1.5 QUANTITIVE PID TUNING METHODS

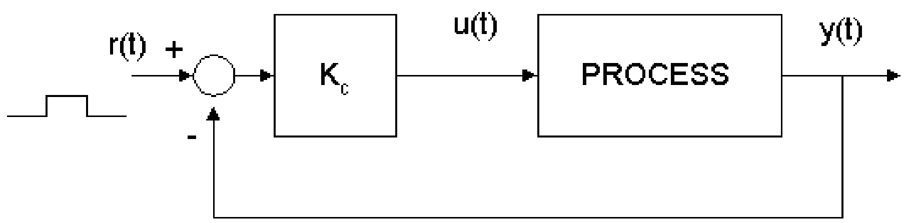
- Tuning PID parameters is not a trivial task in general.
- Various tuning methods have been proposed for different model descriptions and performance criteria.

1.5.1 CONTINUOUS CYCLING METHOD

Frequently called Ziegler-Nichols method since it was first proposed by Ziegler and Nichols (1942). Also referred to as loop tuning or the ultimate sensitivity method.

Procedure:

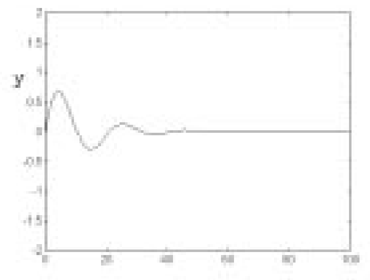
step1 Under P-control, set K_c at a low value. Be sure to choose the right (direct/reverse) mode.



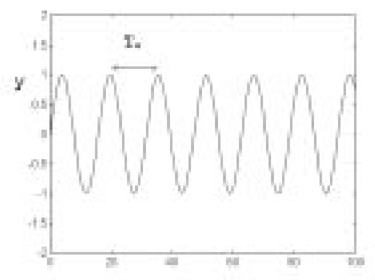
step 2 Increase K_c slowly and monitor $y(t)$ whether it shows oscillating response.
If $y(t)$ does not respond to K_c change, apply a short period of small pulse input on $r(t)$.

step 3 Increase K_c until $y(t)$ shows continuous cycling. Let K_u be K_c at this condition. Also let T_u be the period of oscillation under this condition.

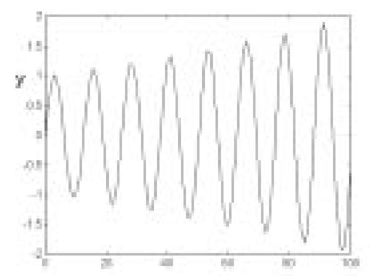
$K_c < K_u$



$K_c = K_u$



$K_c > K_u$



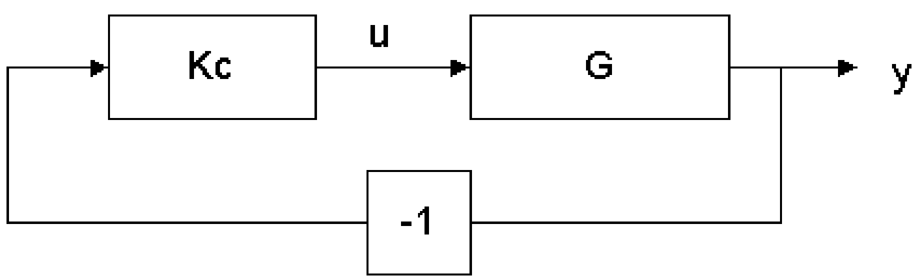
step 4 Calculate and implement PID parameters using the the Ziegler-Nichols tuning tables:

Ziegler-Nichols Controller Settings

Controller	K_c	T_I	T_D
P	$0.5K_u$	$--$	$--$
PI	$0.45K_u$	$T_u/1.2$	$--$
PID	$0.6K_u$	$T_u/2$	$T_u/8$

Remarks :

