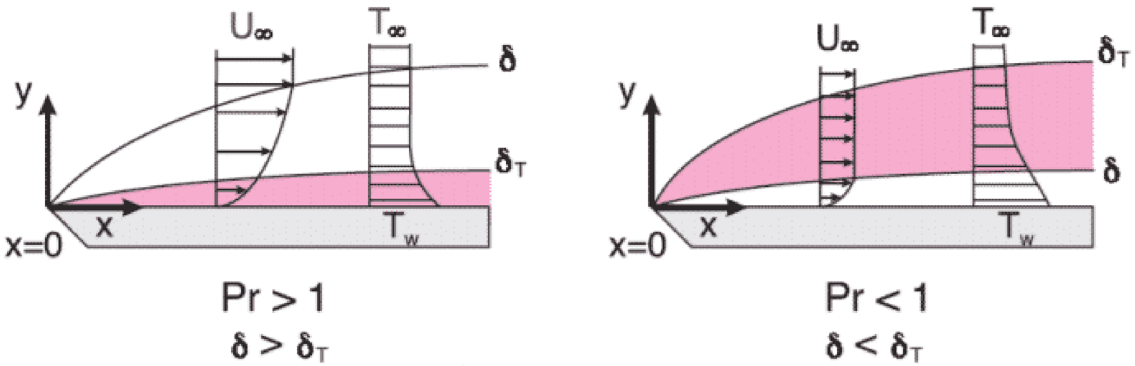
Chapter 12. Heat transfer to fluid without phase change

Prandtl number

$$Pr = \frac{\nu(\text{momentum diffusivity})}{\alpha(\text{thermal diffusivity})} = \left[\frac{(\mu/\rho)}{(k/\rho C_p)}\right] = \frac{\rho C_p}{k}$$

δ: Prandtl boundary layer
δ_T: thermal boundary layer



$Pr < 1 (\Rightarrow \nu < \alpha)$: Liquid metal
$Pr > 1 (\Rightarrow \nu > \alpha)$: Liquid
$Pr \cong 1 (\Rightarrow \nu \cong \alpha)$: Gas

$$q = hA\Delta T$$
$$q/A = h\Delta T$$

1. Dimensional Analysis

Number of dimensional variables ⇢ n
Number of fundamental dimensions ⇢ m

⇨ Number of dimensionless variables ⇢ $n - m$

$$q/A = h\,\Delta T$$

$$q/A = q/A(\rho, C_p, k, \mu, u, \Delta T, D, \beta \times g)$$

$$F(q/A, \rho, C_p, k, \mu, u, \Delta T, D, \beta \times g) = 0$$

where, $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$: coefficient of thermal expansion $[K^{-1}]$

$\beta[= K^{-1}][= \theta^{-1}]$ $g[= m/s^2][= LT^{-2}]$

$\beta g[= LT^{-2}\theta^{-1}]$ $k[= W/m \cdot K][= HT^{-1}L^{-1}\theta^{-1}]$
$q/A[= HT^{-1}L^{-1}]$ $\mu[= g/cm \cdot s][= ML^{-1}T^{-1}]$
$C_p[= J/kg \cdot K][= HM^{-1}\theta^{-1}]$ $\Delta T[= K][= \theta]$
$u[= m/s][= LT^{-1}]$ $D[= m][= L]$

Chapter 12. Heat transfer to fluid without phase change

1. Dimensional Analysis

Number of fundamental dimensions; $[M], [L], [T], [\theta], [H]$ ➔ $m = 5$

- Recurring sets

$\Delta T[= \theta] \rightarrow [\theta] = \Delta T$

$D[= L] \rightarrow [L] = D$

$\rho[= ML^{-3}] \rightarrow [M] = \rho \times [L^3] = \rho D^3$

$\mu[= ML^{-1}T^{-1}] \rightarrow [T] = \mu^{-1}[ML^{-1}] = \mu^{-1}\rho D^3(D^{-1}) = \rho D^2/\mu$

$k[= HT^{-1}L^{-1}\theta^{-1}] \rightarrow [H] = k[T\theta L] = k\left(\frac{\rho D^3}{\mu}\right)(\Delta T)(D) = k(\rho D^3)\Delta T/\mu$

