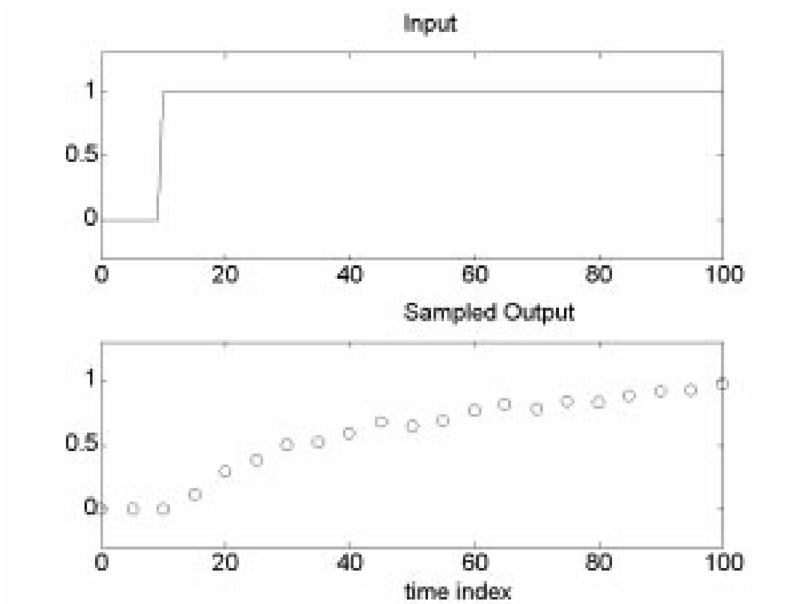
Chapter 3

SYSTEM IDENTIFICATION

The quality of model-based control absolutely relies on the quality of the model.

3.1 DYNAMIC MATRIX IDENTIFICATION

3.1.1 STEP TESTING



Procedure

1. Assume operation at steady-state with

$$\text{controlled var.(CV)}: \quad y(t) = y_0 \quad \text{for} \quad t < t_0$$
$$\text{manipulated var.(MV)}: \quad u(t) = u_0 \quad \text{for} \quad t < t_0$$

2. Make a step change in u of a specified magnitude, Δu for

$$u(t) = u_0 + \Delta u \quad \text{for} \quad t \geq t_0$$

3. Measure $y(t)$ at regular intervals:

$$y_k = y(t_0 + kh) \quad \text{for} \quad k = 1, 2, \ldots, N$$

   where
   h is the sampling interval
   Nh is approximate time required to reach steady state.

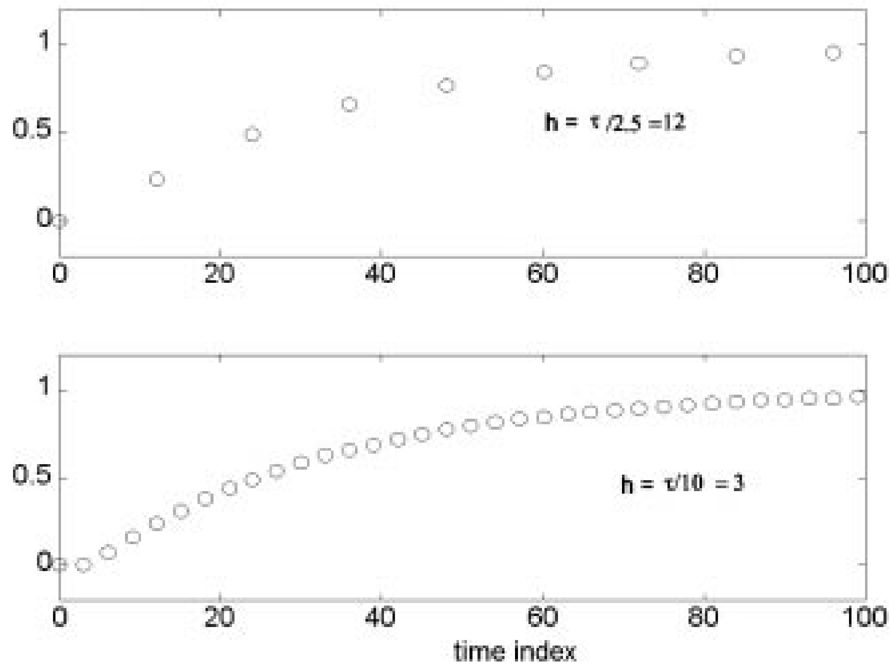
4. Calculate the *step response coefficients* from the data

