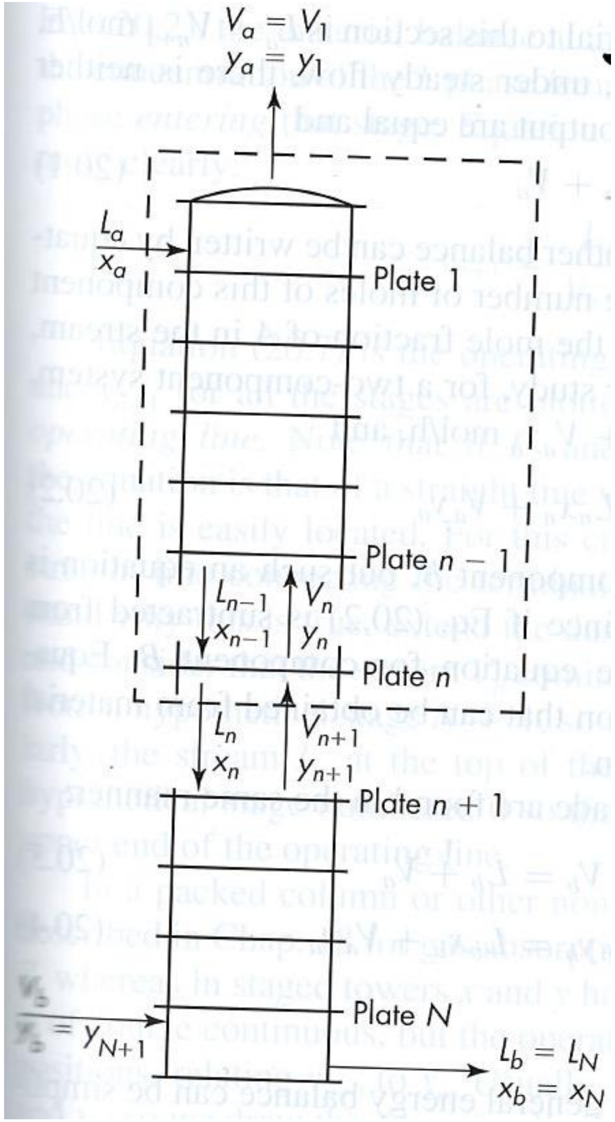
FIGURE 20.2
Countercurrent leaching plant: *A*, launder; *B*, rake; *C*, slurry pump.

20.2 Principles: terminology

- ***V* vs *L* phase**

- **Numbering of stages**
  ✓ Start from where the L phase enters, and increases serially

- **Fractions** (x_i, y_i)
  ✓ mole fractions are commonly used for distillation while mass fractions are often used for extraction.



