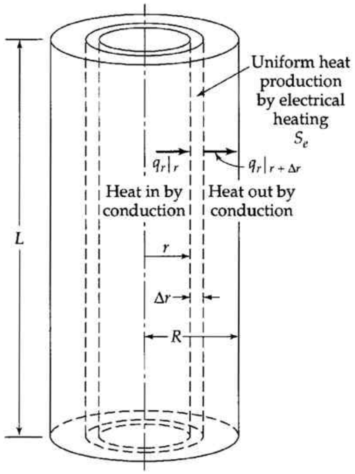
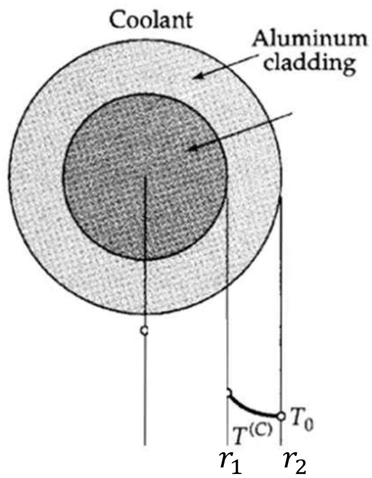
3. Steady-State Conduction

<assumption> : No heat generation

(2) Single layer of cylinder





Heat conduction equation for cylindrical coordination

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\rho C_p}$$

$$0 = \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

1st intg

$$r\frac{dT}{dr} = C_1$$

2nd intg

$$T = C_1 \ln r + C_2$$

B.C.
$@r = r_1, T = T_1$
$@r = r_2, T = T_2$

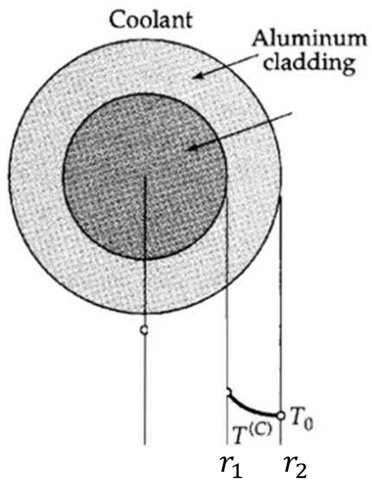
$$C_1 = \frac{T_1 - T_2}{\ln\frac{r_1}{r_2}} \qquad C_2 = T_1 - \frac{T_1 - T_2}{\ln\frac{r_1}{r_2}}\ln r_1$$

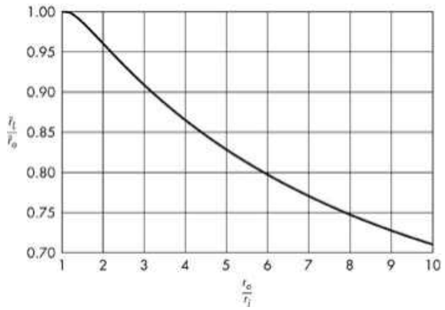
$$T(r) = \frac{T_1 - T_2}{\ln\frac{r_1}{r_2}}\ln r + \left\{T_1 - \frac{T_1 - T_2}{\ln\frac{r_1}{r_2}}\ln r_1\right\}$$

3. Steady-State Conduction

<assumption> : No heat generation

(2) Single layer of cylinder





$$q = -kA\frac{dT}{dr}$$

면적 A 는 일정치 않음 → 평균면적 사용

$$\overline{A_L} = 2\pi\left(\frac{r_1 - r_2}{\ln\frac{r_1}{r_2}}\right)L \quad \overline{r_L} = \frac{r_1 - r_2}{\ln\frac{r_1}{r_2}}$$

$$T(r) = \frac{T_1 - T_2}{\ln\frac{r_1}{r_2}}\ln r + \left\{T_1 - \frac{T_1 - T_2}{\ln\frac{r_1}{r_2}}\ln r_1\right\}$$

$$\frac{T_1 - T(r)}{T_1 - T_2} = \frac{\ln\frac{r}{r_1}}{\ln\frac{r_2}{r_1}}$$

3. Steady-State Conduction

(2) Single layer of cylinder

$$q = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr}$$

$$= -k(2\pi rL)\frac{1}{r}\frac{T_1 - T_2}{\ln\frac{r_1}{r_2}}$$

$$\boxed{q = k(2\pi L)\frac{T_1 - T_2}{\ln\frac{r_2}{r_1}}}$$

$$q = k(2\pi L)\frac{T_1 - T_2}{\ln\frac{r_2}{r_1}}\frac{r_2 - r_1}{r_2 - r_1}$$

$$= k\left\{2\pi L\frac{r_2 - r_1}{\ln\frac{r_2}{r_1}}\right\}\frac{T_1 - T_2}{r_2 - r_1}$$

