비선형 모델기반 반복학습제어기의 개발 및 응용

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Development and Application of the Nonlinear Model-based Iterative Learning Controller

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Abstract

In this study, the extended Kalman filter (EKF) based nonlinear model predictive control is reformulated into the structure of iterative learning algorithm. The design of control algorithm is presented, and its validity and effectiveness are proven by performing simulation studies for the quality control of copolymer product in a semibatch methyl methacrylate (MMA)/ methyl acrylate (MA) copolymerization reactor.

Theory

1. *Model*: The nonlinear model is expressed by the following nonlinear differential equation:

$$\dot{x} = f(x_k, u_k) \quad \text{and} \quad y = g(x_k) \tag{1}$$

where x, u and y represent the state vector, the manipulated input vector and the measured output vector, respectively. Subscript k denotes the batch index.

For digital controller design, one can assume that u remains constant between two sampling instants. By defining x_k^t as the state vector at time t of the k^{th} batch, a discrete version of the model, Eq. (1), may be expressed as follows:

$$x_k^t = F_{t_s}\left(x_k^{t-1}, u_k^{t-1}\right) \quad \text{and} \quad \hat{y}_k^t = g(x_k^t) + v_k \tag{2}$$

where F_{ts} represents the terminal state vector obtained by integrating the ordinary differential equation, Eq. (1), for one sample interval (t_s) with the initial condition of x_k^{t-1} and the constant input of $u = u_k^{t-1}$. It is assumed that the measurements of y_k^t are corrupted by measurement noise v_k ,which is the white noise with covariance of R^v.

2. *State Estimation*: The state of x_k^t is estimated from the extended Kalman filter (EKF). Since the procedure for the implementation of EKF is a straightforward extension of the optimal linear filter, the detailed procedure will not be presented here. For details, one may refer to Lee and Ricker (1994).

3. *One step ahead prediction*: Here, we express the prediction of controlled outputs as a function of the current (estimated) state and the future input sequence. To avoid any confusion in notation, x_k^t will be used in place of the state estimation $x_k^{t|t}$ in the sequel. At time t of the k^{th} batch, one step ahead prediction is calculated by the model

$$x_k^{t+1} = F_{t_s}(x_k^t, u_k^t) \tag{3}$$

In order to recast Eq. (3) into the form of iterative learning controller, it is approximated by linearizing $F_{t_s}(x_k^t, u_k^t)$ at the state and input of the previous batch, i.e., at x_{k-1}^t and u_{k-1}^t, with the assumption that these values are not much different from the current ones, i.e., x_k^t and u_k^t. This feature makes our algorithm different from that of Lee and Ricker (1994) in which linearization is performed at the state and input of the previous time step, i.e., x_k^{t-1} and u_k^{t-1}. Hence, we obtain

$$\Delta x_k^{t+1} \cong F_{t_s}(x_{k-1}^t, u_{k-1}^t) + A_{k-1}^t(x_k^t - x_{k-1}^t) + B_{k-1}^t(u_k^t - u_{k-1}^t) \tag{4}$$

where $\Delta x_k^{t+1} = x_k^{t+1} - x_{k-1}^{t+1} = x_k^{t+1} - F_{t_s}(x_{k-1}^t, u_{k-1}^t)$

$$A_{k-1}^t = \exp\left(\tilde{A}_{k-1} t_s\right), \quad \tilde{A}_{k-1} = \left.\frac{\partial f}{\partial x}\right|_{x=x_{k-1}^t, u=u_{k-1}^t} \quad \text{and} \quad B_{k-1}^t = \int_0^{t_s} \exp\left(\tilde{A}_{k-1}\tau\right) d\tau \cdot \tilde{B}_{k-1}, \quad \tilde{B}_{k-1} = \left.\frac{\partial f}{\partial u}\right|_{x=x_{k-1}^t, u=u_{k-1}^t}$$