$$s_k = \frac{y_k - y_0}{\Delta u} \quad \text{for} \quad k = 1, \ldots, N$$

Discussions

1. Choice of sampling period

   - For modeling, best h is one such that $N = 30 \sim 40$.
     Ex : If $g(s) = Ke^{-ds}/(\tau s + 1)$,
     then settling time $\approx 4\tau + d$
     Therefore, $h \approx \frac{4\tau+d}{N} = \frac{4\tau+d}{40} = 0.1\tau + 0.025d$



- May be adjusted depending upon control objectives.

2. Choice of step size (Δu)

- too small :

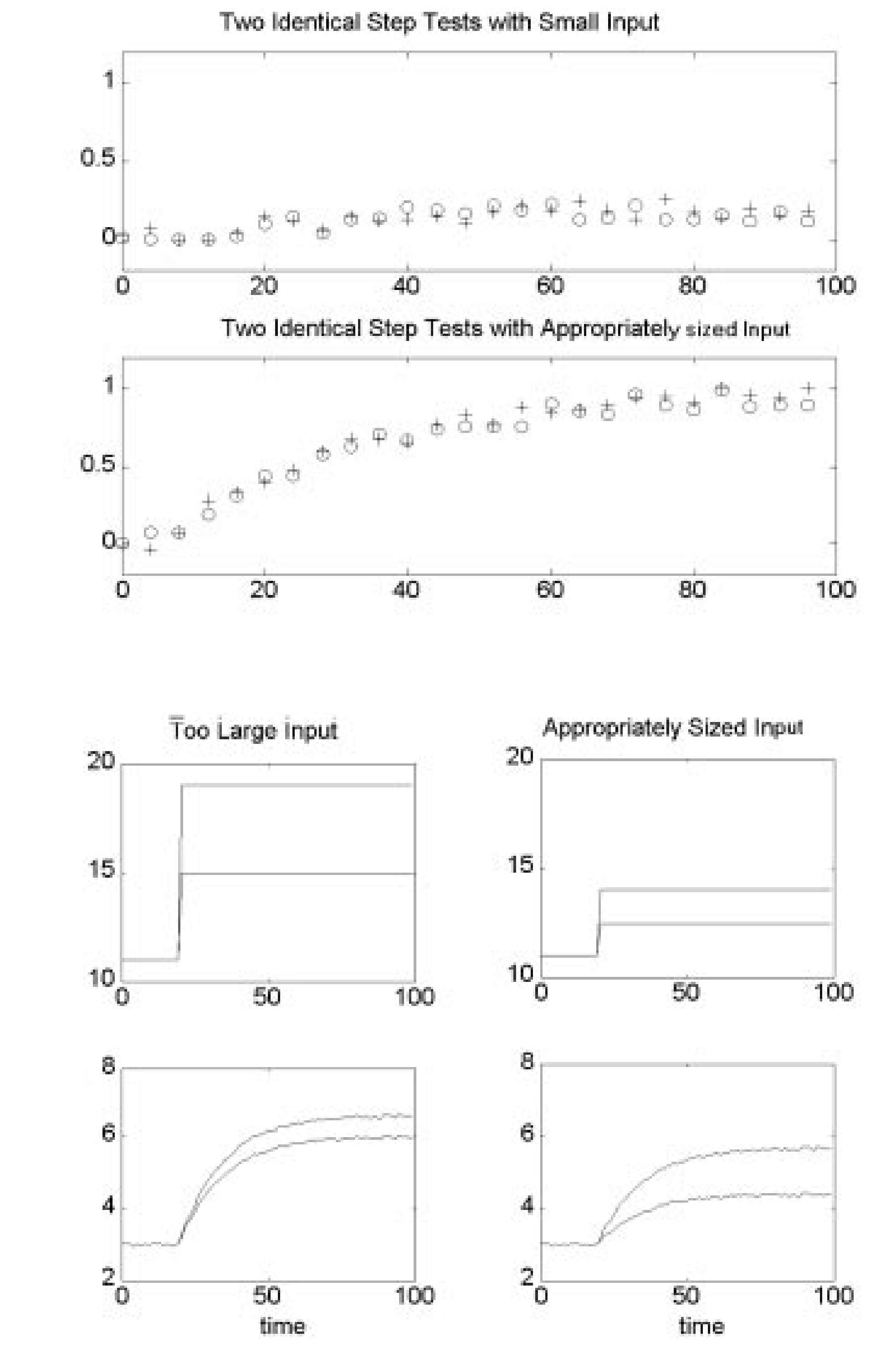
  May not produce enough output change

  Low signal to noise ratio

- too big :

  Shift the process to an undesirable condition

  Nonlinearity may be induced.