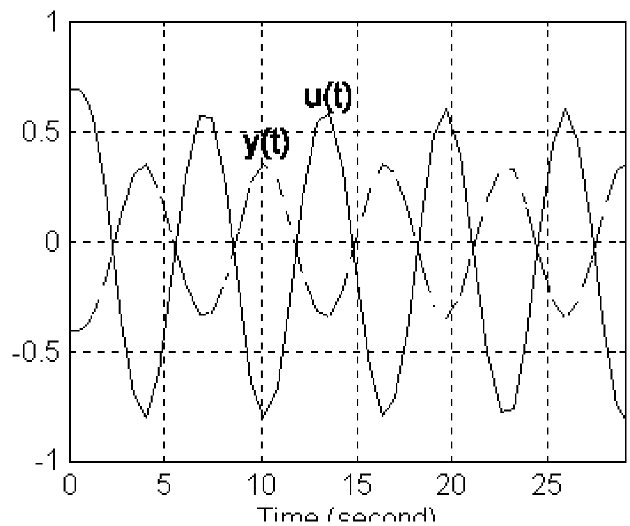
- We call
  - K_u ultimate gain
  - T_u ultimate period ($\omega_{co} = 2\pi/T_u$ critical frequency)
- Ziegler-Nichols tuning is based on the process characteristic at a single point where the closed-loop under P-control shows continuous cycling.



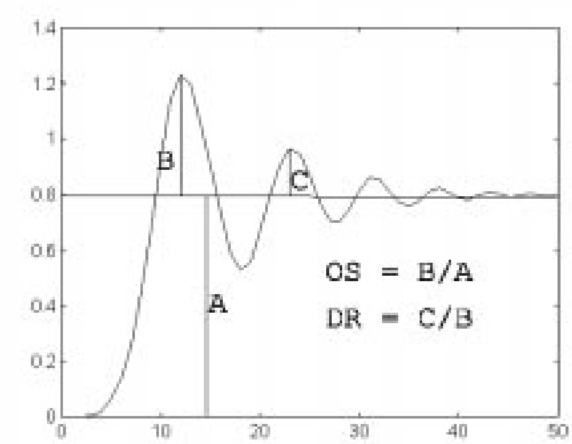
At ω_{co},

$$|G_c K_c|_{\omega_{co}} = 1$$

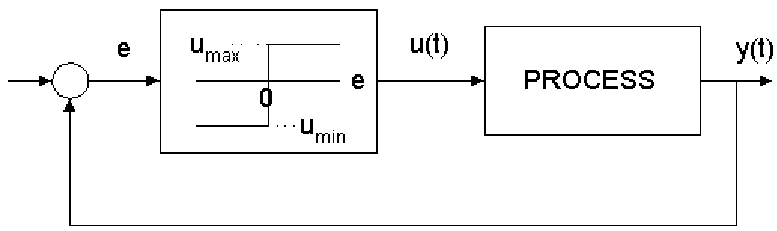
$y(t)$ is 180^o (phase lag) behind $u(t)$.



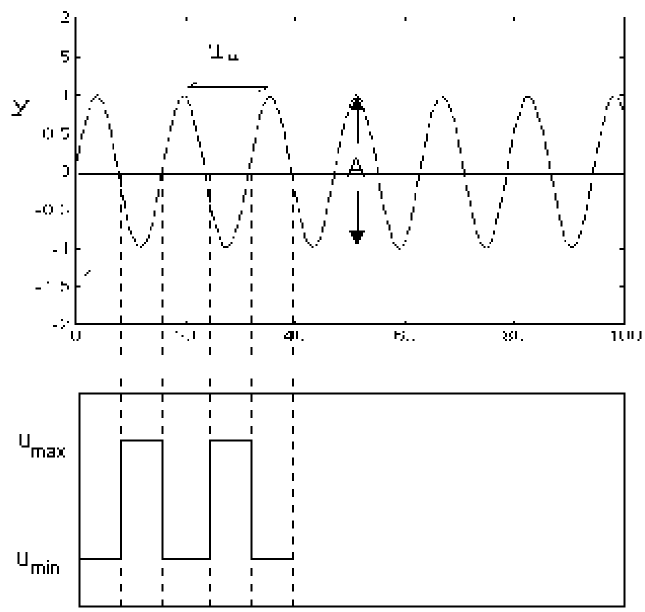
- K_u is the largest K_c for closed-loop stability under P-control. When $K_c = 0.5K_u$ under P-control, the closed-loop approximately shows 1/4 (Quarter) decay ratio response. This roughly gives 50% overshoot.



- An alternative way to emperically find K_u and T_u is to use relay feedack control (sometimes called bang-bang control).