4. *p step ahead prediction*: By repeating the preceding steps, one can obtain the p step ahead prediction of the state Δx_k^{t+p} and all the predicted terms can be rearranged in the form of matrix as follows:

$$\Delta x_k^{t+p|t} = \begin{bmatrix} \Delta x_k^{t+1} \\ \Delta x_k^{t+2} \\ \vdots \\ \Delta x_k^{t+p} \end{bmatrix} = \begin{bmatrix} A_{k-1}^{t} \\ A_{k-1}^{t+1} A_{k-1}^{t} \\ \vdots \\ \prod_{i=0}^{p-1} A_{k-1}^{t+i} \end{bmatrix} \Delta x_k^t + \begin{bmatrix} B_{k-1}^{t} & 0 & 0 & 0 \\ A_{k-1}^{t+1} B_{k-1}^{t} & B_{k-1}^{t+1} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=1}^{p-1} A_{k-1}^{t+i} B_{k-1}^{t} & \prod_{i=2}^{p-1} A_{k-1}^{t+i} B_{k-1}^{t+1} & & B_{k-1}^{t+p-1} \end{bmatrix} \begin{bmatrix} \Delta u_k^{t} \\ \Delta u_k^{t+1} \\ \vdots \\ \Delta u_k^{t+p-1} \end{bmatrix} \tag{5}$$

5. *Controlled output prediction*: To derive the controlled output prediction, the controlled output function $\left(y^c\right)_k^{t+1} = h\left(x_k^{t+1}\right)$ is linearized with respect to x_{k-1}^{t+1} to obtain

$$\left(y^c\right)_k^{t+1} = h\left(x_k^{t+1}\right) \cong h\left(x_{k-1}^{t+1}\right) + H_{k-1}^{t+1}\left(x_k^{t+1} - x_{k-1}^{t+1}\right) \quad \text{where} \quad H_{k-1}^{t+1} = \left.\frac{\partial h}{\partial x}\right|_{x=x_{k-1}^{t+1}} \tag{6}$$

Continuing on with the same method for the controlled output function, one can derive the controlled output prediction equation as

$$\left(y^c\right)_k^{t+p} \cong h\left(x_{k-1}^{t+p}\right) + H_{k-1}^{t+p} \Delta x_k^{t+p} \tag{7}$$

The combination of Eq. (5) with Eq. (7) provides the prediction equation as follows:

$$Y_k^{t+p|t} = H_{k-1}^{t+p|t} + \tilde{H}_{k-1}^{t+p|t} \tilde{A}_{k-1}^{t+p|t} \Delta x_k^t + \tilde{H}_{k-1}^{t+p|t} \tilde{B}_{k-1}^{t+p|t} \Delta U_k^{t+p|t} \tag{8}$$

where

$$H_{k-1}^{t+p|t} \triangleq \begin{bmatrix} h\left(x_{k-1}^{t+1}\right) \\ h\left(x_{k-1}^{t+2}\right) \\ \vdots \\ h\left(x_{k-1}^{t+p}\right) \end{bmatrix}, \quad \tilde{H}_{k-1}^{t+p|t} \triangleq \begin{bmatrix} H_{k-1}^{t+1} & 0 & 0 & 0 \\ 0 & H_{k-1}^{t+2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & H_{k-1}^{t+p} \end{bmatrix}, \quad \Delta U_k^{t+p|t} = \begin{bmatrix} \Delta u_k^{t} \\ \Delta u_k^{t+1} \\ \vdots \\ \Delta u_k^{t+p-1} \end{bmatrix}$$

$$\tilde{A}_{k-1}^{t+p|t} \triangleq \begin{bmatrix} A_{k-1}^{t} \\ A_{k-1}^{t+1} A_{k-1}^{t} \\ \vdots \\ \prod_{i=0}^{p-1} A_{k-1}^{t+i} \end{bmatrix}, \quad \tilde{B}_{k-1}^{t+p|t} \triangleq \begin{bmatrix} B_{k-1}^{t} & 0 & 0 & 0 \\ A_{k-1}^{t+1} B_{k-1}^{t} & B_{k-1}^{t+1} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=1}^{p-1} A_{k-1}^{t+i} B_{k-1}^{t} & \prod_{i=2}^{p-1} A_{k-1}^{t+i} B_{k-1}^{t+1} & & B_{k-1}^{t+p-1} \end{bmatrix}$$

6. *Constraints*: In most of the chemical processes, constraints are imposed on both the input and output due to the limit of the device performance or the environmental reason. Here, we consider the input magnitude constraint and the input change constraints with respect to both batch and time indices. Controlled output constraint is not considered in this study because a severe disturbance may cause the output to violate the constraint and result in an infeasible solution of the optimization problem.