- Trial and error is needed to determine the optimum step size.

3. Choice of number of experiments

- Averaging results of multiple experiments reduces impact of disturbances on calculated s_k's
- Multiple experiments can be used to check model accuracy by cross-validation.
  Data sets for Identification $\leftrightarrow$ Data set for Validation

4. An appropriate method to detect steady state is requried.
5. While the steady state (low frequency) charateristics are accurately identified, high frequency dynamics may be inaccurately charactierized.

3.1.2 PULSE TESTING

Procedure

1. Steady operation at y_0 and u_0.
2. Send a pulse of size δu lasting for 1 sampling period.
3. Calculate pulse response coefficients

$$h_k = \frac{y_k - y_0}{\delta u} \quad \text{for} \quad k = 1, \ldots, N$$

4. Calculate the step response coefficients as a cumulative sum of h_k.

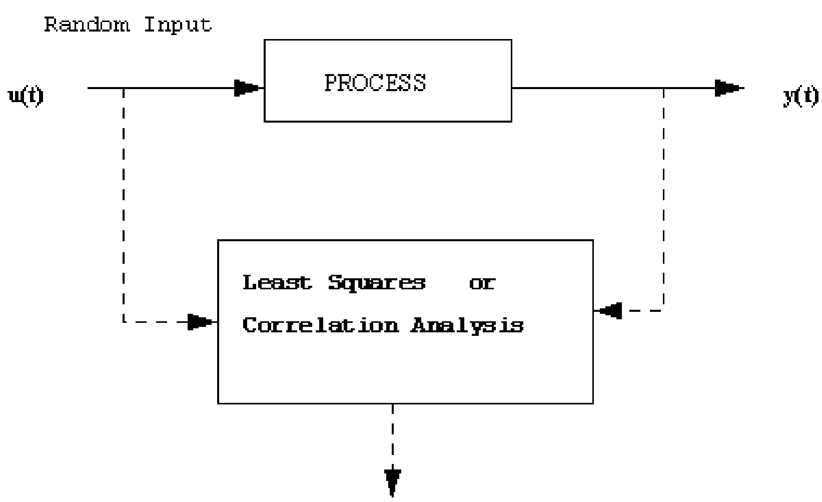
$$s_k = \sum_{i=1}^{k} h_i \quad \text{for} \quad k = 1, 2, \ldots, N$$

Discussions

1. Select h and N as for the step testing.
2. Usually need $\delta u \gg \Delta u$ for adequate S/N ratio.
3. Multiple experiments are recommended for the same reason as in the step testing.
4. An appropriate method to detect steady state is requried.
5. Theoretically, pulse is a perfect (unbiased) excitation for linear systems.

