Mass transfer

Lecture 07: *McCabe-Thiele Method*

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Learning objectives

- Calculate *q* and construct the operating line for the given feed condition.
- Become capable of constructing operating lines for the continuous fractionation, and calculating the number of ideal plates needed.
- Analyze how the reflux ratio affects the number of plates needed, and determine the optimal ratio for a given separation.

Today's outline

- **McCabe-Thiele method**
  - ✓ Effect of feed condition
  - ✓ Feed line
  - ✓ Construction of operating lines, feed plate location
  - ✓ Heating and cooling requirements
  - ✓ Example 21.2
  - ✓ Minimum number of plates & minimum reflux
  - ✓ Invariant zone & optimum reflux ratio
  - ✓ Example 21.3

21.3 Effect of feed condition

- ***q* represents the fraction of liquid in the feed stream.**

$$q = \frac{\textit{heat needed to evaporate 1 mol of feed at entering conditions}}{\textit{molar latent heat of vaporization of feed}}$$

$$= \frac{H_V - H_F}{H_V - H_L} = \frac{(H_V - H_L) + (H_L - H_F)}{H_V - H_L} = 1 + \frac{H_L - H_F}{H_V - H_L}$$

H_V: enthalpy at the dew point
H_F: enthalpy of feed at the entrance
H_L: enthalpy at the boiling point

✓ $q = 1 - f$

✓ q can also be calculated as shown in below: **(homework)**

$q = 1 + \frac{C_{pL}(T_b - T_F)}{\lambda}$ (cold liquid) vs. $q = 1 + \frac{C_{pV}(T_F - T_d)}{\lambda}$ (Superheated vapor)

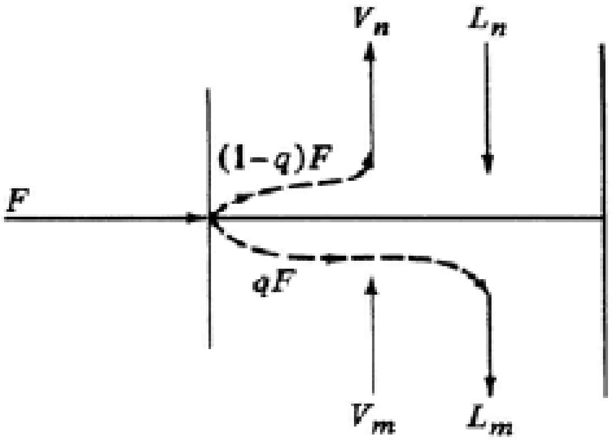
where C_{pL}, C_{pV} represent specific heats of liquid and vapor, respectively; T_F, T_b, and T_d are feed, bubble-point, and dew-point temperature; λ denotes heat of vaporization

21.3 Feed line

- **Most columns operate w/ the feed as liquid @ ~ b.p.**

- ***q* can be used to further analyze the material balance:**
  - ✓ $L_m = L_n + qF$
    $V_n = V_m + (1 - q)F$
  - ✓ At the feed, two operating lines intersect:
    $$V_n y = L_n x + Dx_D$$
    $$V_m y = L_m x - Bx_B$$
  - ✓ Subtraction and rearrangement gives:
    $$(V_m - V_n)y = (L_m - L_n)x - (Dx_D + Bx_B)$$
    $$Fx_F = Dx_D + Bx_B$$

$$y = \frac{q}{q-1}x - \frac{x_F}{q-1}$$



21.3 Operating lines

- **The simplest method of plotting the operating lines consists of the following 3 steps:**