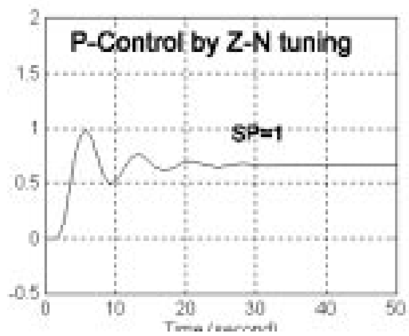
Under relay feedback, the following repsonse is obtained:

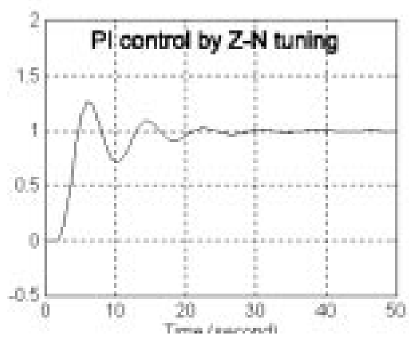


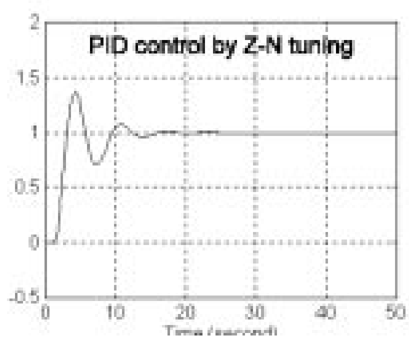
$$K_u = \frac{4(u_{max} - u_{min})}{\pi A}, \quad T_u: \quad \text{from the repsonse}$$

Typical Responses of Z-N Tuning

$$\text{Process:} \quad G(s) = \frac{1}{(s+1)^4}$$





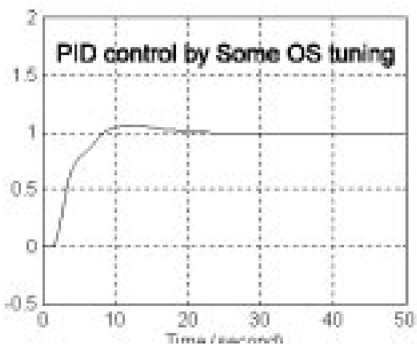


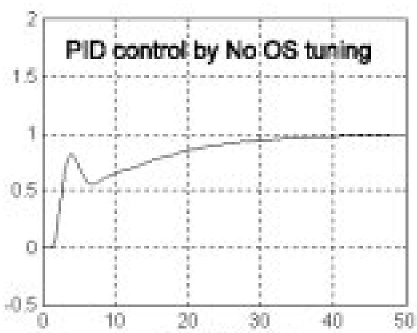
- Since 50% overshoot is considered too oscillatory in chemical process control, the following modified Ziegler-Nichols settings have been proposed for PID contollers:

Modified Ziegler-Nichols Settings

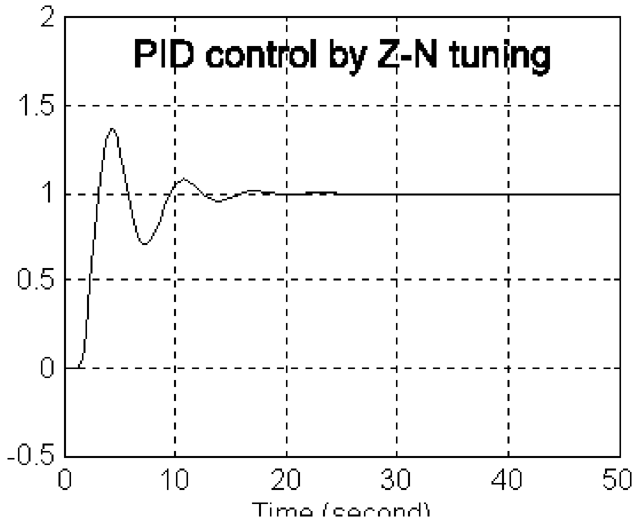
	K_c	T_I	T_D
Original(1/4 decay ratio)	$0.6K_u$	$T_u/2$	$T_u/8$
Some Overshoot	$0.33K_u$	$T_u/2$	$T_u/3$
No Overshoot	$0.2T_u$	T_u	$T_u/3$