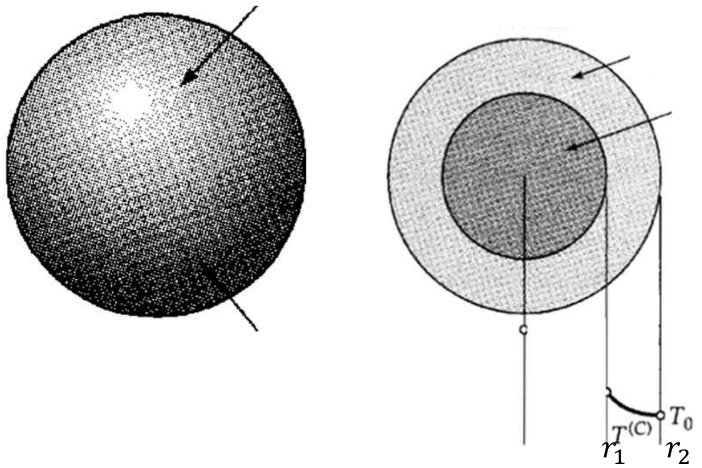
$$= k\left\{2\pi\left(\frac{r_2 - r_1}{\ln\frac{r_2}{r_1}}\right)L\right\}\frac{\Delta T}{\Delta r}$$

평균
- 산술평균 : $\frac{a+b}{2}$
- 기하평균 : $\frac{ab}{\sqrt{ab}}$
- 로그평균 : $\frac{b-a}{\text{h}\left[\frac{b}{a}\right]}$

3. Steady-State Conduction

<assumption> : No heat generation

(3) Sphere



Heat conduction equation for spherical coordination

$$\frac{\partial T}{\partial t} = \alpha \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right\} + \frac{\dot{q}}{\rho C_p}$$

$$0 = \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)$$

1st intg

$$r^2 \frac{dT}{dr} = C_1$$

2nd intg

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

B.C.

$@r = r_1, T = T_1$

$@r = r_2, T = T_2$

$$C_1 = \frac{T_1 - T_2}{\frac{1}{r_2} - \frac{1}{r_1}} \qquad C_2 = T_1 + \frac{1}{r_1} \left[\frac{T_1 - T_2}{\frac{1}{r_2} - \frac{1}{r_1}} \right] = T_1 - \frac{T_1 - T_2}{1 - \frac{r_1}{r_2}}$$

$$T(r) = -\frac{1}{r} \left(\frac{T_1 - T_2}{\frac{1}{r_2} - \frac{1}{r_1}} \right) + \left\{ T_1 - \frac{T_1 - T_2}{1 - \frac{r_1}{r_2}} \right\}$$

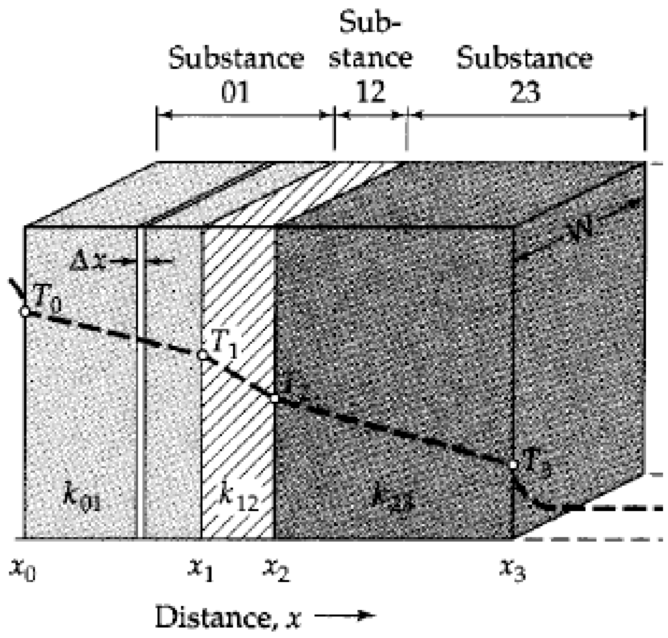
$$\frac{T_1 - T(r)}{T_1 - T_2} = \frac{\frac{1}{r_1} - \frac{1}{r}}{\frac{1}{r_1} - \frac{1}{r_2}}$$

