부록 A: 위험의 통계학적 기초개념

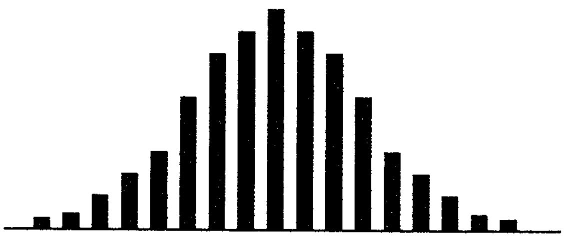
❑ 평균(mean) 또는 기대값(expected value)

- 중심위치를 나타내는 척도

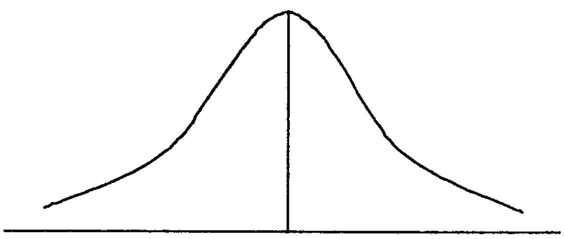
$$E(X) = \sum_{i=1}^{n} x_i f(x_i) \qquad (\textit{discrete distribution})$$

$$\int_{-\infty}^{\infty} x f(x) dx \qquad (\textit{continuos distribution})$$

Discrete Distribution



Continuous Distribution



❑ 분산(variance)과 표준편차(standard deviation)

- 분산 : 중심위치로부터 떨어진 정도를 나타내는 척도
- 표준편차 : 확률변수와 같은 단위에서의 산포에 대한 척도

$$\mathrm{VAR}(x) = E[(x-\mu)^2]$$

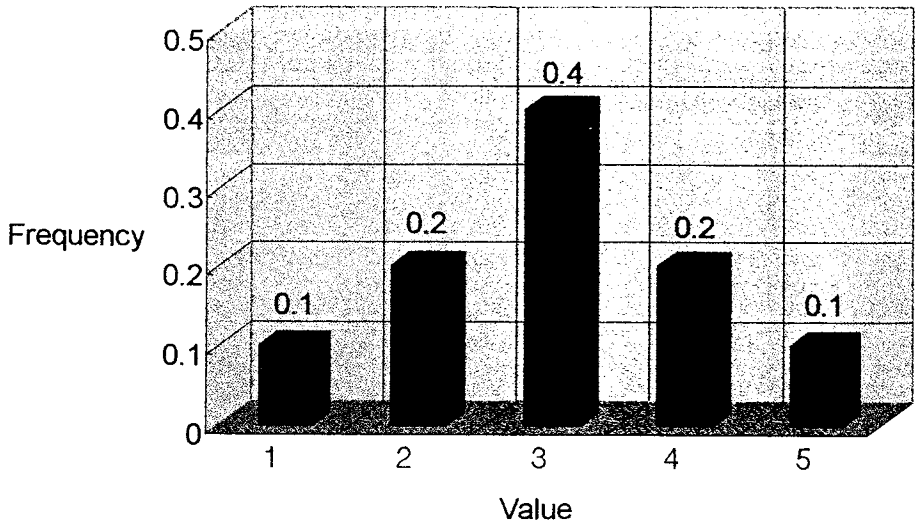
$$= \sum_{i=1}^{n} (x_i - \mu)^2 f(x_i) \qquad (\textit{discrete distribution})$$

$$\int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx \qquad (\textit{continuos distribution})$$

$$\sigma(x) = \sqrt{VAR(x)} = \sqrt{E[(x-\mu)^2]}$$

μ : mean, expected value

❑ Sample Case



– 평균

X(값)	1	2	3	4	5	합계
$f(x)$ (빈도)	0.1	0.2	0.4	0.2	0.1	1
$xf(x)$	0.1	0.4	1.2	0.8	0.5	3