(1) Locate the feed line

$$y = \frac{q}{q-1}x - \frac{x_F}{q-1}$$

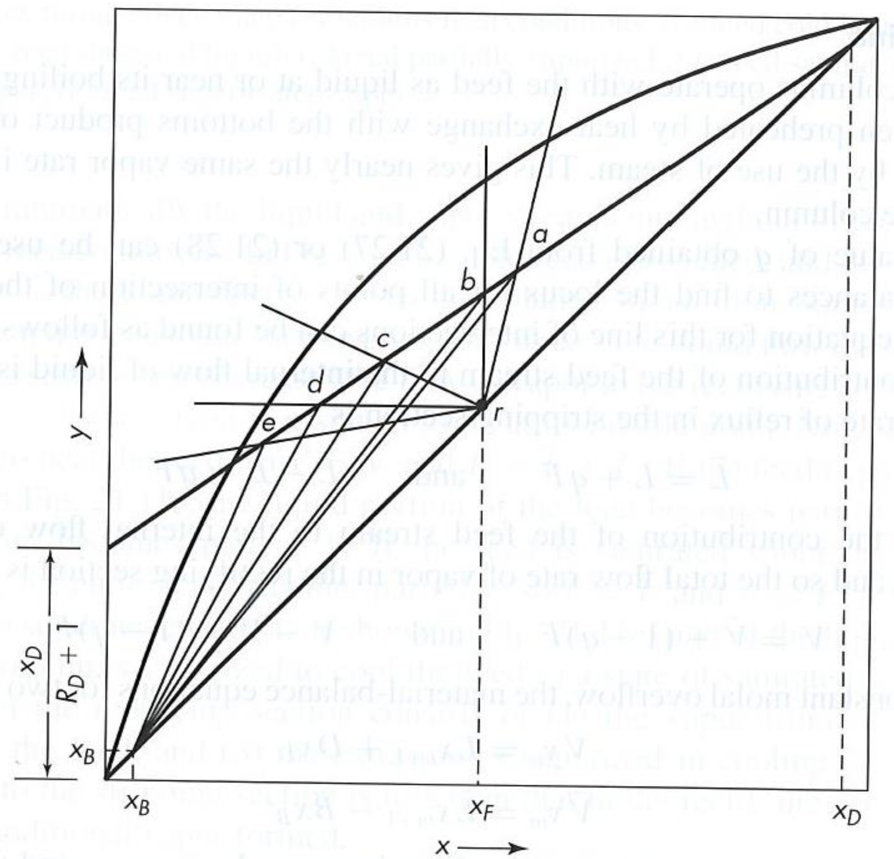
(2) Plot the operating line for the rectifying section

- $x_D/(R_D+1)$
- (x_D, x_D)

(3) Plot the operating line for the stripping section

- x_B, x_B
- Intersection w/ the rectifying @ ???

✓ **What about lines a~e?**



21.3 Feed plate location

- **Use the operating lines to calculate the number of ideal plates need.**
  - ✓ **start from top vs bottom?**

- **Locate the feed plate such that the number of plates is the smallest:**
  - ✓ **After the feed plate, you move to the operating line of the different section**