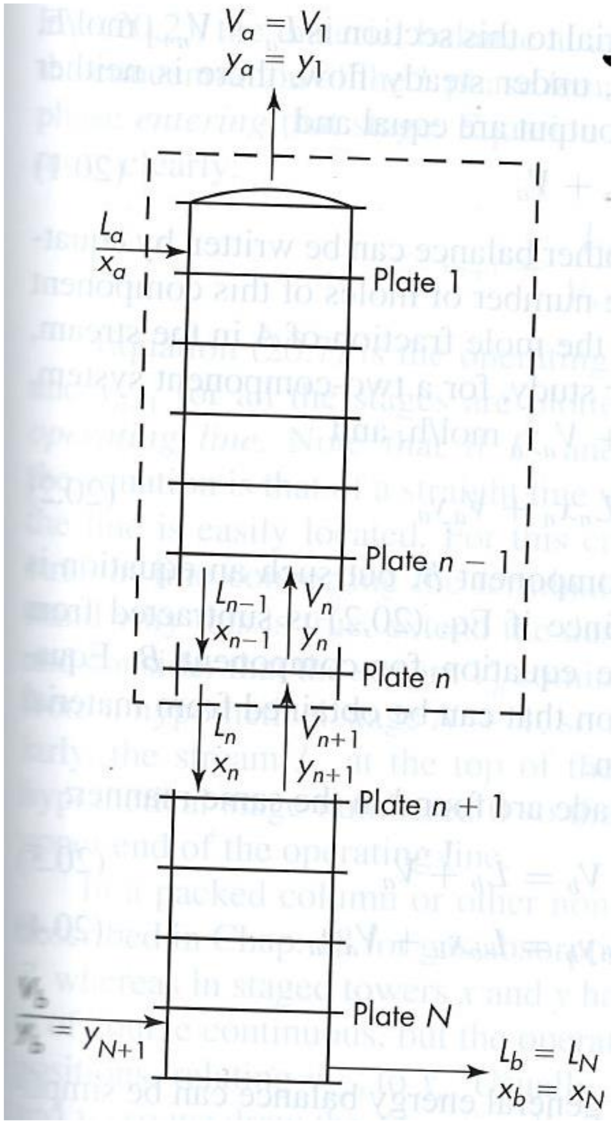
20.2 Material balances

- **From the first to n^th stage,**
  - ✓ at steady state, without chemical rxn,

$$L_a + V_{n+1} = L_n + V_a$$

$$L_a x_a + V_{n+1}\ y_{n+1} = L_n x_n + V_a\ y_a$$

  - ✓ the above is for a 2-component system
  - ✓ how about the eqn. for the other comp.?



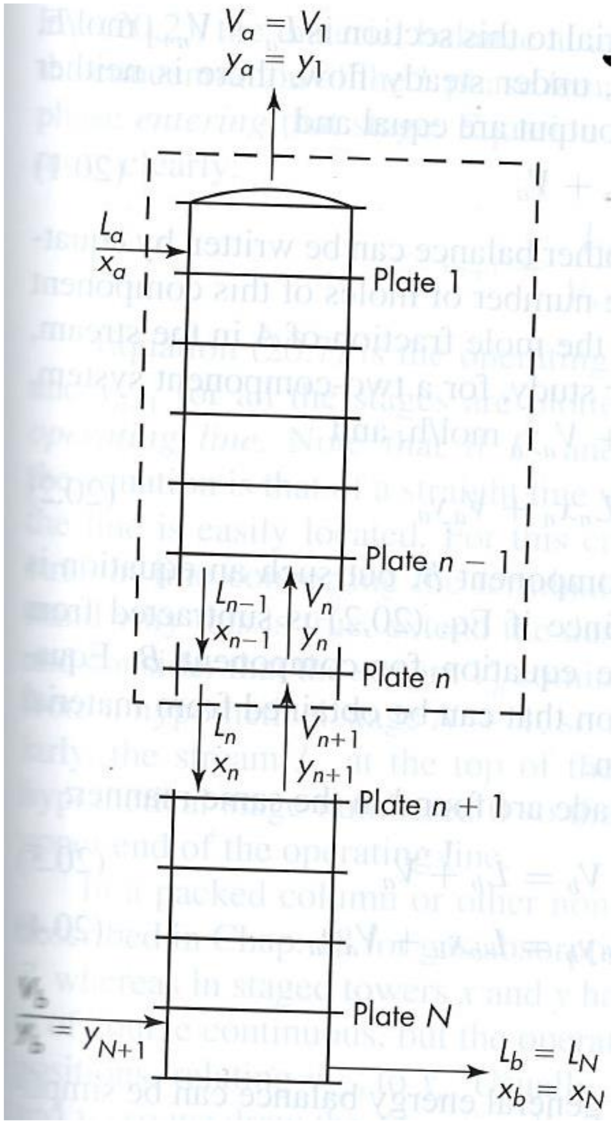
20.2 Energy balances

- **From the first to n^th stage,**
  - ✓ Simplification is possible if you neglect mechanical potential and kinetic energy
  - ✓ for **adiabatic, workless** process,

$$L_a H_{L,a} + V_{n+1} H_{v,n+1} = L_n H_{L,n} + V_a H_{v,a}$$

$$L_a x_a + V_{n+1}\ y_{n+1} = L_n x_n + V_a\ y_a$$

  - ✓ the above is for a 2-component system
  - ✓ **Homework**: derivation



20.3 Operating line diagram

- **Graphical methods based on material balances and equilibrium can be used for many 2-component systems.**

