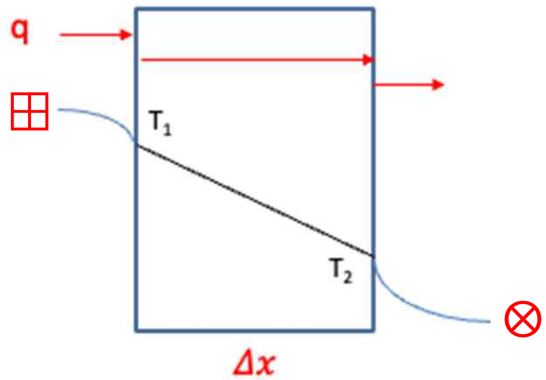
Chapter 11. Principle of heat flow in fluid

convection : Natural convection & Forced convection

1. Newton's equation of cooling



$$q \propto A\,\Delta T$$

$$q = h\,A\,\Delta T \qquad \Delta T = ⊞ - ⊗$$

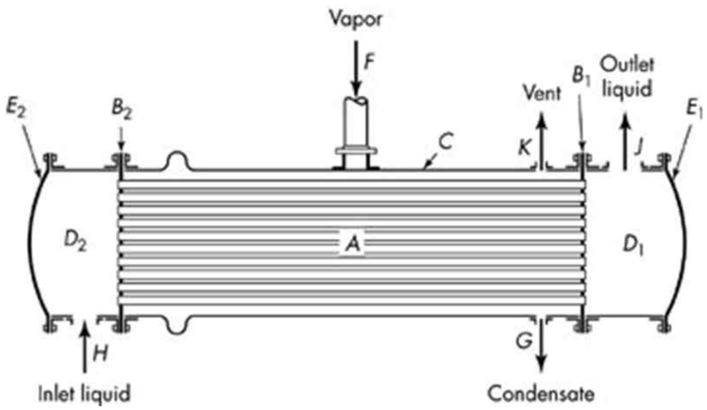
where, h is heat transfer coefficient

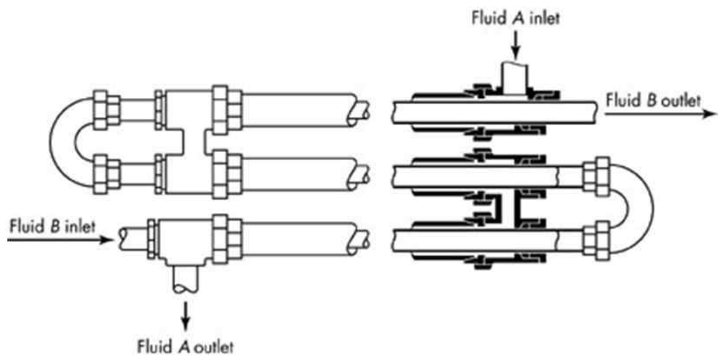
$h = h$(geometry, physical property of fluid, fluid velocity…)

$$\left[\frac{\mathrm{W}}{\mathrm{m^2\,°C}}\right], \left[\frac{\mathrm{Btu}}{\mathrm{ft^2\ hr\ °F}}\right]$$

$$q = \frac{\Delta T}{R_{conv}} \qquad R_{conv} = \frac{1}{h\,A}$$

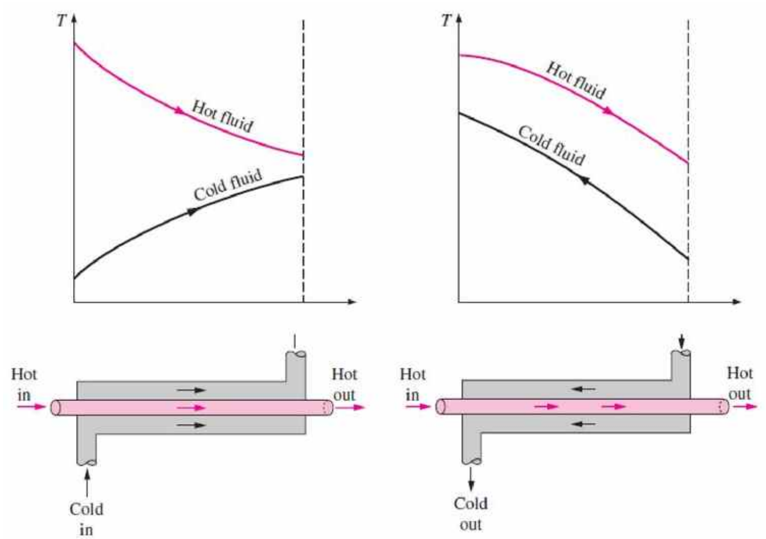
2. Types of Heat Exchanger (HEX)