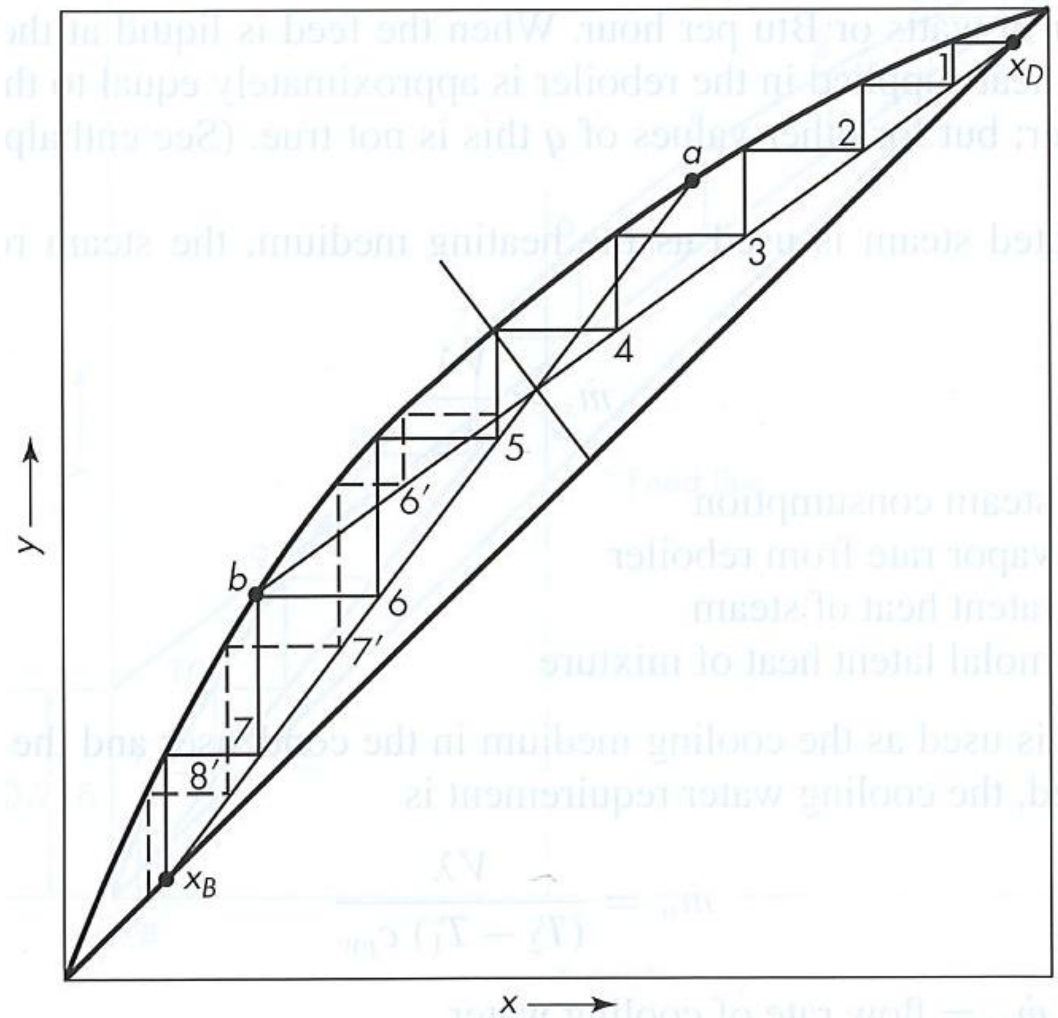


FIGURE 21.13
Optimum feed plate location: ——, with feed on plate 5 (optimum location); ------, with feed on plate 7.

21.3 Heating & cooling

- **One can consider the column as being adiabatic.**
  - ✓ Heat effects are confined to the condenser and reboiler.
  - ✓ Heat added to the reboiler (q_r) is used to create the vapor flow:

$$q_r = \bar{V}\,\lambda$$

where $\bar{V}$ is the vapor rate from the reboiler

  - ✓ If the saturated steam is used as the heating medium, the steam required at the reboiler is

$$\dot{m}_s = \frac{\bar{V}\,\lambda}{\lambda_s}$$

where λ_s is the latent heat of steam

  - ✓ If water is used as the cooling medium and the condensate is not subcooled, the water required at the condenser is

$$\dot{m}_w = \frac{V\,\lambda}{(T_2 - T_1)C_{p,w}}$$

where $T_2 - T_1$ is the temperature rise of the cooling water

21.3 Continuous fractionating

Ex. 21.2. A continuous fractionating column is to be designed to separate 30,000 kg/h of a mixture of 40 wt% benzene and 60 wt% toluene into an overheated product containing 97 wt% benzene and a bottom containing 98 wt% toluene. A reflux ratio of 3.5 mol to 1 mol of product is used. The molal latent heats of benzene and toluene are 7,360 and 7,960 cal/mol. Benzene and toluene form a nearly ideal system with relative volatility of about 2.5; the equilibrium curve is shown in below. The feed has a boiling point of 95 ºC @ 1 atm. (**In-class & homework**)

(a) Calculate the moles of overheated product and bottom product per hour.