3.1.3 RANDOM INPUT TESTING

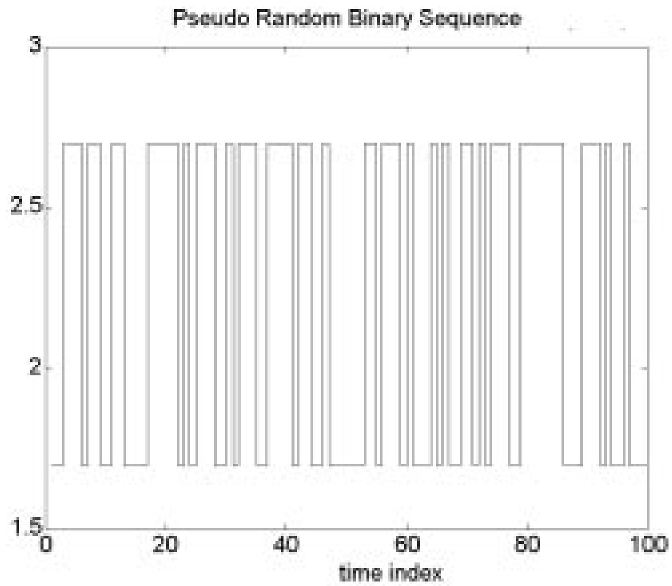
Concept



$\{h_k\}$ or $\{A, B, C\}$ or $G(s)$

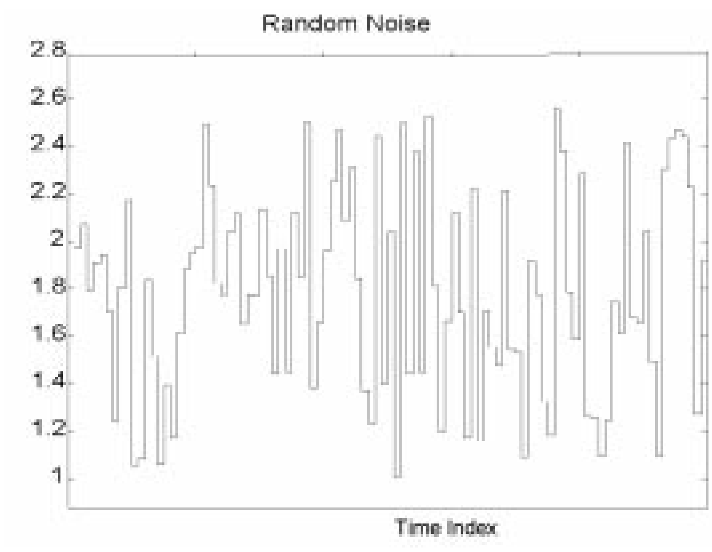
Type of Inputs

1. Pseudo-Random Binary Signal(PRBS)



In MATLAB, ≫ `u=u0+del*2*sign(rand(100,1))-0.5;`
or ≫ `u=mlbs(12);`

2. Random Noise



In MATLAB, ≫ `u=u0+del*2*(rand(100,1)-0.5);`

Data Analysis - Least Squares Fit

Given $\{u_1, u_2, \ldots, u_M\}$ and $\{y_1, y_2, \ldots, y_M\}$, determine the best fit FIR(finite impulse response) model $\{h_1, h_2, \ldots, h_N\}$.

Consider

$$y_k = h_1 u_{k-1} + h_2 u_{k-2} + \ldots + h_N u_{k-N} + d_k$$

Assume the effects of initial condition are negligible.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} u_0 & u_{-1} & \ldots & u_{1-N} \\ u_1 & u_0 & \ldots & u_{2-N} \\ \vdots & \vdots & \vdots & \vdots \\ u_{M-1} & u_{M-2} & \ldots & u_{M-N} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_M \end{bmatrix}$$

$$\mathbf{y} = \mathbf{U}\mathbf{h} + \mathbf{d}$$

The least squares solution which minimizes

$$(\mathbf{y} - \mathbf{U}\mathbf{h})^T(\mathbf{y} - \mathbf{U}\mathbf{h}) = \sum_{i=1}^{M} \left(y_i - \sum_{j=1}^{N} h_j u_{i-j} \right)^2$$

is

$$\hat{\mathbf{h}} = \left(\mathbf{U}^T\mathbf{U}\right)^{-1} \mathbf{U}^T\mathbf{y}$$

In MATLAB, $\gg$ `hhat=y\U;`

Discussions

1. Random input testing, if appropriately designed, gives better models than the step or pulse testing does since it can equally excite low to high frequency dynamics of the process.
2. If $\mathbf{U}^T\mathbf{U}$ is singular, the inverse doesn't exist and identification fails. $\rightarrow$ *persistent excitation* condition.