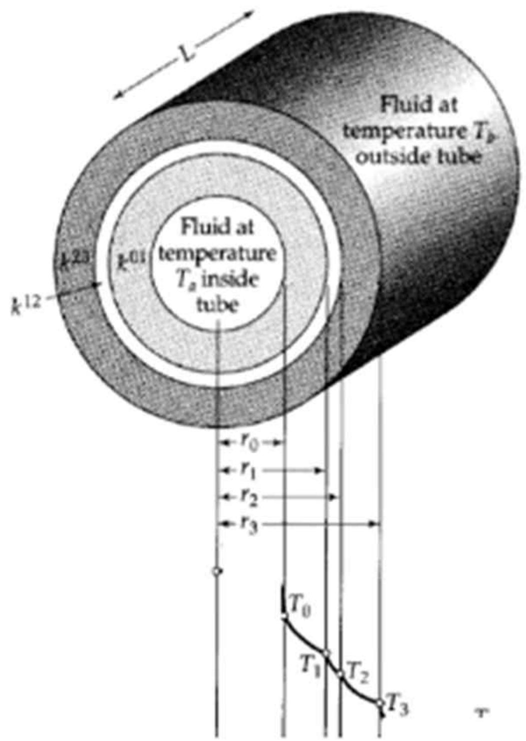
Temperature profile in spherical coordination

3. Steady-State Conduction

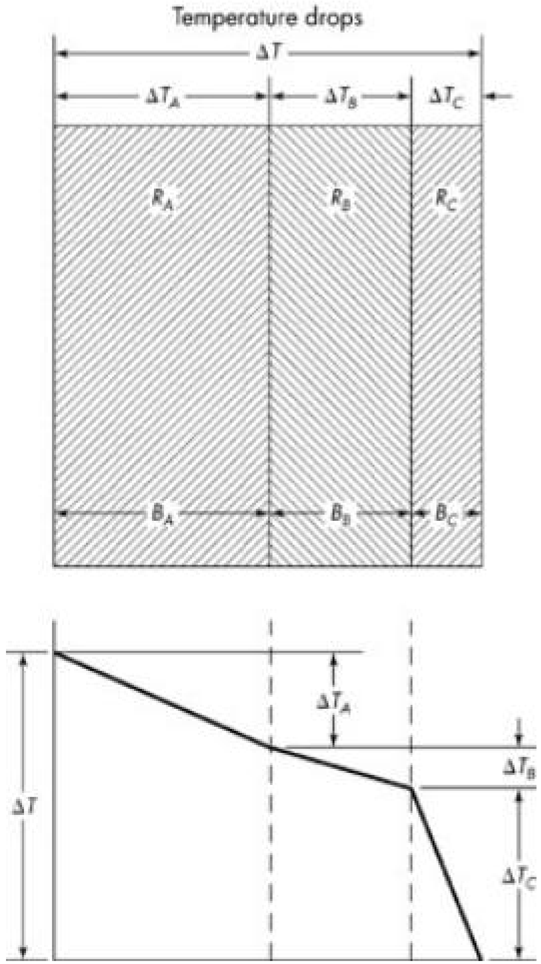
<assumption> : No heat generation

(4) Multi-layer wall





*** Slab resistances in series**



$$\Delta T = \Delta T_A + \Delta T_B + \Delta T_C$$

$$q = \frac{\Delta T_A}{R_A} = \frac{\Delta T_B}{R_B} = \frac{\Delta T_C}{R_C}$$

Thermal resistance

- Slab

$$R_A = \frac{\Delta x_A}{k_A\, A}, R_B = \frac{\Delta x_B}{k_B\, A}, R_C = \frac{\Delta x_C}{k_C\, A},$$