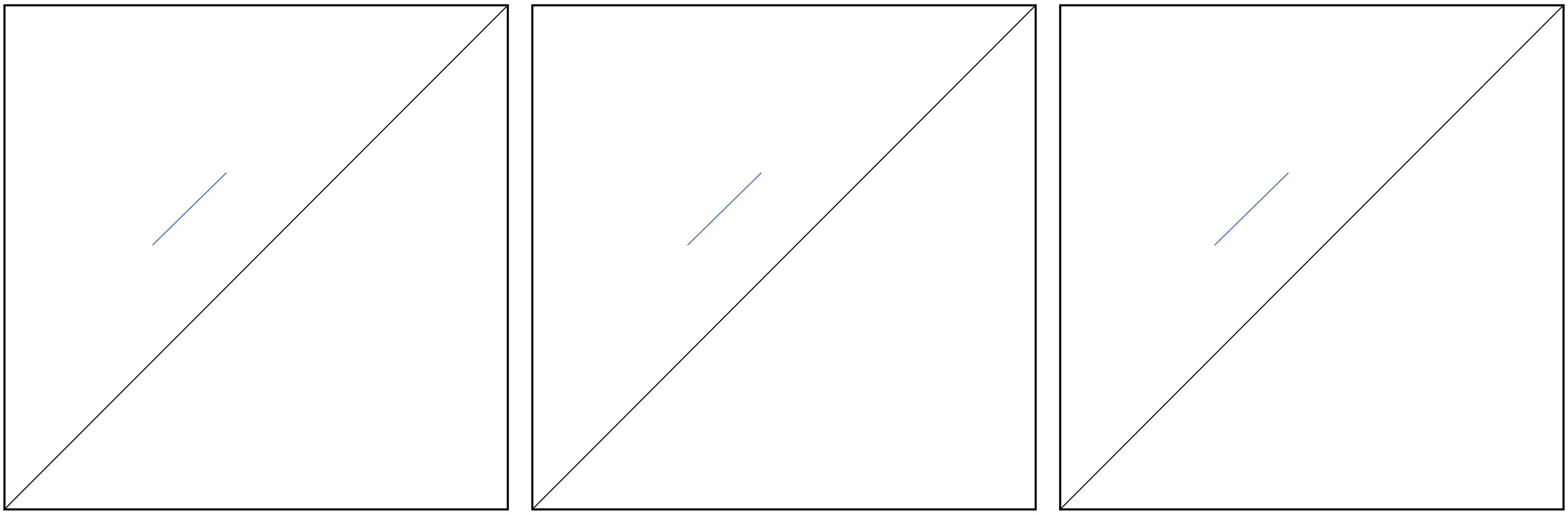
(b) Determine the number of ideal plates and the position of the feed for three different conditions of the feed.

(c) If steam is used for heating, how much steam is required per hour?

(d) If cooling water enters the condenser at 25 ºC and leaves at 40 ºC, how much cooling water is required per hour?

Benzene: C_6H_6, Toluene: C_7H_8

21.3 Continuous fractionating



21.3 Minimum number of plates

- **The number of plates becomes minimum at total reflux:**
  - ✓ Why is this the case? (Explain graphically)
  - ✓ $R_D \rightarrow \infty$, meaning the slope of the rectifying line becomes 1
  - ✓ $F = D = B = 0$.
  - ✓ The operating line becomes the diagonal line, and no feed line.
  - ✓ For ideal mixtures, a simple method is available for calculating the number of plates needed:

Using relative volatility, $y = \frac{\alpha\, x}{1+(\alpha-1)x}$ where x, y, are equilibrium conc.

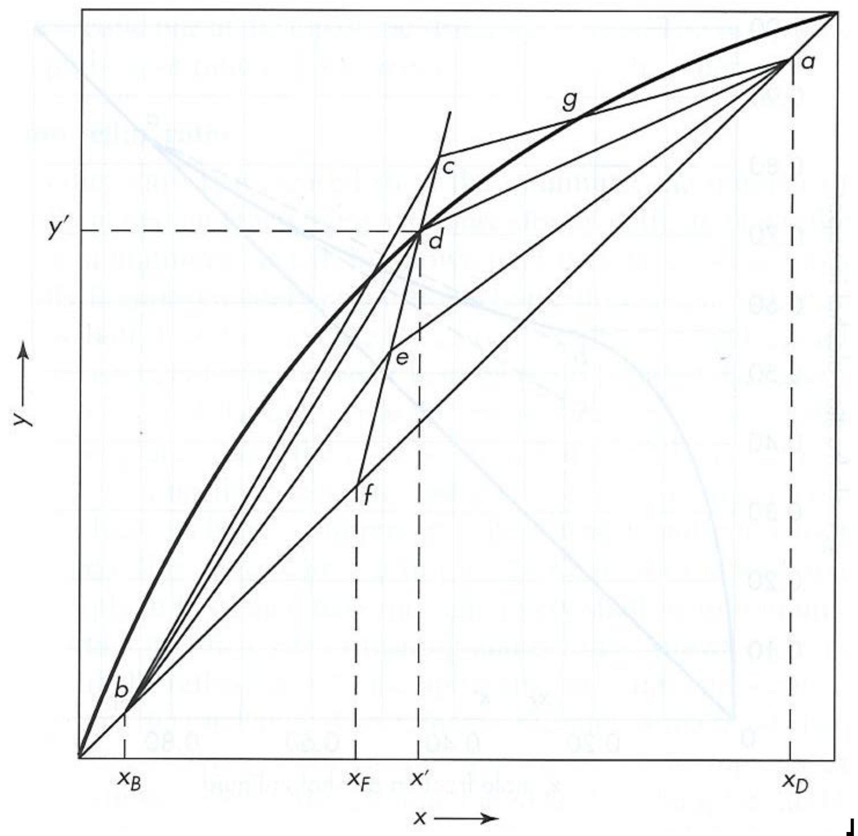
And going through a series of mathematical manipulations,