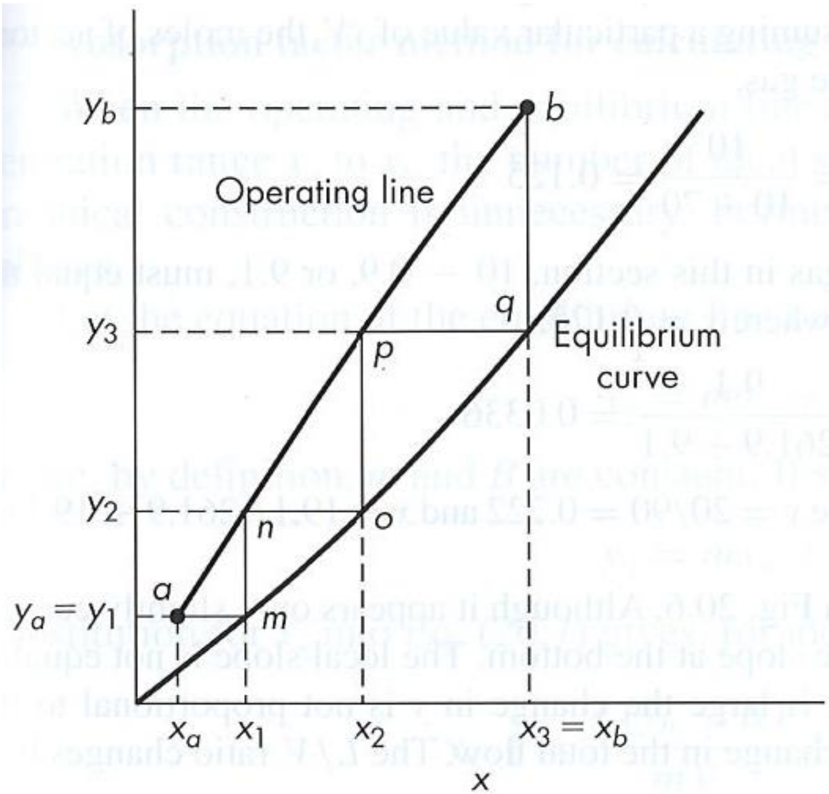
- **Compositions of 2 phases are represented in a 2D graph:**

✓ Rewrite the mat'l balance eqn for a component as in below:

$$y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{V_a\, y_a - L_a\, x_a}{V_{n+1}}$$

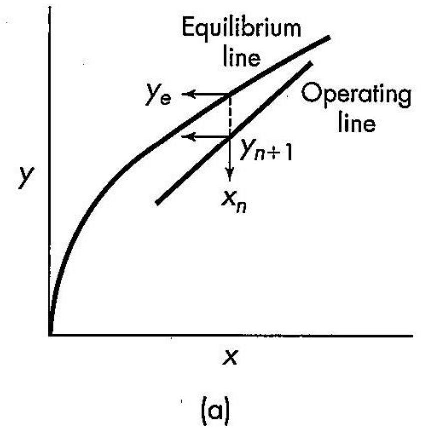
✓ the above gives the **operating line** as shown on the right; is it always a line?

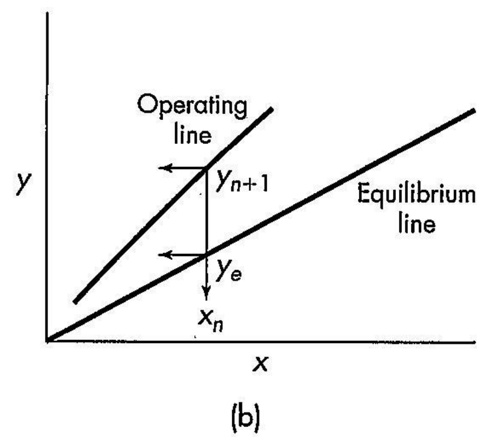
✓ What does the **equilibrium curve** represent?