- Cylinder

$$R_A = \frac{\Delta r_A}{k_A\, \overline{A_{L,A}}}, R_B = \frac{\Delta r_B}{k_B\, \overline{A_{L,B}}}, R_C = \frac{\Delta r_C}{k_C\, \overline{A_{L,C}}}$$

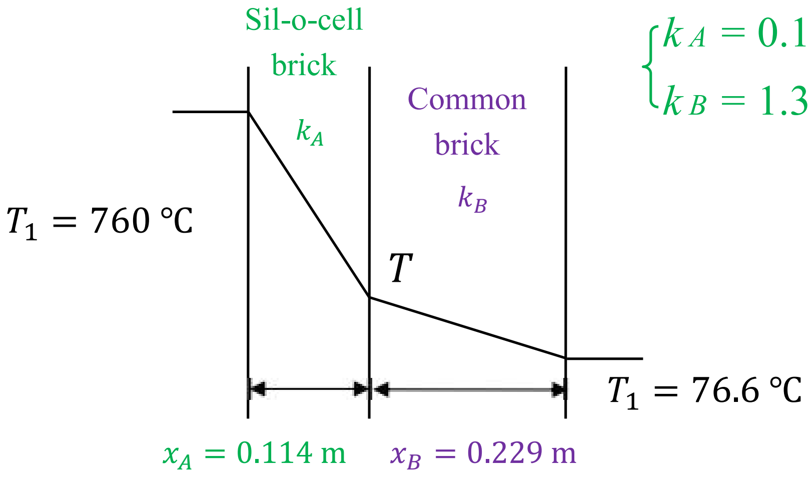
- Sphere

$$R_A = \frac{\Delta r_A}{k_A\, \overline{A_{G,A}}}, R_B = \frac{\Delta r_B}{k_B\, \overline{A_{G,B}}}, R_C = \frac{\Delta r_C}{k_C\, \overline{A_{G,C}}}$$

$$q = \frac{\Delta T_A}{R_A} = \frac{\Delta T_B}{R_B} = \frac{\Delta T_C}{R_C} = \frac{\Delta T_B + \Delta T_B + \Delta T_C}{R_A + R_B + R_C}$$

$$= \frac{\sum \Delta T_i}{\sum \Delta R_i} = \frac{\text{Overall driving force}}{\text{Overall thermal resistance}}$$

Ex. 10.2) A flat furnace wall constructed of a layer of Sil-o-cel brick backed by a common brick



$$\begin{cases} k_A = 0.138 \text{ W/m°C} \\ k_B = 1.38 \text{ W/m°C} \end{cases}$$

(a) Heat loss through the wall, q = ?

Assume A = 1 m^2

$$R_A = \frac{x_A}{k_A} = 0.826 \quad R_B = \frac{x_B}{k_B} = 0.159$$

$$R = R_A + R_B = 0.985 \text{ m}^2 \text{ °C/W} \qquad \Delta T = 683.4 \text{ °C}$$

$$q/A = 683.4/0.985 = 693.81 \text{ W/m}^2$$

$q = 693.81$ W

(b) Temperature of the interface between the two bricks

$$\Delta T/R = \Delta T_A/R_A = 683.4/0.985 = \Delta T_A/0.826 \qquad \Delta T_A = 573.08 \text{ °C} \qquad \therefore T = T_1 - \Delta T_A = 186.9 \text{ °C}$$

(c) In case that the contact between the two bricks is poor and the contact resistance is 0.088 m 2 o C/W, the heat loss q = ?

$$R = 0.985 + 0.088 = 1.073 \text{ m}^2 \text{ °C/W}$$

$$\therefore q = \frac{\Delta T}{R} = 636.9\ W$$

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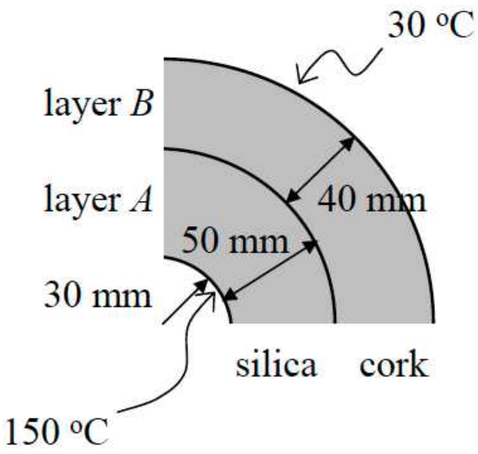
Ex. 10.3) A tube of 60 mm OD insulated with a 50 mm silica foam layer and a 40 mm cork layer