1. Dimensional Analysis

남은 변수들 ; $u, C_p, \beta g, q/A$

1) $u[= LT^{-1}]$

$$\pi_1 = \frac{u}{[LT^{-1}]} = \frac{u}{D[LT^{-1}]} = \frac{u}{D\left(\frac{\rho D^2}{\mu}\right)^{-1}} = \frac{\rho u D}{\mu}$$

$\pi_1 = Re$

2) $C_p[= HM^{-1}\theta^{-1}]$

$$\pi_2 = \frac{C_p}{[=HM^{-1}\theta^{-1}]} = \frac{C_p}{[(k\rho D^3)(\rho D^3)^{-1}\Delta T^{-1}]} = \frac{C_p\mu}{k}$$

$\pi_2 = Pr$

3) $\beta g[= LT^{-2}\theta^{-1}]$

$$\pi_3 = \frac{\beta g}{[LT^{-2}\theta^{-1}]} = \frac{\beta g}{\left[D\left(\frac{\rho D^2}{\mu}\right)^{-2}\Delta T^{-1}\right]} = \frac{\beta g \Delta T \rho^2 D^3}{\mu^2}$$

$\pi_3 = Gr$ (Grashof number)

4) $q/A[= HT^{-1}L^{-2}]$

$$\pi_4 = \frac{q/A}{[HT^{-1}L^{-2}]} = \frac{q/A}{\left[\left(\frac{k\rho D^3\Delta T}{\mu}\right)\left(\frac{\rho D^2}{\mu}\right)^{-1} D^{-2}\right]} = \frac{hD}{k}$$

$\pi_4 = Nu$ (Nusselt number)

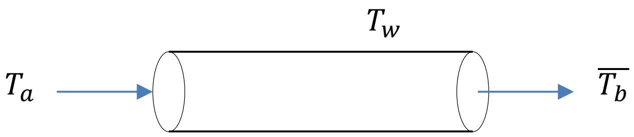
$$Nu = \frac{(q/A)_{convection}}{(q/A)_{con\ duction}}$$

$$= \frac{h\Delta T}{k\frac{\Delta T}{\Delta x}} = \frac{h}{k}\Delta x$$

2. Forced convection

1) Turbulent flow

$$Nu = Nu(Re, Pr)$$



- Empirical equation

$$Nu = 0.023 Re^{0.8} Pr^{1/3} \phi^{0.14}$$

where, $\phi = \mu / \mu_w$

$$Nu = \frac{h_i D_i}{k},\ Pr = \frac{C_p \mu}{k},\ Re = \frac{\rho u D}{\mu},$$

$$\frac{h_i D_i}{k} = 0.023 \left(\frac{\rho u D_i}{\mu}\right)^{0.8} \left(\frac{C_p \mu}{k}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{1.4}$$

2. Forced convection

2) Laminar flow

$$Nu = Nu(Gz)$$

$$Gz = \frac{\dot{m}C_p}{k\ L} = \frac{(\rho\bar{u}s)C_p}{k\ L} = \frac{(\rho\bar{u}\frac{\pi}{4}D^2)C_p}{k\ L}\frac{\mu}{\mu}$$

$$= \left(\frac{C_p\mu}{k}\right)\left(\frac{\rho\bar{u}D}{\mu}\right)\left(\frac{\pi}{4}\frac{D}{L}\right)$$

$Pe[= Pr \cdot Re]$ (Peclet Number)

$$Gz = Pe\left(\frac{\pi}{4}\frac{D}{L}\right) = Pr \cdot Re\left(\frac{\pi}{4}\frac{D}{L}\right)$$