20.3 Operating line diagram

- **The position of the operating line relative to the equilibrium line determines the direction of mass transfer and how many stages are needed for separation.**



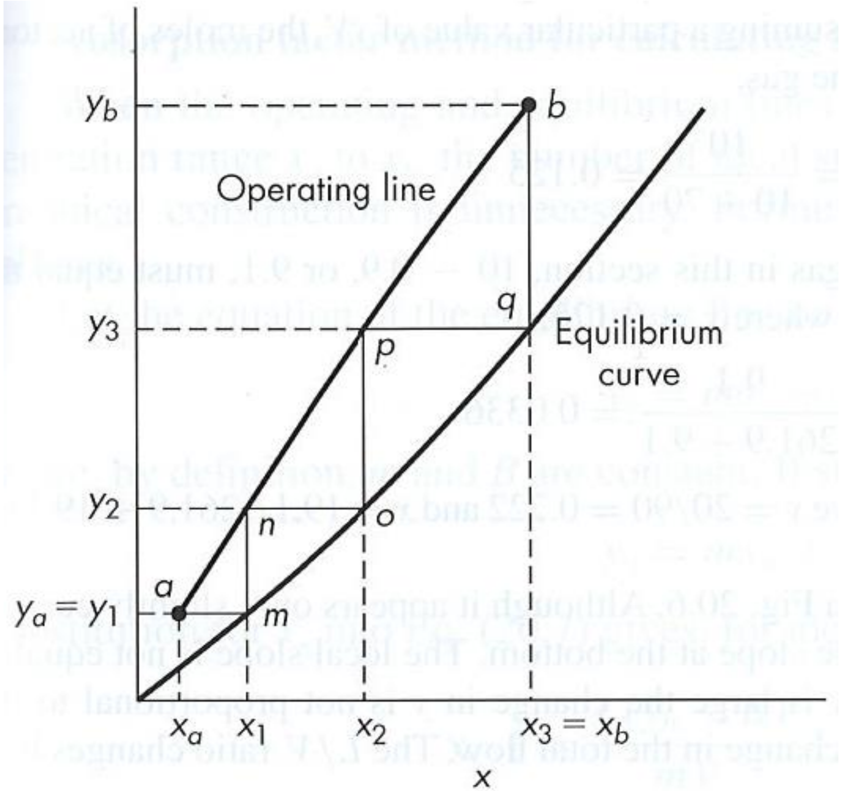


✓ Discussion and answer:

- What happens to the component with the lower bp in the stage?
- Is this the rectifying or stripping section?
- What is the relationship between the slope of operating line and number of stages needed?

20.3 Ideal contact stages

- **The V phase leaving the stage is in equilibrium with the L phase leaving the same stage.**
  - ✓ stage/plate efficiency is used to correct for non-ideality



20.3 # of ideal contact stages

- **How many ideal stages are needed to convert y_b to y_a and x_a to x_b simultaneously?**

✓ for given y_a, x_a is found.