$$N_{min} = \frac{\ln[x_D(1-x_B)/x_B(1-x_D)]}{\ln \alpha_{AB}} \qquad \text{(Fenske equation)}$$

if α changes much, use the geometric mean.

21.3 Minimum reflux

- **The number of plates becomes infinite at minimum reflux:**
  - ✓ All actual columns operate at the *intermediate* reflux ratio.
  - ✓ This is when either or both of the operating lines touch the equilibrium curve, meaning infinite steps needed to cross the lines.
  - ✓ If R_{Dm} is the minimum reflux ratio, the slope of the operating line for the rectifying is $R_{Dm}/(R_{Dm}+1)$, and it becomes ???



21.3 Minimum reflux

- **The previous eqn. is not always true. For example,**

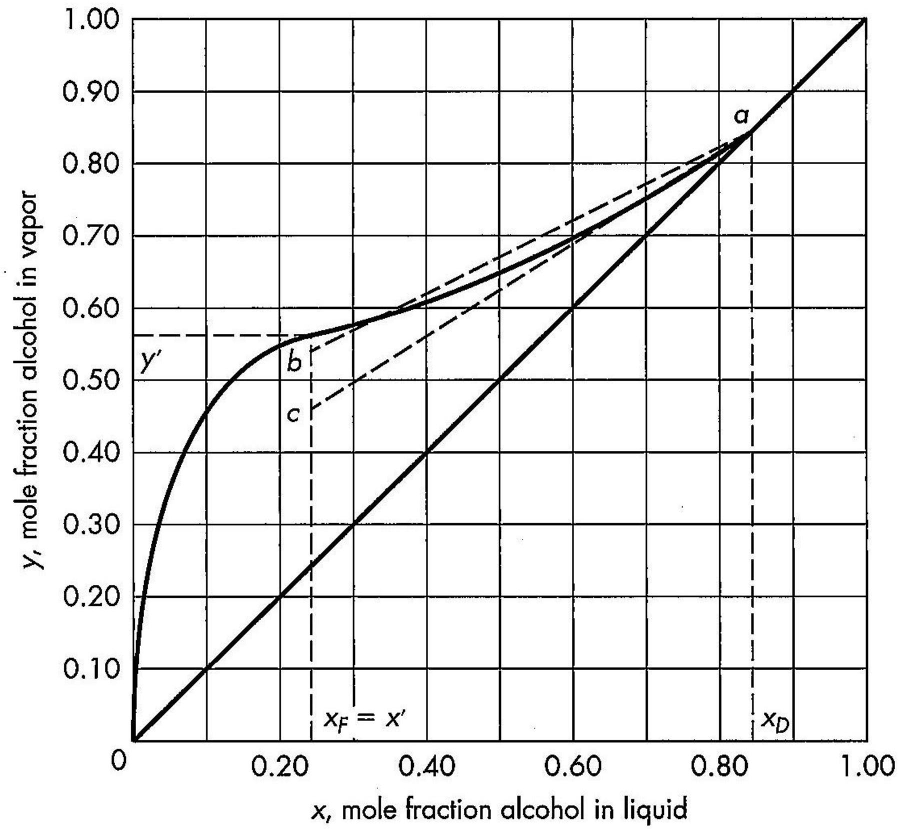
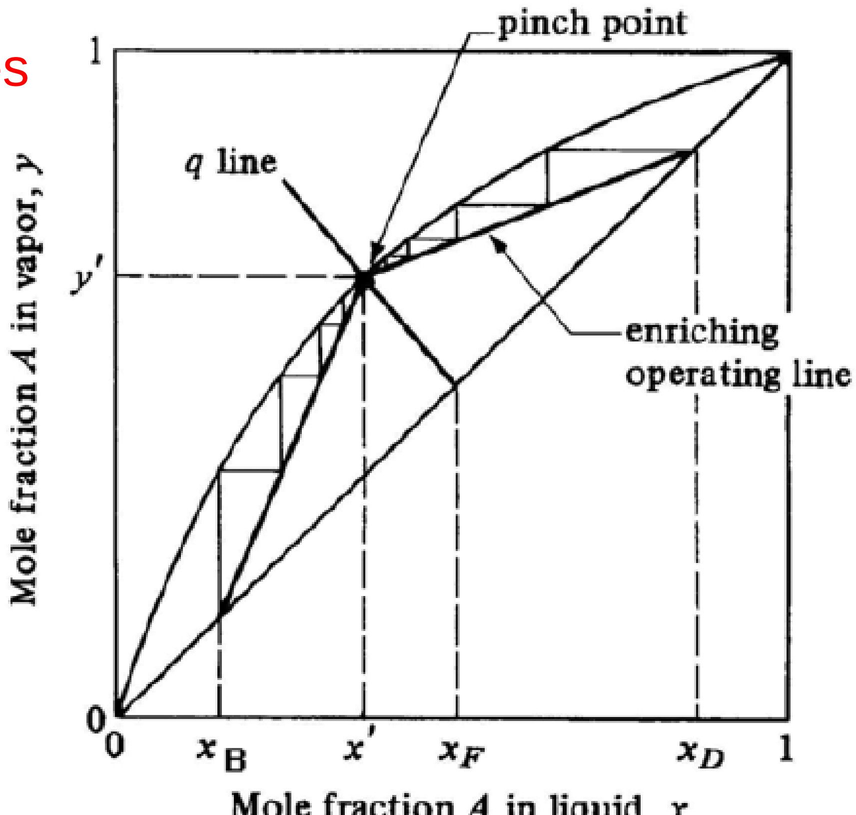