Calculate the heat loss q of pipe in W/m ?



$$\begin{cases} k_A = 0.055 \text{ W/m°C} \\ k_B = 0.05 \text{ W/m°C} \end{cases}$$

silica foal layer, $\bar{r}_L = \dfrac{80-30}{\ln(80/30)}$

cork layer, $\bar{r}_L = \dfrac{120-80}{\ln(120/80)}$

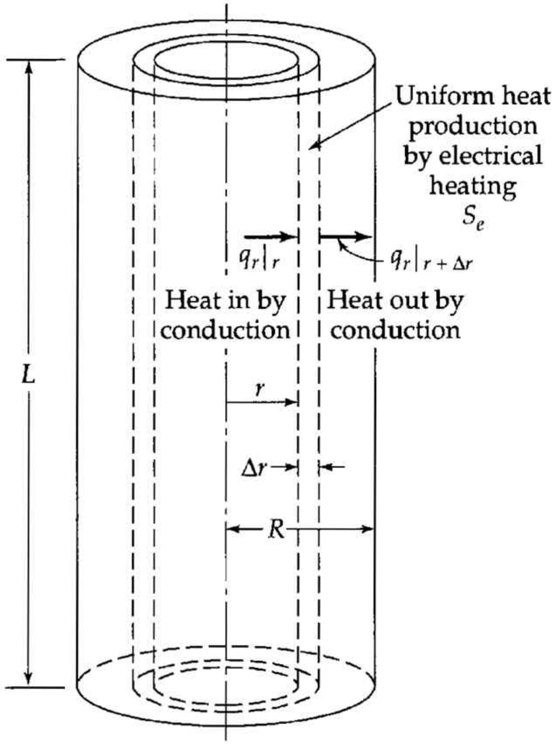
$$\bar{r}_L = \frac{r_o - r_i}{\ln(r_o / r_i)}$$

$$q_A = \frac{k_A(2\pi\, \overline{r_{L,A}}\, L)(T_i - T)}{x_A} \qquad q_B = \frac{k_B(2\pi\, \overline{r_{L,B}}\, L)(T - T_o)}{x_B}$$

$$\Rightarrow \quad q_A = 0.3522\, L(T_i - T)$$
$$q_B = 0.7748\, L(T - T_o)$$

At steady state, $q = q_A = q_B$ $\therefore q / L = 29.1$ W

4. Steady Conduction with electrical heat generation



$$\frac{\partial T}{\partial t} = \alpha\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right\} + \frac{\dot{q}}{\rho C_p}$$

$$0 = \frac{k}{\rho C_p}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right\} + \frac{\dot{q}}{\rho C_p}$$

$$k\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right\} = -\dot{q}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{\dot{q}}{k}r$$

1st intg

$$r\frac{dT}{dr} = -\frac{\dot{q}}{2k}r^2 + C_1$$

$$\frac{dT}{dr} = 1\frac{\dot{q}}{2k}r + \frac{C_1}{r}$$

2nd intg

$$T = -\frac{\dot{q}}{4k}r + C_1 \ln r + C_2$$

B.C.

$@r = 0, T = \infty$

$@r = R, T = T_w$

$C_1 = 0 \quad C_2 = T_w - \frac{\dot{q}}{4k}R^2$

4. Steady Conduction with electrical heat generation

Temperature profile

$$T(r) = \frac{\dot{q}}{4k}\left(R^2 - r^2\right) = \frac{\dot{q}}{4k}R^2\left(1 - \frac{r^2}{R^2}\right)$$

For $\dot{q}$

$$\dot{q} = V_e I = \frac{V_e^2}{\rho_e}\left(= \frac{I^2}{K_e}\right)$$

$$T(r) = T_w + \frac{I^2 R^2}{4kK_e}\left(1 - \frac{r^2}{R^2}\right)$$

For $q_{\text{out}} = -kA \left.\frac{dT}{dr}\right|_{r=R}$

$$= -k(2\pi RL)\left\{-\frac{\dot{q}R}{2k}\right\}$$

$$= \dot{q}(2\pi R^2 L)$$