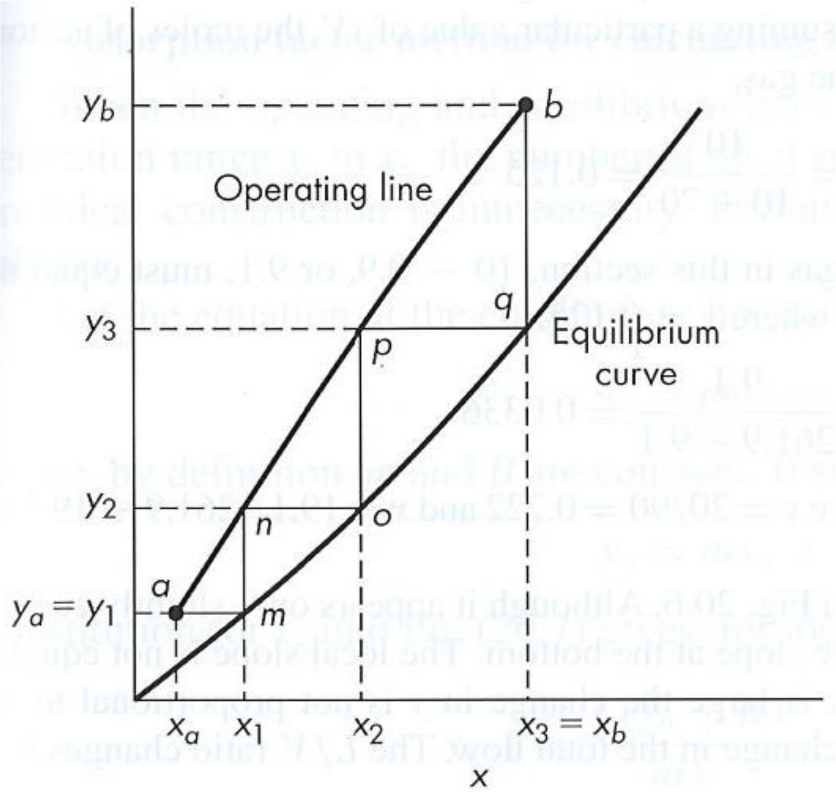
✓ If ideal, move to m.   Why?

✓ In the subsequent stage, y_2 is found by moving to n. Why?

✓ Repeat *these motions* until the target y_b is reached.

✓ How many stages were needed?

✓ How about for a rectifying section? Differences?



20.3 McCabe-Thiele method

- **This graphical method was first used to find the number of ideal stages in distillation columns, and is known as McCabe-Thiele method.**
  - ✓ What if the end point is not exactly met?

Ex. 20.1 By means of a plate column, acetone is absorbed from its mixture with air in a nonvolatile absorption oil. The entering gas contains 30 mol% acetone, and the entering oil is acetone-free. Of the acetone in the air, 97% is to be absorbed, and the concentrated liquor at the bottom is to contain 10% acetone. The equilibrium relationship is $y_e = 1.9x_e$. Plot the operating lines to determine the number of ideal stages. (**Homework**)

20.4 Equipment at glance

- **How do these trays look like?**
  - ✓ What determines what to use?

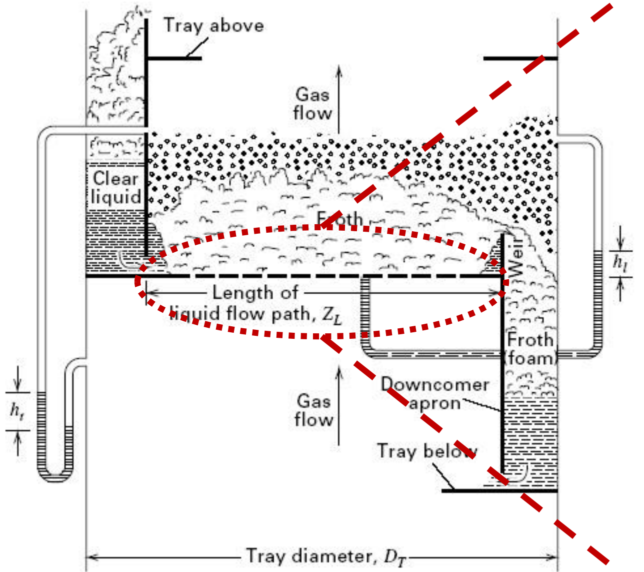
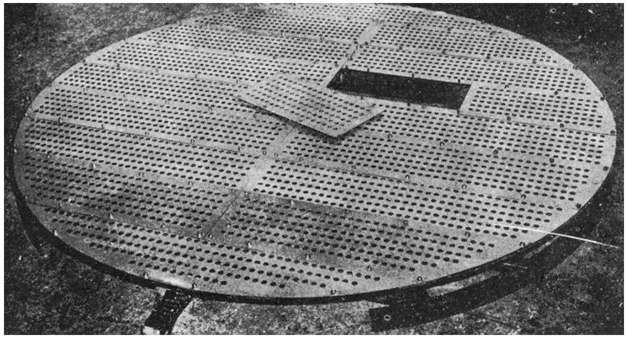
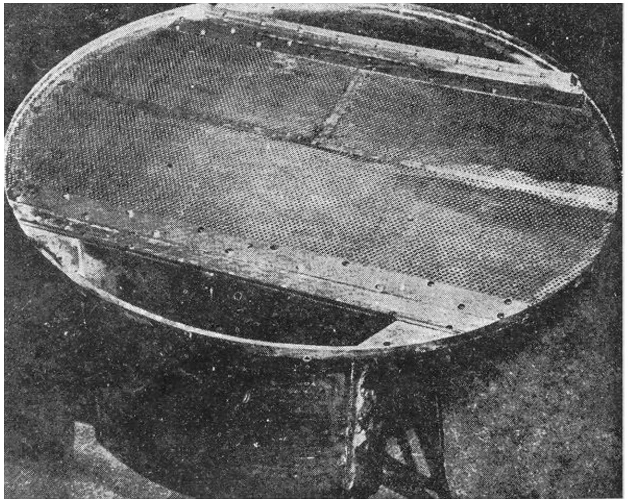