3. When the number of coefficients is large, $\mathbf{U}^T\mathbf{U}$ can be easily singular (or nearly singular). To avoid the numerical, a regularization term is addded the the cost function. → *ridge regression*

$$\min_{\mathbf{h}} \left[(\mathbf{y} - \mathbf{U}\mathbf{h})^T (\mathbf{y} - \mathbf{U}\mathbf{h}) + \alpha \mathbf{h}^T \mathbf{h} \right]$$

$$\rightarrow \hat{\mathbf{h}} = \left(\mathbf{U}^T \mathbf{U} + \alpha \mathbf{I} \right)^{-1} \mathbf{U}^T \mathbf{y}$$

4. *Unbiasedness*: If $d(\cdot)$ and/or $u(\cdot)$ is zero-mean and $u(i)$ is uncorrelated with $d(j)$ for all (i, j) pairs (these conditions are easily satisfied.), the estimate is unbiased.

$$\hat{\mathbf{h}} = \left(\mathbf{U}^T \mathbf{U} \right)^{-1} \mathbf{U}^T \left(\mathbf{U}\mathbf{h} + \mathbf{d} \right) = \mathbf{h} + \left(\mathbf{U}^T \mathbf{U} \right)^{-1} \mathbf{U}^T \mathbf{d}$$

   Since

$$E\{ \left(\mathbf{U}^T \mathbf{U} \right)^{-1} \mathbf{U}^T \mathbf{d} \} = 0$$

   we have

$$E\{\hat{\mathbf{h}}\} = \mathbf{h}$$

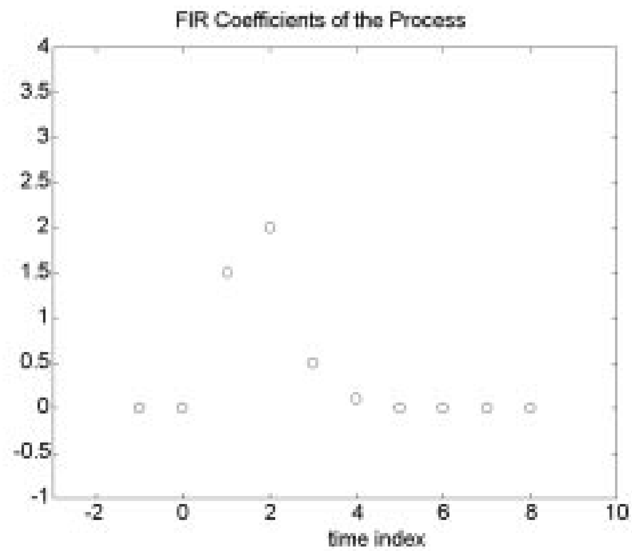
5. *Consistency*: In addition to the unbiasedness,

$$\hat{\mathbf{h}} \rightarrow \mathbf{h} \text{ (or, equivalently } E\left\{ (\hat{\mathbf{h}} - \mathbf{h})(\hat{\mathbf{h}} - \mathbf{h})^T \right\} \rightarrow 0) \text{ as } M \rightarrow \infty.$$

6. Extension to MIMO identification is straightforward.
   The above properties are inherited to the MIMO case, too.

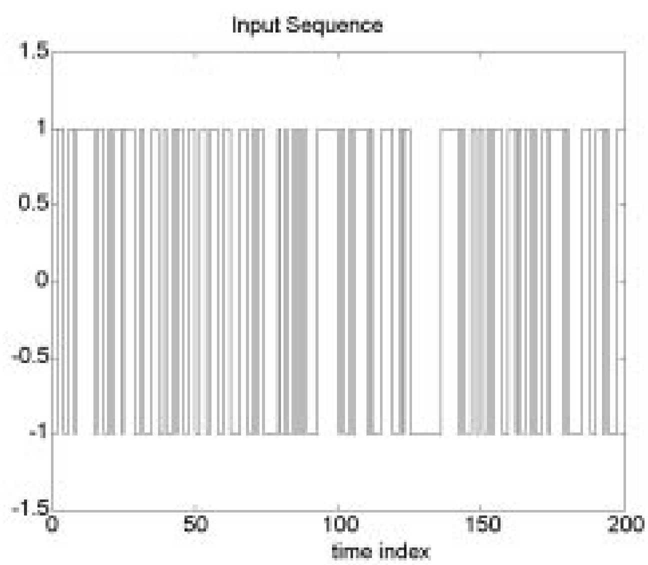
Example

- Process : $h = [h_1,\ h_2,\ h_3,\ h_4,\ h_5,\cdots] = [1.5\ 2.0\ 5.5\ 0.1\ 0\ \ \cdots]$