FIGURE 21.18
Equilibrium diagram (system ethanol-water).

21.3 Invariant zone

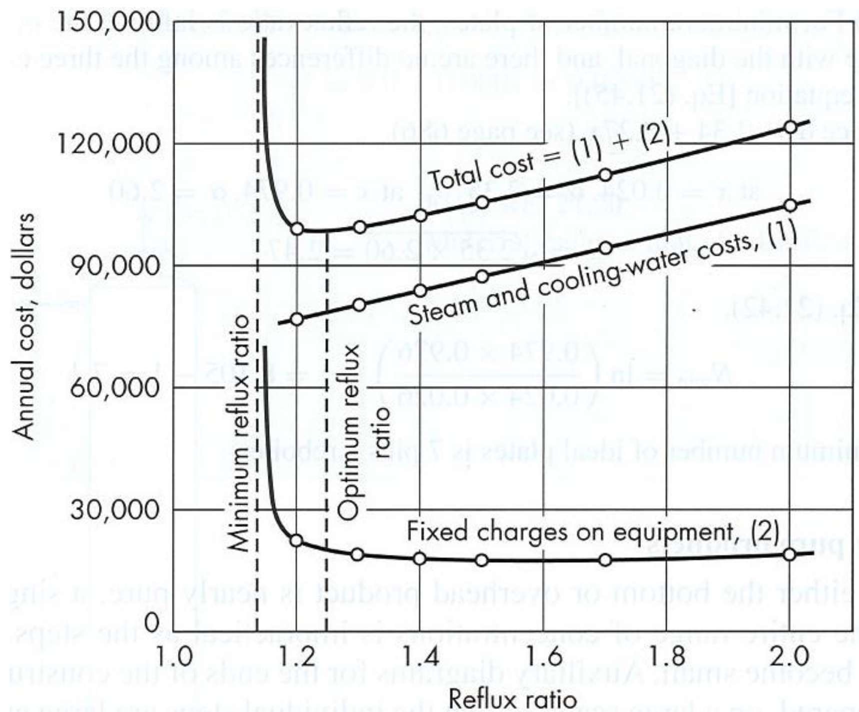
- **There can exist an infinite number of ideal plates in which no changes occur w.r.t. liquid/vapor concentration.**
  - ✓ $x_{n+1} = x_n$ and $y_{n-1} = y_n$
  - ✓ The term *pinch point* is also used.
  - ✓ Where (top? Bottom? Middle?) does the zone form when the min. reflux ratio is used?