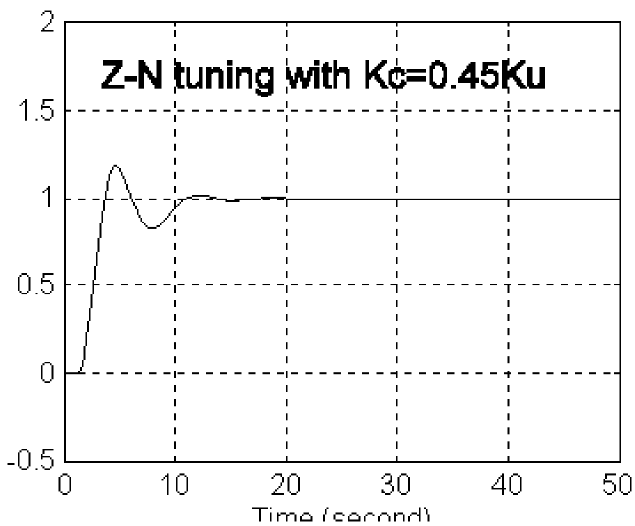
$$\text{Process:} \quad G(s) = \frac{1}{(s+1)^4}$$

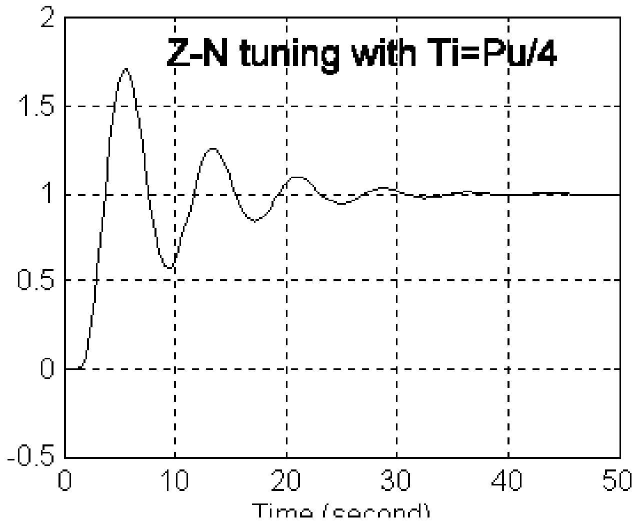


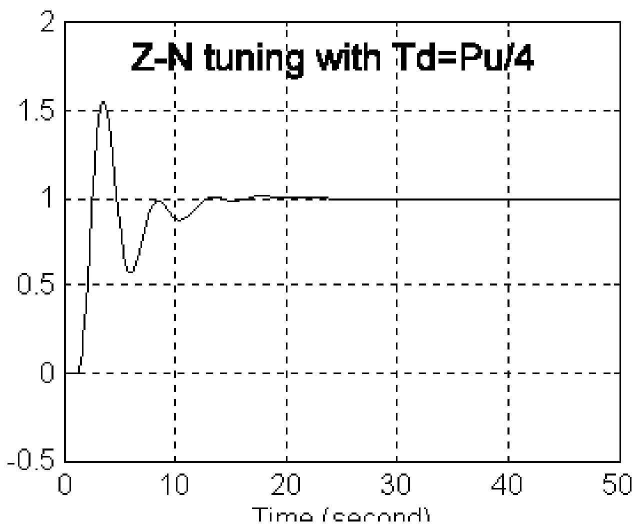


Effects of Tuning Parameters: $G(s) = 1/(s+1)^4$

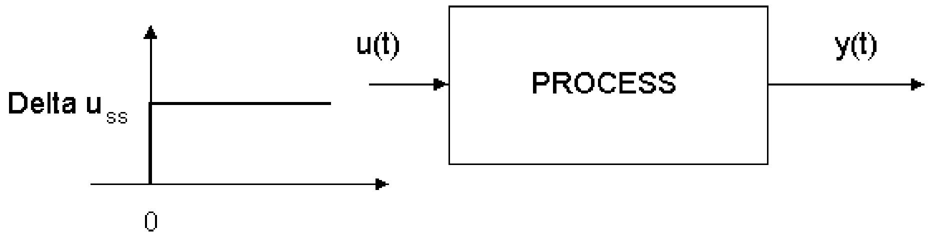




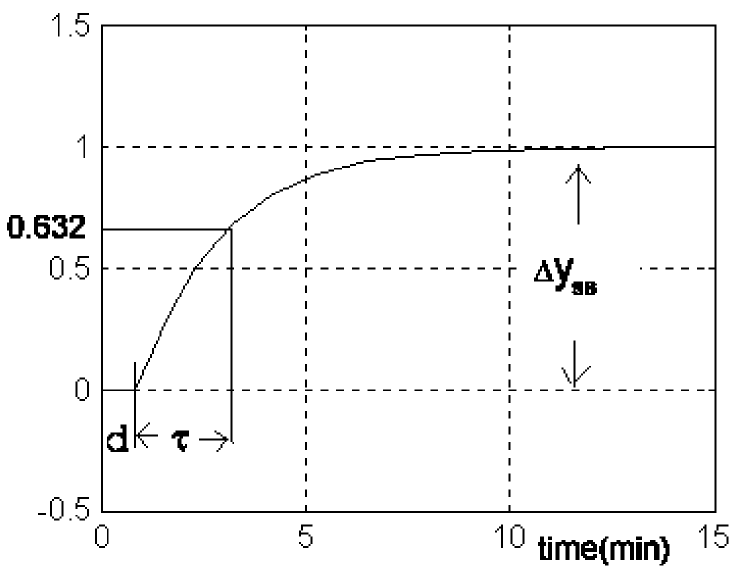

Z-N tuning with Ti=Pu/4
2
1.5
1
0.5
0
-0.5
0
10
20
30
40
50


Z-N tuning with Td=Pu/4
2
1.5
1
0.5
0
-0.5
0
10
20
30
40
50

1.5.2 REACTION-CURVE-BASED METHOD