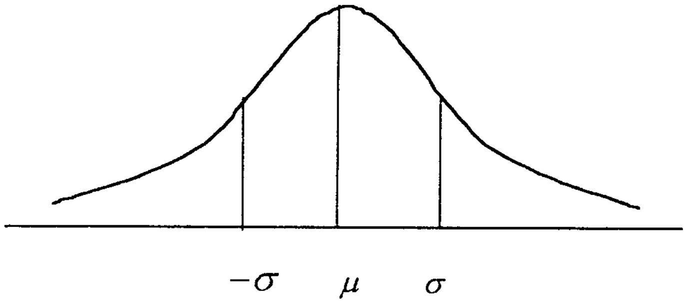
$$E(X) = \sum_{i=1}^{n} x_i f(x_i) = 3$$

– 표준편차

$x-\mu$	-2	-1	0	1	2	합계
$(x-\mu)^2$	4	1	0	1	4	10
$f(x)$	0.1	0.2	0.4	0.2	0.1	1
$(x-\mu)^2 f(x)$	0.4	0.2	0	0.2	0.4	1.2

$$[\sigma(x)]^2 = \sum (x-\mu)^2 f(x) = 1.2 \qquad \sigma(x) = \sqrt{1.2} = 1.1$$

❑ normal distribution



$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

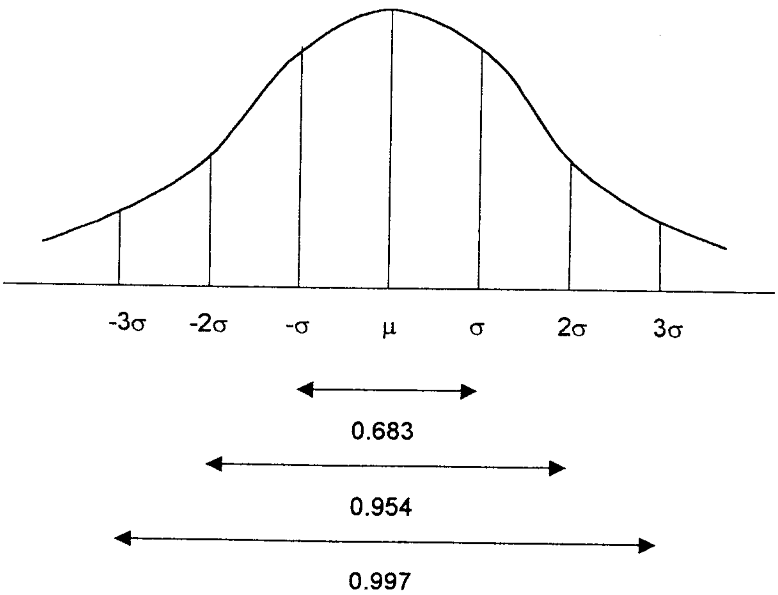
❑ 평균이 μ이고 분산이 σ^2인 Normal Distribution

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

$$P\{\mu - \sigma < X < \mu + \sigma\} = 0.683$$

$$P\{\mu - 2\sigma < X < \mu + 2\sigma\} = 0.954$$

$$P\{\mu - 3\sigma < X < \mu + 3\sigma\} = 0.997$$



부록 B : 3 차 적률(3 order moment)과 왜도(skewness)

❑ 3 차 적률

$$m_3 = \sum_{i=1}^{n} (x_i - \mu)^3 f_i$$

❑ 왜도(skewness)

– 평균값을 지나는 수직선에 대한 비대칭성의 정도를 나타냄

$$\int_{-\infty}^{\infty} (x-\mu)^3 f(x) dx$$

$$\alpha = \frac{m_3}{\sigma_3} = \frac{1}{\sigma_3} E[(x-\mu)^3]$$

$\alpha < 0$: 우측 기울어짐 (좌측으로 꼬리가 길다)

$\alpha = 0$: 좌우대칭

$\alpha > 0$: 좌측으로 기울어짐 (우측으로 꼬리가 길다)