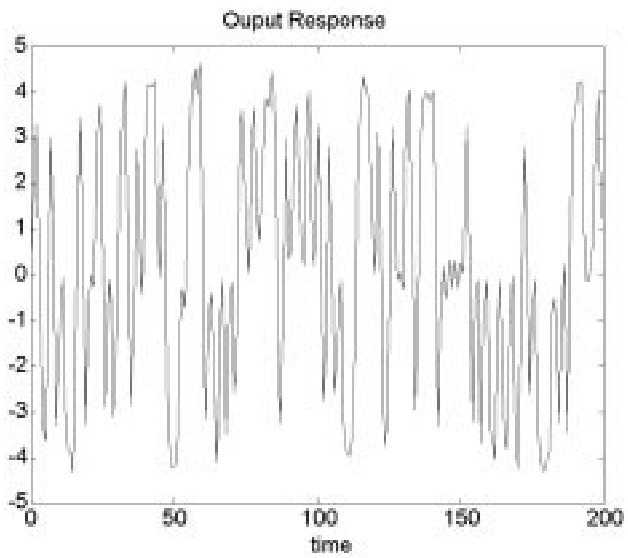


- Input : PRBS with

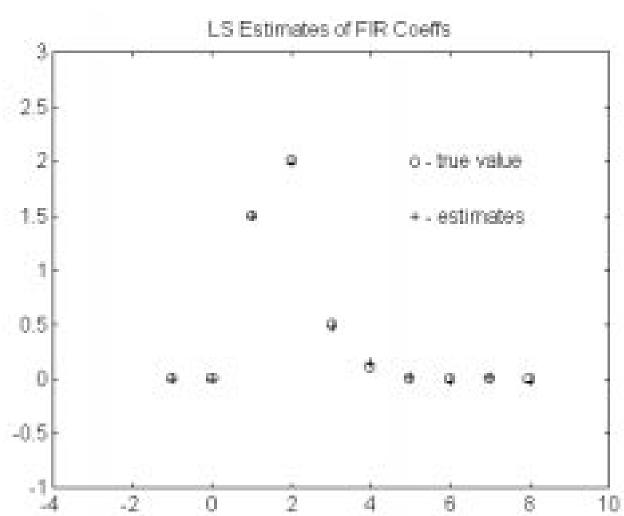
$$N = 200 \quad \bar{u} = 0$$



• The resulting output response corrupt with measurement noise with $\sigma_n^2 = 0.25^2$ is



• Estimates of $\{h_j\}$ appear as



3.1.4 DATA PRETREATMENT

The data need to be processed before they are used in identification.

(a) **Spike/Outlier Removal**

- Check plots of data and remove *obvious* outliers (*e.g.*, that are impossible with respect to surrounding data points). Fill in by interpolation.
- After modeling, plot of actual *vs* predicted output (using measured input and modeling equations) may suggest additional outliers. Remove and redo modelling, if necessary.
- But don't remove data unless there is a clear justification.

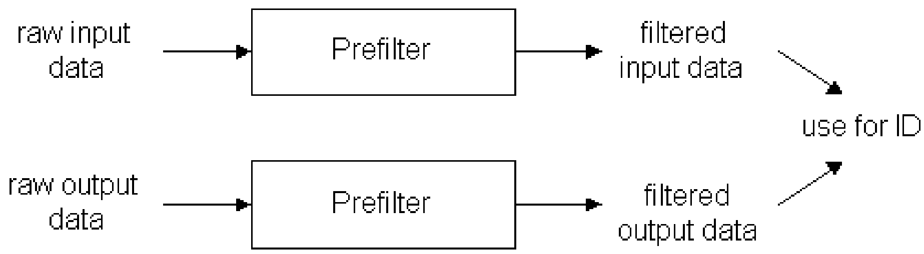
(b) **Bias Removal and Normalization**

- The input/output data are biaesd by the nonzero steady state and also by disturbance effects. To remove the bias, difference is taken for the input/output data. Then the differenced data is conditioned by scaling before using in identification.

$$\left.\begin{array}{rcl} y(k) & = & (y^{proc}(k) - y^{proc}(k-1))/c_y \\ u(k) & = & (u^{proc}(k) - u^{proc}(k-1))/c_u \end{array}\right\} \rightarrow \text{identification}$$

(c) **Prefiltering**

- If the data contain too much frequency components over an undesired range and/or if we want to obtain a model that fits well the data over a certain frequency range, data prefiltering (via digital filters) can be done.



The two filters should be same.