Not all but many SISO(single-input single-output) chemical processes show step responses which can be well approximated by that of the **First-Order Plus Dead Time(FOPDT)** process.



The FOPDT process is represented by three parameters

- K_p steady state gain defined by $\Delta y_{ss}/\Delta u_{ss}$
- d dead time (min), no response during this period

- τ time constant, represents the speed of the process dynamics.

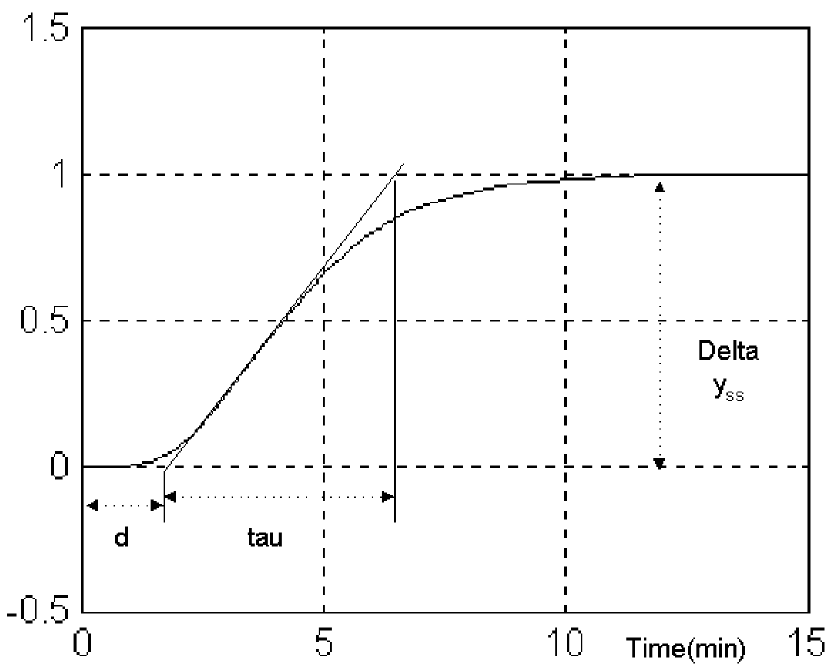
$$G(s) = \frac{K_p e^{-ds}}{\tau s + 1}$$