* Contact methods
- Counter current flow
- Co-current flow
- Cross current flow

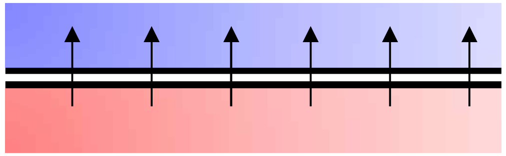
Double pipe HEX



a) co-current flow   b) counter-current flow

T_c $T_c + dT_c$

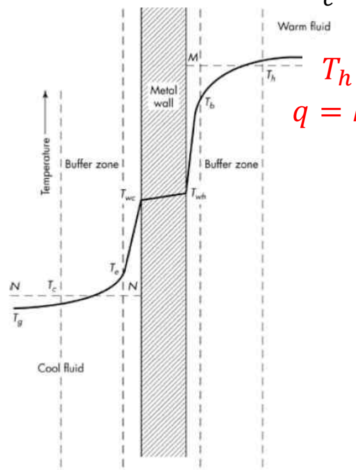
H_c $H_c + dH_c$



l $l + dl$

T_h $T_h + dT_h$

H_h $H_c + dH_h$



T_h

$q = h_h A \Delta T_h$

T_c

$q = h_c A \Delta T_c$

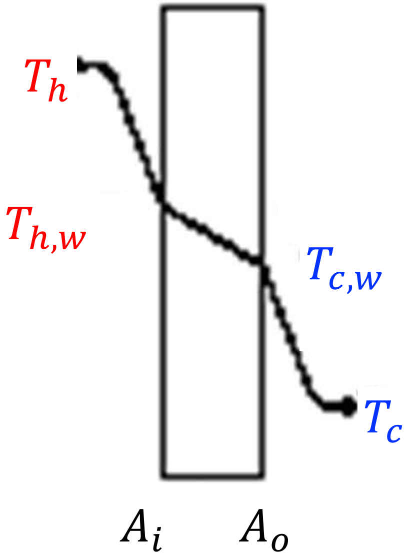
$$q = h_h A \frac{\Delta T_w}{x_w}$$

3. Rate of heat transfer

1) Overall heat transfer coefficient

<Assumptions>

1) $\dot{m}_h, \dot{m}_c$: constant
2) C_{ph}, C_{pc}: constant



$$q = U\,A\,(T_h - T_c)$$

$T_h - T_c$: Overall local temperature difference
U : local overall heat transfer coefficient

Differential rate of heat transfer through infinitesimal cross-sectional area, dA

$$\delta q = U_i dA_i \Delta T = U_o dA_o \Delta T$$

$$dA_i = \pi D_i dl$$
$$dA_o = \pi D_o dl$$

$$U_i dA_i = U_o dA_o$$

$$\frac{U_i}{U_o} = \frac{dA_o}{dA_i} = \frac{D_o}{D_i}$$

3. Rate of heat transfer

2) Heat balance

Hot fluid side

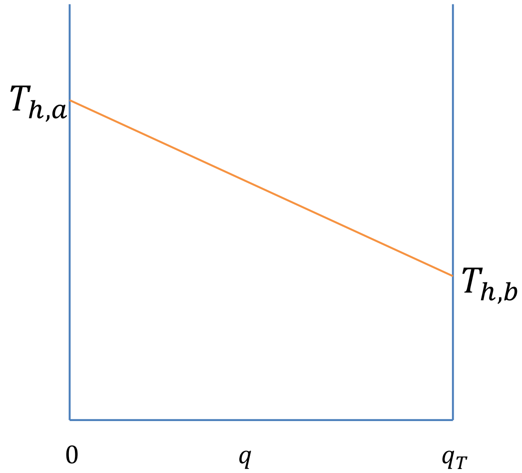
$$H_h = (H_h + dH_h) + \delta q$$
$$dH_h = -\delta q$$

