B. Fick's law of diffusion

$$J_A = -D_{AB}\frac{dC_A}{dz} \;\&\; J_B = -D_{BA}\frac{dC_B}{dz}$$

Newton's law of viscosity

$$\tau = -\mu\frac{du}{dz}$$

$$\tau = -\nu\frac{d(\rho u)}{dz}$$

Fourier's law

$$q = -k\frac{dT}{dz}$$

$$q = -\alpha\frac{d(\rho C_p T)}{dz}$$

ν : momentum diffusivity

α : thermal diffusivity

D : mass diffusivity

1) Relation between D_{AB} and D_{BA}

① for gases

- $C_A + C_B = C\left(= \frac{P}{RT}\right)$

$$\frac{dC_A}{dz} + \frac{dC_B}{dz} = 0 \Rightarrow \frac{dC_A}{dz} = -\frac{dC_B}{dz}$$

- $J_A + J_B = C_A(u_A - u_0) + C_B(u_B - u_0)$

$$= C_A u_A - C_A u_0 + C_B u_B - C_B u_0$$
$$= C_A u_A + C_B u_B - (C_A u_0 + C_B u_0)$$
$$= N_A + N_B - (C_A + C_B)u_0 = 0$$

- Thus

$$J_A + J_B = 0$$
$$J_A + J_B = \left(-D_{AB}\frac{dC_A}{dz}\right) + \left(-D_{BA}\frac{dC_B}{dz}\right)$$
$$= -(D_{AB} - D_{BA})\,\frac{dC_A}{dz} = 0$$

$$\therefore D_{AB} = D_{BA} = D_v$$

② for liquids (ρ is constant)

- $$\rho_A + \rho_B = \text{constant}$$
  $$\frac{d\rho_A}{dz} + \frac{d\rho_B}{dz} = 0 \Rightarrow \frac{d\rho_A}{dz} = -\frac{d\rho_B}{dz}$$

$$j_A = -D_{AB}\frac{d\rho_A}{dz}$$
$$j_B = -D_{BA}\frac{d\rho_B}{dz}$$

- $$\begin{aligned} j_A + j_B &= \rho_A(u_A - u) + \rho_B(u_B - u) \\ &= \rho_A u_A - \rho_A u + \rho_B u_B - \rho_B u \\ &= \rho_A u_A + \rho_B u_B - (\rho_A + \rho_B)u \\ &= n_A + n_B - \rho u = n - n = 0 \end{aligned}$$
- Thus
  $$j_A + j_B = \left(-D_{AB}\frac{d\rho_A}{dz}\right) + \left(-D_{BA}\frac{d\rho_B}{dz}\right)$$
  $$= -(D_{AB} - D_{BA})\frac{d\rho_A}{dz} = 0$$

$$\therefore D_{AB} = D_{BA} = D_v$$