Figure 6.3 Details of a contacting tray in a trayed tower. [Adapted from B.F. Smith, *Design of Equilibrium Stage Processes*, McGraw-Hill, New York (1963).]



Perforated tray



Sieve tray

20.4 Equipment at glance

- **The choice of tray type is related to the preferred vapor-liquid flow regimes.**
  - ✓ major parameters: mixing, efficiency, contact, flowrate

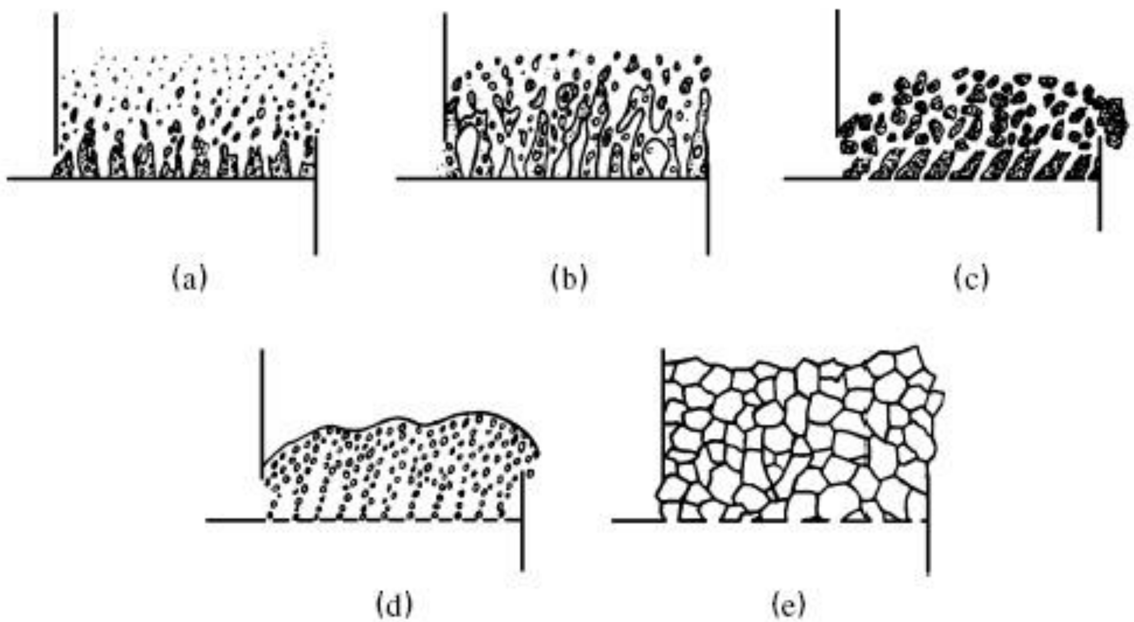


Figure 6.4 Possible vapor–liquid flow regimes for a contacting tray: (a) spray; (b) froth; (c) emulsion; (d) bubble; (e) cellular foam.