$dH_h = m_h C_{ph} dT_h$

$$\delta q = -\dot{m}_h C_{ph} dT_h$$
$$q = -\dot{m}_h C_{ph}\left(T_h - T_{h,a}\right) = \dot{m}_h C_{ph}\left(T_{h,a} - T_h\right)$$

Total heat transfer rate

$$q_T = m_h C_{ph}\left(T_{h,a} - T_{h,b}\right)$$



3. Rate of heat transfer

2) Heat balance

Cold fluid side

$$(H_c + dH_c) + \delta q = H_c$$
$$H_c = -\delta q$$

$$dH_c = \dot{m}_c C_{pc} dT_c = -\delta q$$

$$\int \delta q = \int_{T_{ca}}^{T_c} -\dot{m}_c C_{pc} dT_c$$

$$q = -\dot{m}_c C_{pc} d\left(T_c - T_{c,a}\right) = \dot{m}_c C_{pc}\left(T_{c,a} - T_c\right)$$

Total heat transfer rate

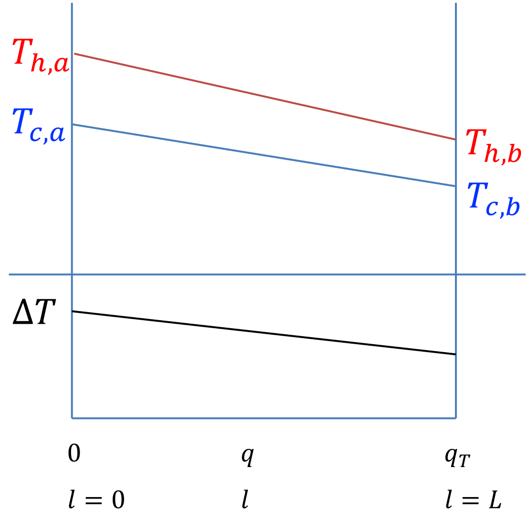
$$q_T = \dot{m}_c C_{pc}\left(T_{c,a} - T_{c,b}\right) = -\Delta H_{T,h} = \Delta H_{T,c}$$

3. Rate of heat transfer

2) Heat balance

Overall heat balance

$$(q_T =)\dot{m}_c C_{pc}\left(T_{c,a} - T_{c,b}\right) = \dot{m}_h C_{ph}\left(T_{h,a} - T_{h,b}\right)$$
$$-\Delta H_{T,h} = \Delta H_{T,c}$$
$$-\left\{\dot{m}_h C_{ph}\left(T_{h,b} - T_{h,a}\right)\right\} = \dot{m}_c C_{pc}\left(T_{c,a} - T_{c,b}\right)$$



$$slope = \frac{d(\Delta T)}{\delta q} = \frac{\Delta T_2 - \Delta T_1}{q_T} \qquad \delta q = U dA \Delta T$$

$$\boxed{\frac{d(\Delta T)}{U dA \Delta T} = \frac{\Delta T_2 - \Delta T_1}{q_T}}$$

where, $q_T = \dot{m}_c C_{pc}\left(T_{c,a} - T_{c,b}\right) = \dot{m}_h C_{ph}\left(T_{h,a} - T_{h,b}\right)$

3. Rate of heat transfer

Case I, U is constant

$$\int_{\Delta T_1}^{\Delta T_2} \frac{1}{U}\frac{d(\Delta T)}{\Delta T} = \int_0^{A_T} \frac{\Delta T_2 - \Delta T_1}{q_T} dA$$

$$\frac{1}{U}\ln\frac{\Delta T_2}{\Delta T_1} = \frac{\Delta T_2 - \Delta T_1}{q_T} A_T$$

$$\therefore q_T = U A_T \frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}}$$

$$\boxed{q_T = U A_T \overline{\Delta T_L}}$$

$$\overline{\Delta T_L} = \frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}}$$ Log mean temperature difference(LMDT)

