17.5 Dimensional Analysis

Number of dimensional variables ⇢ n
Number of fundamental dimensions ⇢ m

➡ Number of dimensionless variables ⇢ $n-m$

$N_A = k_c \Delta C$: rate of mass transfer

- . Concentration difference
- . Physical properties
- . Degree of turbulence
- . Geometry

$$k_c = k_c(\rho, \mu, D_v, u, D) = F$$

$$F(k_c, \rho, \mu, D_v, u, D) = 0 \qquad n = 6$$

Number of fundamental dimensions; $[M], [L], [T]$ ➔ $m = 3$

$k_c[= m/s][= LT^{-1}]$
$\rho[= kg/m^3][= ML^{-3}]$
$\mu[= g/cm \cdot s][= ML^{-1}T^{-1}]$
$D_v[= m^2/s][= L^2T^{-1}]$
$u[= m/s][= LT^{-1}]$
$D[= m][= L]$

17.5 Dimensional Analysis

Number of fundamental dimensions; $[M], [L], [T]$ ➔ $m = 3$

- Recurring sets

$D[= L] \rightarrow [L] = D$

$\rho[= ML^{-3}] \rightarrow [M] = \rho \times [L^3] = \rho D^3$

$u[= LT^{-1}] \rightarrow [T] = u^{-1}[L] = u^{-1}D$

17.5 Dimensional Analysis

Remaining variables ; k_c, D_v, μ

1) $k_c[= LT^{-1}]$

$$\pi_1 = \frac{k_c}{[LT^{-1}]} = \frac{k_c}{D(u^{-1}D)^{-1}} = \frac{k_c}{u}$$

2) $D_v[= L^2T^{-1}]$

$$\pi_2 = \frac{C_p}{[L^2T^{-1}]} = \frac{D_v}{(D^2)(u^{-1}D)^{-1}} = \frac{D_v}{D{\cdot}u}$$

3) $\mu[= ML^{-1}T^{-1}]$

$$\pi_3 = \frac{\mu}{[ML^{-1}T^{-1}]} = \frac{\mu}{[(\rho D^3)(D^{-1})(u^{-1}D)^{-1}]} = \frac{\mu}{\rho u D}$$

17.5 Dimensional Analysis

Modification of dimensionless groups

1) $\pi_1' = \frac{\pi_1}{\pi_2} = \frac{k_c/u}{D_v/D \cdot u} = \frac{k_c D}{D_v}$ $\pi_1' = Sh$

2) $\pi_2' = \pi_2^{-1} \cdot \pi_3 = \left(\frac{D_v}{D \cdot u}\right)\left(\frac{\mu}{\rho u D}\right) = \frac{\mu/\rho}{D_v} = \frac{\nu}{D_v}$ $\pi_2' = Sc$

3) $\pi_3' = \pi_3^{-1} = \left(\frac{\mu}{\rho u D}\right)^{-1} = \frac{\mu}{\rho u D}$ $\pi_3' = Re$

$$\therefore G(\pi_1, \pi_2, \pi_3) \Rightarrow G(\pi_1', \pi_2', \pi_3') = 0$$

Thus,

$$G(Sh, Sc, Re) = 0 \quad \rightarrow \quad Sh = Sh(Sc \cdot Re)$$

$$Sh = Sh(Sc \cdot Re)$$

$$Sh = \frac{k_c D}{D_v}$$

17.5 Dimensional Analysis

For heat transfer

$$Nu = Nu(Pr, Re, Gr)$$

$$Nu = \frac{hD}{k}$$

$$Pr = \frac{\nu}{\alpha} = \frac{C_p \mu}{k}$$

$$Sh = \frac{k_c D}{D_v} = \frac{\text{convective mass transfer coefficient}}{\text{diffusive mass transfer coefficient}}$$

$$Sh = \frac{k_c D}{D_v}$$

$$Sh = 0.023\, Re^{0.8} Sc^{\frac{1}{3}} \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

For turbulent flow

$$Sh = 1.76\, Gz_M^{\frac{1}{3}}$$

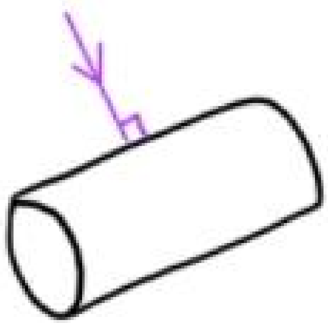
For laminar flow

$$Gz_M = \frac{\dot{m}}{D_v \rho L} = \frac{\pi}{4} Re \cdot Sc \frac{D}{L}$$

$$Gz_H = \frac{\dot{m} C_p}{k \cdot L} = \frac{\pi}{4} Re \cdot Pr \frac{D}{L}$$

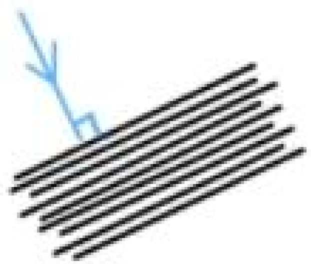
17.6 Case studies

Flow normal to cylinders



$$Sh = 0.61 Re^{1/2} Sc^{1/3}$$

Flow normal to many cylinders



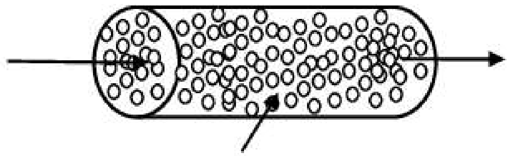
$$Sh = 1.28 Re^{0.4} Sc^{1/3}$$

Flow normal to sphere

$Sh \sim 2.0$ for small Re

$Sh = 2.0 + 0.6 Re^{1/2} Sc^{1/3}$ for $Re \leq 1{,}000$

Packed bed



$$Sh = 1.17 Re^{0.585} Sc^{1/3}$$

For void fraction of ~0.4