21.3 Optimum reflux ratio

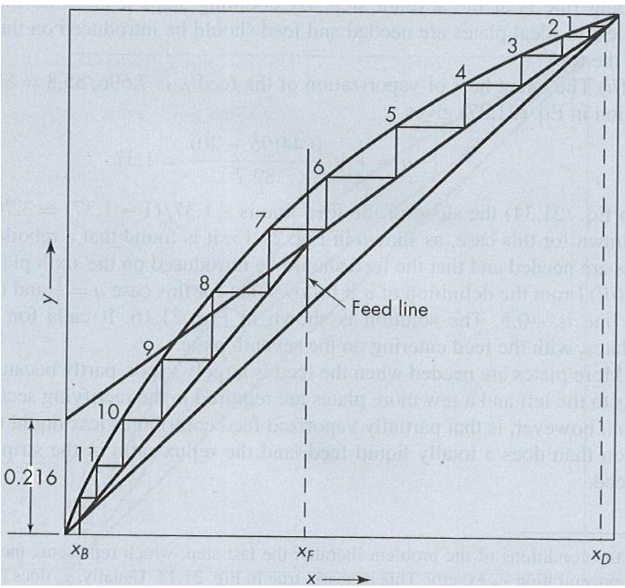
- **Cross-sectional area of the column is often proportional to the vapor flow rate.**

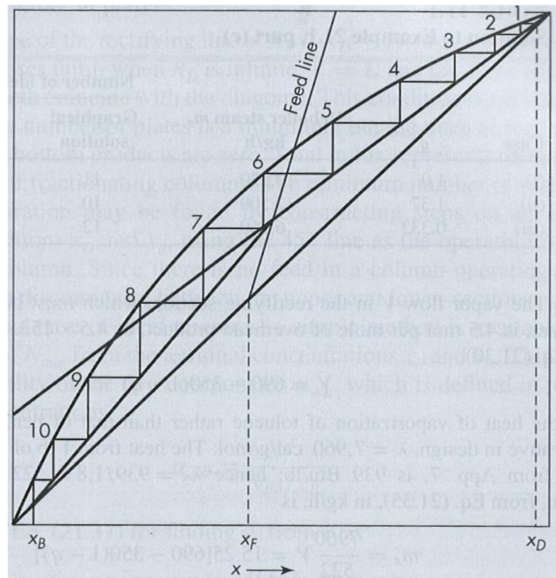
- **As reflux ratio increases, V & L ↑ as well as column diameter; number of plates needed increase/decrease?**
  - ✓ Total cost is proportional to #plates x cross-sectional area
  - ✓ Optimal reflux ratio is the one resulting in the min total cost!

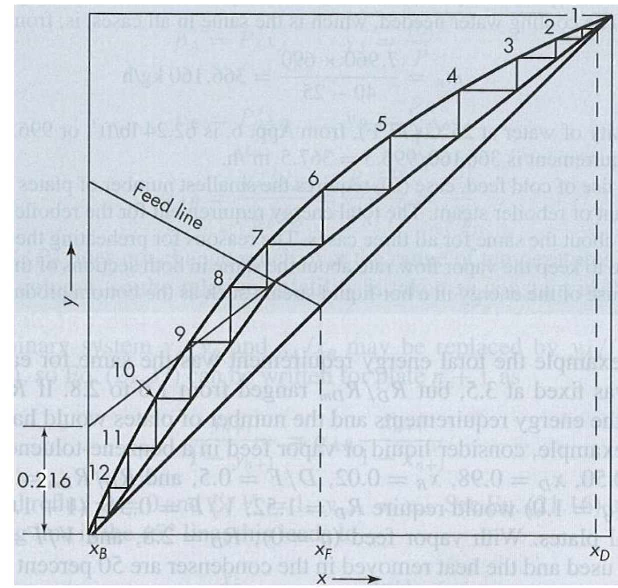


21.3 Minimum ratio and plates

Ex. 21.3. What are (a) the minimum reflux ratio and (b) the minimum number of plates for cases (b)(i), (b)(ii), and (b)(iii) of Ex. 21.2? (**In-class**)







q = 0.33, 1, 1.37