If $\dot{m}$ and C_p are constant

$$\frac{d(\Delta T)}{\delta q} = \frac{\Delta T_2 - \Delta T_1}{q_T - 0}$$

← $\delta q = ud\bar{A}\Delta T$

3. Rate of heat transfer

Case II, U is **NOT** constant

$$U = U(\Delta T) = a + b\Delta T$$

$$\frac{d(\Delta T)}{U dA \Delta T} = \frac{\Delta T_2 - \Delta T_1}{q_T}$$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{a + b\Delta T - b\Delta T}{a(a + b\Delta T)\Delta T} d(\Delta T) = \int_0^{A_T} \frac{\Delta T_2 - \Delta T_1}{q_T} dA$$

$$\frac{1}{a}\int_{\Delta T_1}^{\Delta T_2} \left[\frac{a + b\Delta T}{(a + b\Delta T)\Delta T} - \frac{b\Delta T}{(a + b\Delta T)\Delta T}\right] d(\Delta T)$$

$$= \frac{1}{a}\int_{\Delta T_1}^{\Delta T_2} \left[\frac{1}{\Delta T} - \frac{b\Delta T}{(a + b\Delta T)\Delta T}\right] d(\Delta T) = \frac{\Delta T_2 - \Delta T_1}{q_T} A_T$$

$$\frac{1}{a}\left\{\ln\frac{\Delta T_2}{\Delta T_1} - \ln\frac{a + b\Delta T_2}{a + b\Delta T_1}\right\} = \frac{\Delta T_2 - \Delta T_1}{q_T} A_T$$

$$\ln\frac{\Delta T_2(a + b\Delta T_1)}{\Delta T_1(a + b\Delta T_2)} = \frac{a\Delta T_2 - a\Delta T_1}{q_T} A_T$$

$$\ln\frac{\Delta T_2 U_1}{\Delta T_1 U_2} = \frac{\Delta T_2 U_1 - \Delta T_1 U_2}{q_T} A_T$$

$$\Delta T_2 U_1 = \Delta T_2 a + b\Delta T_1 \Delta T_2$$
$$- \quad \Delta T_1 U_2 = \Delta T_1 a + b\Delta T_2 \Delta T_1$$

$$\Delta T_2 U_1 - \Delta T_1 U_2 = a\Delta T_2 a - a\Delta T_1$$

$$\therefore q_T = A_T \frac{U_1 \Delta T_2 - U_2 \Delta T_1}{\ln \frac{U_1 \Delta T_2}{U_2 \Delta T_1}}$$

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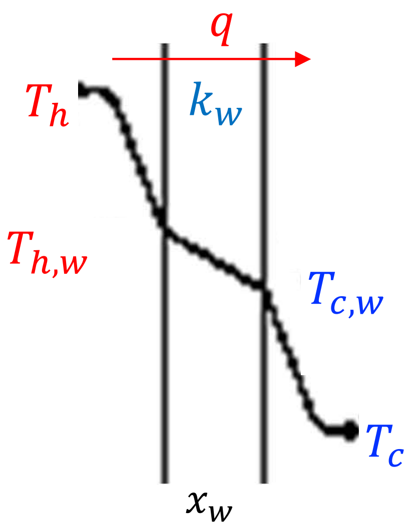
4. Relation between overall heat transfer coefficient and individual heat transfer coefficient

1) For clean surface (no deposit)



$$\delta q = \frac{\Delta T_i}{R_i} \rightarrow \begin{pmatrix} \Delta T_i = T_h - T_{h,w} \\ R_i = 1/(h_i dA_i) \end{pmatrix}$$

$$\delta q = \frac{\Delta T_w}{R_w} \rightarrow \begin{pmatrix} \Delta T_w = T_{h,w} - T_{c,w} \\ R_i = x_w/k_w d\overline{A_L} \end{pmatrix}$$

$$\delta q = \frac{\Delta T_o}{R_o} \rightarrow \begin{pmatrix} \Delta T_o = T_{c,w} - T_c \\ R_o = 1/(h_o dA_o) \end{pmatrix}$$

$$\delta q = \frac{T_h - T_c}{R_i + R_w + R_o} = \frac{\Delta T}{R_i + R_w + R_o} \text{ -------- A}$$

$$\delta q = U_i dA_i \Delta T = U_o dA_o \Delta T \text{ ------- B}$$

4. Relation between overall heat transfer coefficient and individual heat transfer coefficient

1) For clean surface (no deposit)