Fourier's number

$$Fo = \frac{\alpha t}{s^2} = \frac{\alpha t}{R^2} = \frac{4\alpha t}{D^2} = \frac{4kt}{\rho C_p D^2} = \frac{4kL}{\rho C_p D^2 \bar{u}}$$

$$\therefore Gz = \frac{\pi}{Fo}$$

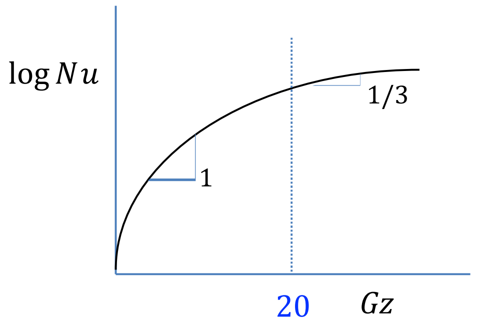
2. Forced convection

2) Laminar flow

if $Gz > 20$

$$Nu = 2Gz^{1/3}\phi_v^{0.14}$$

$$= 2(\Pr Re\frac{\pi}{4}\frac{D}{L})^{\frac{1}{3}}\phi_v^{0.14}$$

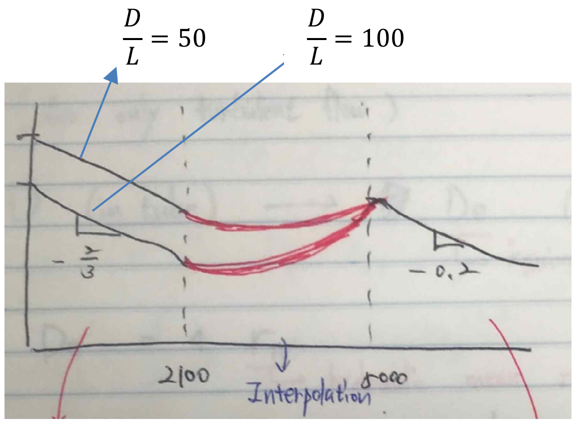


$$NuRe^{-1}Pr^{-1} = 2\left(\frac{\pi}{4}\right)^{\frac{1}{3}} Pr^{-\frac{2}{3}}Re^{-\frac{2}{3}}\phi_v^{0.14}\left(\frac{D}{L}\right)^{\frac{1}{3}}$$

$$St = 1.86\, Pr^{-\frac{2}{3}}Re^{-\frac{2}{3}}\phi_v^{0.14}\left(\frac{D}{L}\right)^{\frac{1}{3}} = 1.86\, Re^{-\frac{2}{3}}\left(\frac{D}{L}\right)^{\frac{1}{3}}$$

2. Forced convection

3) Transition region (2,100 < Re < 50,000)



3. Forced convection in Non-circular duct

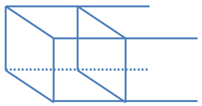
for only turbulent flow

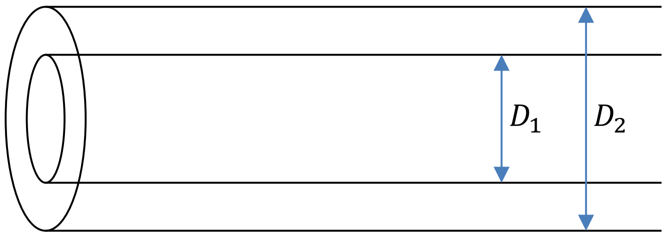
D(tube) → D_e(non-circular tube), equivalent diameter

$$D_e = 4r_H = 4\left(\frac{\text{crossectional area}}{\text{wetted perimeter}}\right)$$

Ex. 가로 3cm, 세로 2cm 의 duct 의 D_e?

$$r_H = \frac{2 \times 3\text{cm}^2}{2 \times (3+2)\text{cm}} = 0.6\ cm \qquad D_e = 4r_H = 2.4\ cm$$





$D_e = D_2 - D_1$

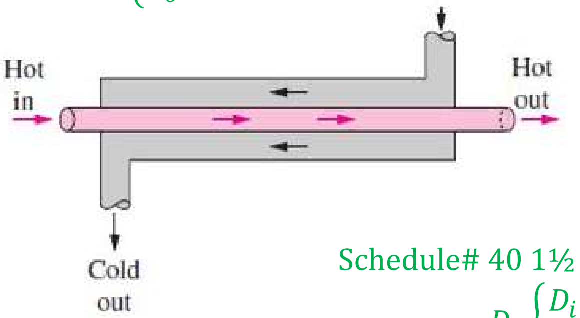
Ex 12-2

$\frac{7}{8}$" 16 BMG copper tube (App 6)