[Reproduced by permission from M.J. Lockett, *Distillation Tray Fundamentals*, Cambridge University Press, London (1986).]

20.5 Absorption factor method

- **It can be used when both operating and equilibrium lines are straight.**
  - ✓ Equilibrium line: $y_n = mx_e + B \;\; \rightarrow y_n = mx_n + B$
  - ✓ Operating line: substitution to the mat'l balance eqn. (p11) gives

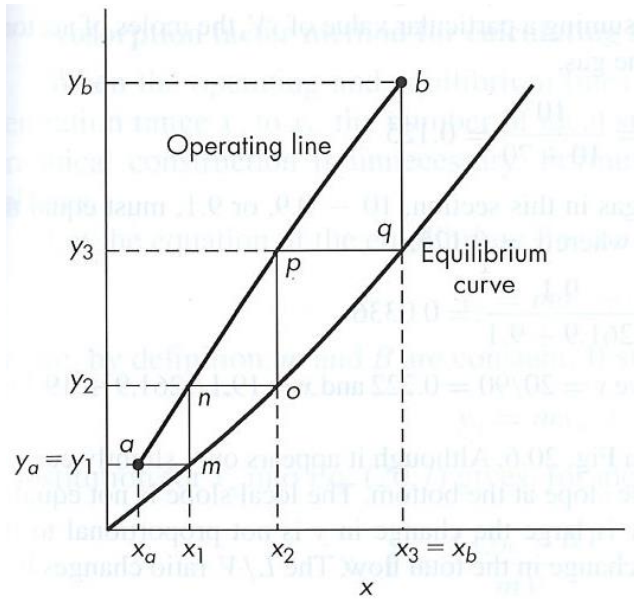
$$y_{n+1} = \frac{L\,(y_n - B)}{m\,V} + y_a - \frac{L\,x_a}{V} \qquad \text{eqn. (20.10)}$$

where absorption factor $A \equiv \frac{L}{m\,V}$ (ratio of the two slopes), and L and V are assumed to be constant

  - ✓ Using A, the above eqn. can be rewritten

$$y_{n+1} = A\,y_n - A(m\,x_a + B) + y_a$$
$$= A\,y_n - Ay_a^* + y_a$$

  - ✓ what is the difference between y_a^* and y_a?



20.5 Kremser equation

- **Using the eqn, one can calculate yns as follows:**
  - ✓ $y_2 = y_a(1 + A) - Ay_a^*$
    $y_3 = y_a(1 + A + A^2) - y_a^*(A + A^2)$
    $y_{n+1} = y_a(1 + A + \cdots + A^n) - y_a^*(A + A^2 + \cdots + A^n)$
  - ✓ For the entire cascades (N stages) where $y_{n+1} = y_{N+1} = y_b$,

$$y_b = y_a\left(\frac{1-A^{N+1}}{1-A}\right) + y_a^*\left(\frac{A-A^{N+1}}{1-A}\right) \qquad \text{eqn. (20.17)}$$

where absorption factor $A \equiv \frac{L}{m\,V}$ (ratio of the two slopes)

  - ✓ The above eqn is called as **Kremser equation.**

20.5 Calculation of N – V phase

- **For the n^th stage,**
  - ✓ $y_b = A\, y_N - A y_a^* + y_a$
    $= A\, y_b^* - A y_a^* + y_a \;\rightarrow y_a = y_b - A(y_b^* - y_a^*)$
  - ✓ Using the above eqn as well as eqn. 20.17, you can solve for N:

$$N = \frac{\ln[(y_b - y_b^*)/(y_a - y_a^*)]}{\ln A} = \frac{\ln[(y_b - y_b^*)/(y_a - y_a^*)]}{\ln[(y_b - y_a)/(y_b^* - y_a^*)]}$$

✓ What if the two lines are parallel, i.e., A = 1?