From A & B,

$$\frac{1}{U_i dA_i} = R_i + R_w + R_o = \frac{1}{h_i dA_i} + \frac{x_w}{k_w d\overline{A_L}} + \frac{1}{h_o dA_o}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{x_w dA_i}{k_w d\overline{A_L}} + \frac{1}{h_o}\frac{dA_i}{dA_o}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{x_w D_i}{k_w \overline{D_L}} + \frac{1}{h_o}\frac{D_i}{D_o}$$

$$\delta q = U_i dA_i \Delta T \rightarrow \frac{\Delta T}{1/(U_i dA_i)} \text{ ---- A}$$

$$= U_o dA_o \Delta T \rightarrow \frac{\Delta T}{1/(U_o dA_o)} \text{ ---- C}$$

$$\delta q = \frac{\Delta T}{\frac{1}{h_i dA_i} + \frac{x_w}{k_w d\overline{A_L}} + \frac{1}{h_o dA_o}}$$

$$\boxed{\frac{1}{U_i} = \frac{1}{h_i} + \frac{x_w D_i}{k_w \overline{D_L}} + \frac{1}{h_o}\frac{D_i}{D_o}}$$

$$\boxed{\frac{1}{U_o} = \frac{1}{h_i}\frac{D_o}{D_i} + \frac{x_w D_o}{k_w \overline{D_L}} + \frac{1}{h_o}}$$

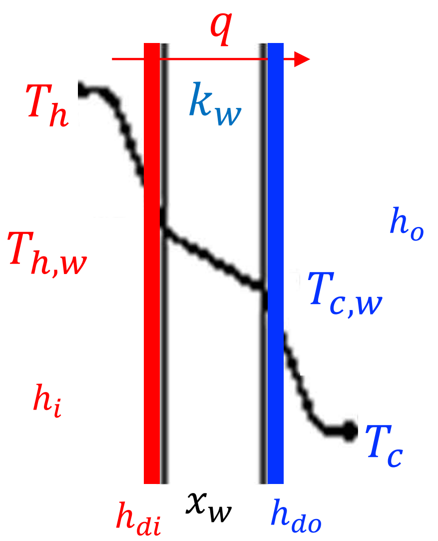
4. Relation between overall heat transfer coefficient and individual heat transfer coefficient

2) with scale deposit

<Assumptions>

1) $dA_i \cong dA_{di}$

2) $dA_o \cong dA_{do}$



$$\delta q = \frac{\Delta T}{R_{total}}$$

$$\Delta T = T_h - T_c$$

$$R_{total} = R_i + R_{di} + R_w + R_{do} + R_o$$

$$R_i = \frac{1}{h_i dA_i} \qquad R_{di} = \frac{1}{h_{di} dA_i}$$

$$R_o = \frac{1}{h_o dA_o} \qquad R_{do} = \frac{1}{h_{do} dA_o}$$

$$R_w = \frac{x_w}{k_w d\overline{A_L}}$$

$$\delta q = \frac{\Delta T}{\frac{1}{h_i dA_i} + \frac{1}{h_{di} dA_i} + \frac{x_w}{k_w d\overline{A_L}} + \frac{1}{h_{do} dA_o} + \frac{1}{h_o dA_o}}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{1}{h_{di}} + \frac{x_w D_i}{k_w \overline{D_L}} + \frac{1}{h_o}\frac{D_i}{D_o} + \frac{1}{h_{do}}\frac{D_i}{D_o}$$

$$\frac{1}{U_o} = \frac{1}{h_i}\frac{D_o}{D_i} + \frac{1}{hd_i}\frac{D_o}{D_i} + \frac{x_w D_o}{k_w \overline{D_L}} + \frac{1}{h_{do}} + \frac{1}{h_o}$$