$D_1 \begin{cases} D_i = 0745\ \dot{n}\ . \\ D_o = 0.875\ \dot{n}\ . \end{cases}$

$T_{wb} = 60°F$
$\overline{u_w} = 4\ ft/s$

$T_{Ba} = 141°F$
$\overline{u_B} = 5$ ft/s



$T_{Bb} = 79°F$
$\overline{u_B} = 5$ ft/s

$T_{wa} = 80°F$
$\overline{u_w} = 4\ ft/s$

Schedule# 40 1½ $\dot{n}$. Steel pipe (App5)

$D_2 \begin{cases} D_i = 1.610\ \dot{n}\ . \\ D_o = 1.900\ \dot{n}\ . \end{cases}$

$$\overline{T_B} = \frac{141 + 79}{2} = 110\ °F \qquad \overline{T_w} = \frac{60 + 80}{2} = 70\ °F$$

	water (70 °F)	Benzene (110 °F)
$\rho(b\ /ft^3)$	62.3	53.1
$\mu(b\ /ft \cdot hr)$	0.982 cP = 2.34	0.48 cP = 1.16
$k(Btu\ /ft \cdot hr \cdot °F)$	0.346	0.089
$C_p(Btu\ /b\ \cdot °F)$	1.0	0.43

Ex 12-2

h_i (Benzene)

$$Nu = 0.023Re^{0.8}Pr^{1/3}, \phi = 1$$

$$Re_B\left(=\frac{\rho u D}{\mu}\right)_B = \frac{5.31\times(5\times3600)\times(0.745/12)}{1.16} = 5.12\times10^4 \text{ (Turbulent flow)}$$

$$Pr_B\left(=\frac{C_p\mu}{k}\right)_B = \frac{0.43\times1.16}{0.08} = 6.235$$

$$Nu\left(=\frac{h_iD_i}{k}\right)_B = 0.023\,(5.12\times10^4)^{0.8}\times6.235^{1/3} = 232.4$$

$$h_i = \frac{232.4\times k}{D_i} = \frac{232.4\times0.08}{0.745/12} = 299.5\ Btu/ft\cdot hr\cdot{}^\circ F$$

h_i (water)

$$Nu = 0.023Re^{0.8}Pr^{1/3}, \phi = 1$$

$$D_e = D_2 - D_1$$

$$Re_w\left(=\frac{\rho u D_e}{\mu}\right)_w = \frac{62.3\times(4\times3600)\times(0.735/12)}{2.38} = 2.31\times10^4 \text{ (Turbulent flow)}$$

$$Pr_w\left(=\frac{C_p\mu}{k}\right)_w = \frac{1\times2.38}{0.346} = 6.88$$

$$Nu\left(=\frac{h_oD_o}{k}\right)_w = 0.023\,(2.31\times10^4)^{0.8}\times6.88^{1/3} = 135.46$$

$$h_o = \frac{135.46\times k}{D_o} = \frac{153.46\times0.346}{0.735/12} = 866.89\ Btu/ft\cdot hr\cdot{}^\circ F$$

U_o(총괄열전달계수)

$$\frac{1}{U_o} = \frac{1}{h_i}\frac{D_o}{D_i} + \frac{x_wD_o}{k_w\overline{D_L}} + \frac{1}{h_o}$$

$$= \frac{1}{299.5}\frac{0.875/12}{0.745/12} + \frac{1}{866.89}$$

$$= 5.08\times10^{-3}$$

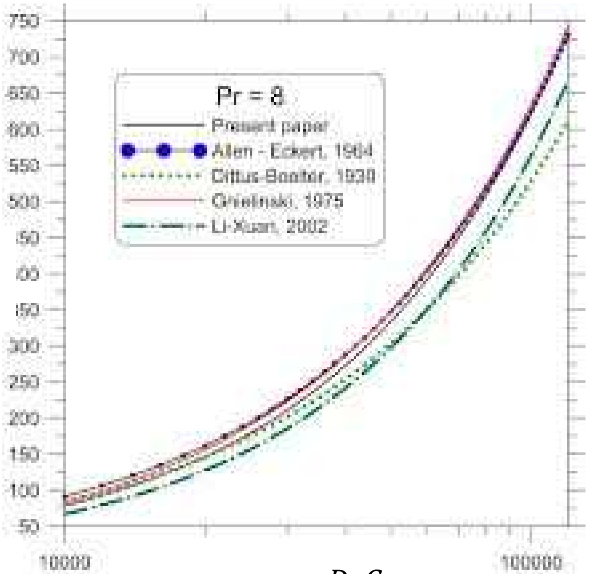
$$U_o = 197.04\ Btu/ft\cdot hr\cdot{}^\circ F$$

