Part III

BACKGROUND FOR ADVANCED ISSUES

Contents

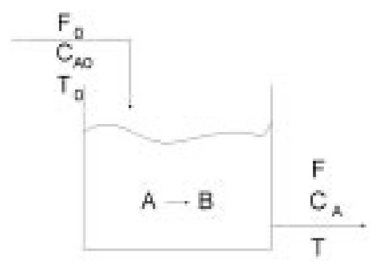
Chapter 1

BASICS OF LINEAR ALGEBRA

1.1 VECTORS

Definition of Vector

Consider a CSTR where a simple exothermic reaction occurs:



A neat way to represent process variables, F, C_A, T, is to stack them in a column.

$$\begin{bmatrix} F \\ C_A \\ T \end{bmatrix}$$

Definition of Vector (Continued)

In general, n tuples of numbers stacked in a column is called vector.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Transpose of a Vector x:

$$x^T = [x_1 \; x_2 \; \cdots \; x_n]$$

Basic Operations of Vectors

a: a scalar, x, y: vectors

Addition:

$$x + y = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{bmatrix}$$

Scalar Multiplication:

$$ax = a \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} ax_1 \\ ax_2 \\ \vdots \\ ax_n \end{bmatrix}$$

Vector Norms

Norm is the measure of vector size.

p norms:

$$\|x\|_p = (|x_1|^p + \cdots + |x_n|^p)^{\frac{1}{p}} \qquad 1 \le p < \infty$$

$$\|x\|_\infty = \max_i |x_i|$$

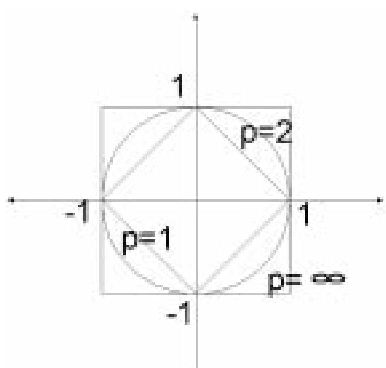
Example:

$$\|x\|_1 = |x_1| + \cdots + |x_n|$$

$$\|x\|_2 = \sqrt{|x_1|^2 + \cdots + |x_n|^2}$$

$$\|x\|_\infty = \max\{|x_1|, \cdots, |x_n|\}$$

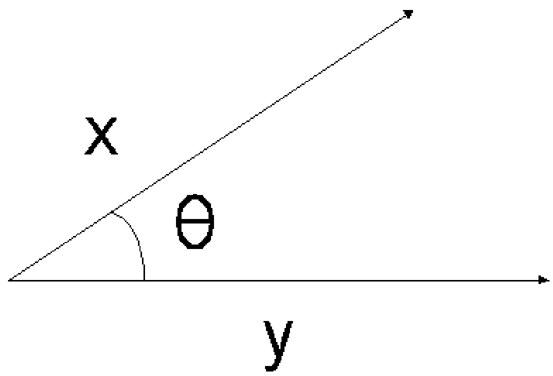
$\|x\|_2$ coincides with the length in Euclidean sense and, thus, is called Eclidean norm. Throughout the lecture, $\|\cdot\|$ denotes $\|\cdot\|_2$.



Unit Ball $\{x : |x|_p \le 1\}$

Inner Product

Inner Product:



$$x \cdot y = x^T y = \|x\| \|y\| \cos\theta$$

$$\Downarrow$$

$$x \cdot y \begin{cases} > 0 & \text{if } \theta \text{ is acute} \\ = 0 & \text{if } \theta \text{ is right} \\ < 0 & \text{if } \theta \text{ is obtuse} \end{cases}$$

Note that two vectors x, y are orthogonal if $x^T y = 0$

Linear Independence and Basis

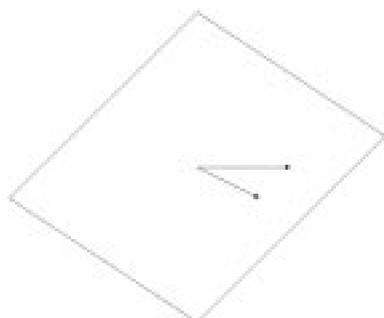
$a_1, \cdots, a_m$: scalars, $x_1, \cdots, x_m$: vectors

Linear Combination:

$$a_1x_1 + a_2x_2 + \cdots + a_mx_m$$

Span: Span of $x_1, \cdots, x_m$ is the set of all linear combination of them, which is a plane in $\mathbf{R}^n$.

$$span\{x_1, x_2, \cdots, x_m\} = \{x = a_1x_1 + a_2x_2 + \cdots + a_mx_m\}$$



Linear Independence: $\{x_1, \cdots, x_m\}$ is called linearly independent if no one of them is in the span of others.

Basis of a Space (S): A set of linearly independent vectors $\{x_1, x_2, \cdots, x_m\}$ such that $S = span\{x_1, x_2, \cdots, x_m\}$