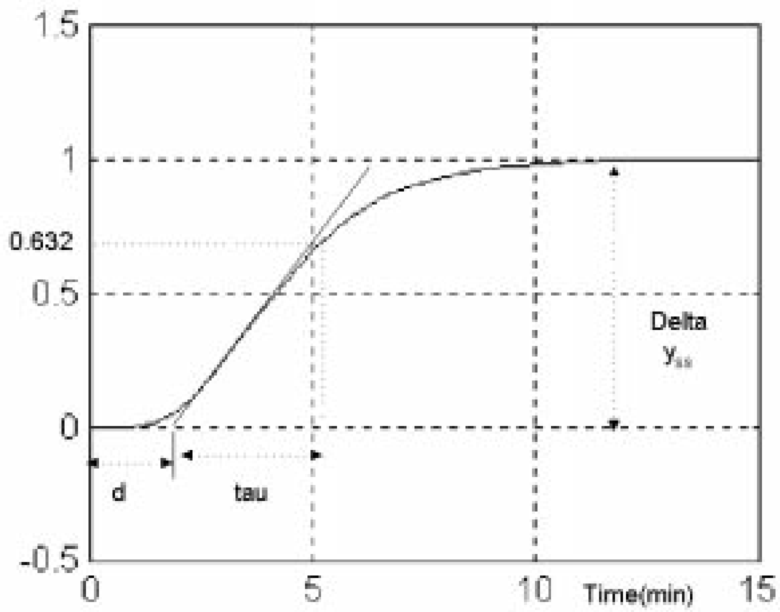
Procedure

step 1 Wait until the process is settled at the desired set point.

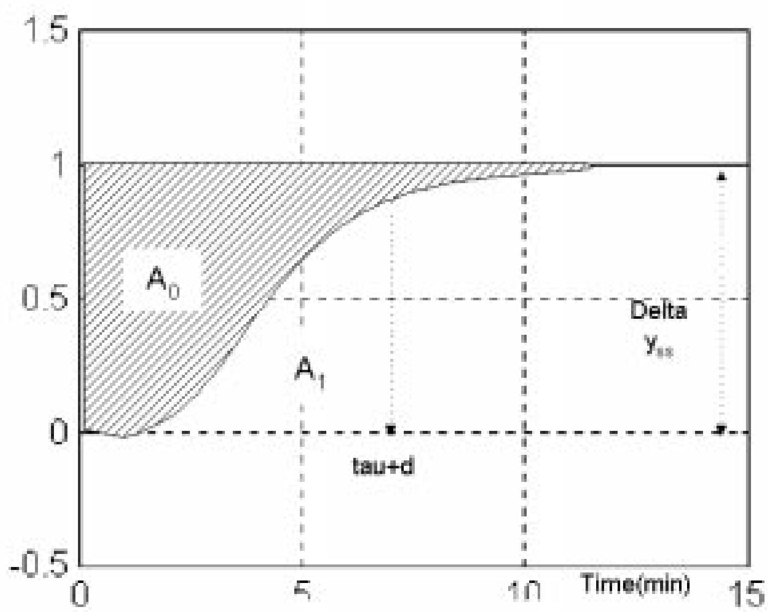
step 2 Switch the A/M toggle to the manual position and increase the CO $(u(t))$ by Δu_{ss} stepwise.

step 3 Record the output reponse and find an approximate FOPDT model using one of the following methods:





- Drawing a tangent is apt to include significant error, especially when the measurement is noisy. To avoid this trouble, the following method is recommended:



1. Obtain $K_p = \Delta y_{ss}/\Delta u_{ss}$.

2. Estimate the area A_0.

3. Let $\tau + d = A_0/K_p$ and estimate the area A_1.

4. Then $\tau = 2.782 A_1/K_p$ and $d = A_0/K_p - \tau$

step 4 Once a FOPDT model is obtained, PID setting can be done based on a tuning rule in the next subsection.

Popular tuning rules are Quater-decay ratio setting and Integral error criterion-based setting.

1.5.3 FOPDT-BASED TUNING RULES

• The following PID tuning rules are applicable for FOPDT processes with $0.1 < d/\tau < 1$.

1/4 Decay Ratio Settings

• Z-N tuning for the FOPDT model.

Controller	K_c	T_I	T_D
P	$(\tau/K_p d)$	$--$	$--$
PI	$0.9(\tau/K_p d)$	$3.33d$	$--$
PID	$1.2(\tau/K_p d)$	$2.0d$	$0.5d$