3.2 BASIC CONCEPTS OF IDENTIFICATION



- u(k) is processed to y(k) by the *process*, *i.e.*, y(k) contains the process information. By treating $\{y_k\}$ together with $\{u(k)\}$, we can extract the process characteristics.
- A multivariable process has directional as well as frequency-dependent characteristics.



Frequency-dependent (Filtering) Characteristic of a Process



Directional Charateristics

The directional gain changes with frequencies.

- To extract all the information, the input should be properly designed to excite all these charateristics uniformly.
- Furthermore, the process variables (inputs as well as outputs) are subject to various random disturbances. To remove the disturbance effects, some kind of averaging is needed. For perfect averaging, infinite number of data should be collected. The excitation input, however, has limited magnitude and duration.
- In reality, it is neither necessary nor feasible to identify all the facets of a process. Depending on the purpose of the model, some characteristics should be accurately identified while the others are not.
- To find an appropriate model to given purposes, the following three elements need to be judiciously selected and/or designed.
 - Model description
 - Experimental condition

- Identification method
- **Ex.** Accurate fit of the step response does not necessarily imply a good model for control.



- (a) open-loop step response
- (b) closed-loop step response



- (a) open-loop step response
- (b) closed-loop step response

3.3 MODEL DESCRIPTION

3.3.1 NONPARAMETRIC MODEL

- Models that are not described by a finite number of parameters.
 - Pulse response model : $\{h_0, h_1, h_2, \cdots\}$
 - Step response model : $\{s_0, s_1, s_2, \cdots\}$
 - Frequency response model : $G(j\omega)$
- Pulse or step response models can be directly identified from pulse or step test.
- The pulse and step tests are very simple to conduct. However, the step test too much emphasizes low-frequency excitation while the pulse test gives too-widely-spread excitation over the whole frequency ranges, hence may not provide an appropriate model adequate to control purpose.
- In general, a parametric model is identified first and then converted to a nonparametric model.

3.3.2 PARAMETRIC METHOD

ARMAX Model

$$y(k) = a_1 y(k-1) + \dots + a_{n_a} y(k-n_a) + b_1 u(k-1) + \dots + b_{n_b} u(k-n_b) + n(k) + c_1 n(k-1) + \dots + c_{n_c} n(k-n_c)$$

or

$$\underbrace{A(q^{-1})y(k)}_{AR} = \underbrace{B(q^{-1})u(k)}_{X} + \underbrace{C(q^{-1})n(k)}_{MA}$$

where $\{n(k)\}$ is a zero-mean i.i.d. (independent and identically distributed) sequence or, equivalently a white noise sequence.

(C(q⁻¹)/A(q⁻¹))n(k) represents the disturbance model. Depending on the nature of the disturbance, a simpler form can be used.
When the disturbance mostly occurs at the input in the form of white noise,



 $A(q^{-1})y(k) = B(q^{-1})u(k) + n(k)$

might be enough.

When the disturbance mostly occurs at the output in the form of white noise,



$$y(k) = \frac{B(q^{-1})}{A(q^{-1})}u(k) + n(k) \quad \to \quad C(q^{-1}) = A(q^{-1})$$

the above output error(OE) model can be an appropriate description.

- The ARMAX model is a kind of general model that can fit most linear discrete-time dynamic systems.
- Many of the well-established classical identification methods such as least squares, extended least squares, generlaized least squares, and output error methods are based on a special case of the ARMAX model or its variants.
- Orders of each term (n_a, n_b, n_c) should be selected by the user. When these values are chosen less than required, parameter estimates may be biased.

When too large values are chosen, more data points are needed for accurate parameter estimation. \rightarrow Not Parsimonious

• In MIMO identification, more paramters than are necessary should be determined since no MIMO canonical ARMAX model with minimum number of paramters has not been known yet.

A generally employed approach for MIMO system identification is to conduct indendent MISO(Multi-Input Single-Output) identification for each output and combine the results.