4. Force convection outside tubes

1) For flow normal to single cylinder

For gases

$$Nu_f = \frac{h_o D_o}{k_f}$$



$$Re_f = \frac{D_o G}{\mu_f}$$

Where, k_f, μ_f →values at mean film temperature

For liquids

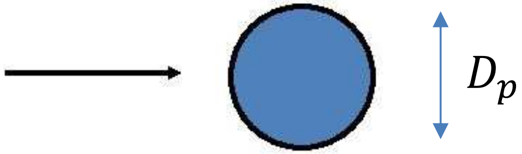
$$\left(\frac{h_o D_o}{k_f}\right)\left(\frac{c_{p,f}\mu_f}{k_f}\right) = 0.35 + 0.56\left(\frac{D_o G}{\mu_f}\right)^{0.52}$$

Nu_f Re_f

Ulsamer's equation

4. Force convection outside tubes

2) For flow past single sphere



$$\frac{h_o D_p}{k_f} = 2.0 + 0.6\left(\frac{D_p G}{\mu_f}\right)^{0.5}\left(\frac{C_{p,f}\mu_f}{k_f}\right)^{\frac{1}{3}}$$

5. Natural convection (free convection)

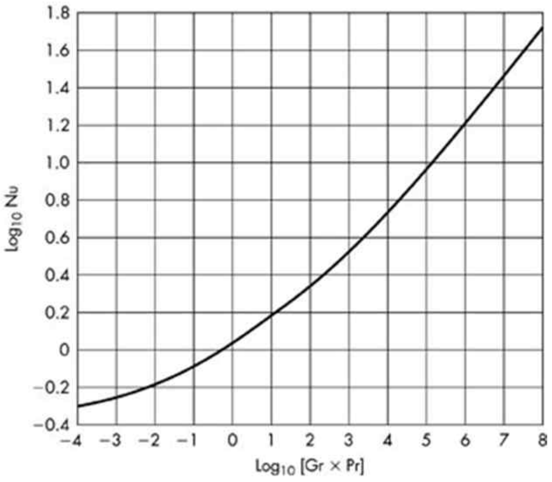
$$Nu = Nu(Pr, Gr)$$

$$Gr = \frac{\beta g(\Delta T)\rho^2 D^3}{\mu^2}$$ Grashof number

* Form of empirical equation

$$Nu = b(Pr \cdot Gr)^n$$

1) For single horizontal cylinder



If $\log(Pr \cdot Gr) > 4$ $Pr \cdot Gr > 10^4$

$b = 0.533, n = 0.25$

$$Nu_f = 0.533(Pr \cdot Gr)^{0.25}$$

$$Nu_f = \frac{h_o D_o}{k_f} \qquad Gr_f = \frac{\beta g(\Delta T)\rho_f^2 D_o^3}{\mu_f^2} \qquad Pr_f = \frac{C_{p,f}\mu_f}{k_f}$$

5. Natural convection (free convection)

2) For air flow vertical and horizontal plane

$$Nu_f = b(Pr \cdot Gr)_f^{\,n}$$

b, n 값은 (표 12.4)

$$Nu_f = \frac{hL}{k_f} \qquad Gr_f = \frac{\beta g(\Delta T)\rho_f^2 L^3}{\mu_f^2}$$

3) Effect of natural convection in laminar flow

If ΔT is large → large ΔT may cause natural convection

$$Nu = 2.0Gz^{\frac{1}{3}}\phi_v^{0.14}\phi_n$$

$$\phi_n = \frac{2.25(1 + 0.01\,Gr^{\frac{1}{3}})}{\log Re}$$

Correction factor duel to natural convection