✓ What if the operating line sits below the equilibrium line? (*recommended homework*)

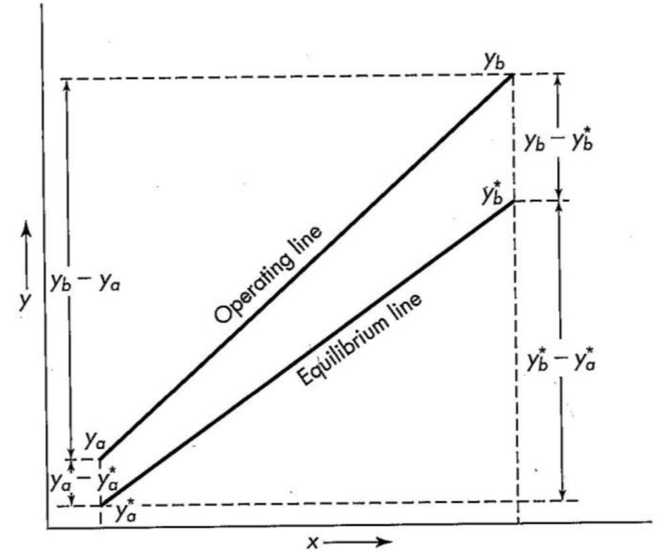


FIGURE 20.8
Concentration differences in Eq. (20.24).

20.5 Calculation of N – L phase

- **For the n^th stage,**
  - ✓ $N = \frac{\ln[(x_a - x_a^*)/(x_b - x_b^*)]}{\ln S} = \frac{\ln[(x_a - x_a^*)/(x_b - x_b^*)]}{\ln[(x_a - x_b)/(x_a^* - x_b^*)]}$

  where $S \equiv \frac{1}{A} = \frac{m\,V}{L}$ and is called the **stripping factor**

  - ✓ What if S =1?

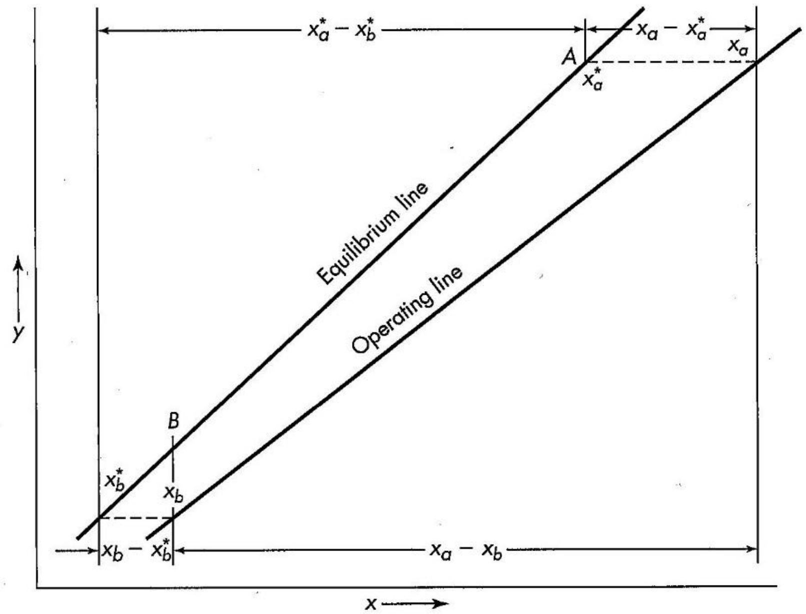


FIGURE 20.9
Concentration differences in Eq. (20.28).

20.5 Multi-stage column design

Ex. 20. 2. Ammonia is stripped from a dilute aqueous solution by countercurrent contact with air in a column containing seven sieve trays. The equilibrium relationship is $y_e = 0.8x_e$, and when the molar flow of air is 1.5 times that of the solution, 90% of the ammonia is removed.

(a) How many ideal stages does the column have and what is the stage efficiency?

(b) What % removal would be achieved if the air rate were increased to 2.0 times the solution rate?

(**homework**)

20.5 Column Design

- **The design goals include the following:**
  - ✓ Maximize separation;
  - ✓ Minimize manufacturing and installation cost;
  - ✓ Minimize energy-operating cost;
  - ✓ Minimize maintenance cost;
  - ✓ Provide operating flexibility