(1) Input magnitude: Manipulation devices such as control valve have their upper and lower bounds in the performance and these bounds are expressed in the form of linear inequalities; i.e.,

$$U_{\min} \le U_k^{t+p|t} = \begin{bmatrix} u_k^t & u_k^{t+1} & \cdots & u_k^{t+p-1} \end{bmatrix}^T \le U_{\max} \tag{9}$$

As the input values in the $(k\text{-}1)^{\text{th}}$ batch have been stored and are available in the k^{th} batch, the constraint on the input magnitude is easily rearranged as follows:

$$U_{\min} - U_{k-1}^{t+p|t} \le \Delta U_k^{t+p|t} \le U_{\max} - U_{k-1}^{t+p|t} \tag{10}$$

(2) Input change in terms of the batch index: To prevent the trajectory of control input from being changed abruptly, the constraint on the input change with respect to the batch index is considered.

$$\Delta U_{\min} \le \Delta U_k^{t+p|t} \le \Delta U_{\max} \tag{11}$$

(3) Input change in terms of the time index: Let us define the increment of the input with respect to the time index as $\delta u_k^t = u_k^t - u_k^{t-1}$. To recast the constraints $\delta u_{\min} \le \delta u_k^{t+l} \le \delta u_{\max}$, $l = 0, \cdots, p\text{-}1$, into the ones in terms of Δu_k^t, the following algebraic manipulation procedure is used:

$$\delta u_k^{t+l} = u_k^{t+l} - u_k^{t+l-1} = u_k^{t+l} - u_{k-1}^{t+l} - u_k^{t+l-1} + u_{k-1}^{t+l-1} + u_{k-1}^{t+l} - u_{k-1}^{t+l-1} = \Delta u_k^{t+l} - \Delta u_k^{t+l-1} + \delta u_{k-1}^{t+l} \tag{12}$$

As Δu_k^{t-1} is available at time *t*, we obtain the following matrix inequality:

$$\delta U_{\min} - \begin{bmatrix} \delta u_{k-1}^{t} \\ \delta u_{k-1}^{t+1} \\ \vdots \\ \delta u_{k-1}^{t+p-1} \end{bmatrix} + \begin{bmatrix} \Delta u_k^{t-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \le \begin{bmatrix} I & 0 & 0 & \\ -I & I & 0 & \\ 0 & -I & I & \\ 0 & 0 & \ddots & \ddots \end{bmatrix} \Delta U_k^{t+p|t} \le \delta U_{\max} - \begin{bmatrix} \delta u_{k-1}^{t} \\ \delta u_{k-1}^{t+1} \\ \vdots \\ \delta u_{k-1}^{t+p-1} \end{bmatrix} + \begin{bmatrix} \Delta u_k^{t-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{13}$$

It is assumed that, at time *t*=1 of every batch, u_k^0 is fixed at a constant value. Hence, Δu_k^0 has a value of zero and $\delta u_{k-1}^1 = u_{k-1}^1 - u_{k-1}^0$ is replaced by $u_{k-1}^1 - u_k^0$.

7. *Calculation of optimal control input sequence*: On the basis of the controlled output prediction equation, Eq. (8), the optimal control input sequence is calculated. The most common choice for the optimization problem is the following quadratic minimization:

$$\min_{\Delta U_k^{t+p|t}} \left\| \Lambda^y \left[Y_k^{t+p|t} - R_k^{t+p|t} \right] \right\|_2^2 + \left\| \Lambda^u \Delta U_k^{t+p|t} \right\|_2^2 \tag{14}$$

where $R_k^{t+p|t} = \left[\left(r_k^{t+1} \right)^T \cdots \left(r_k^{t+p} \right)^T \right]^T$ is the future reference vector for y^c available at time *t* of the k^{th} batch. Λ^y and Λ^u represent the weighting matrices for the controlled output error and the input change in terms of the batch index, respectively. In most cases, these matrices are given in the form of diagonal matrices.

The optimization problem with constraints (10)-(13) can be solved by the quadratic programming (QP) method. From the computed control input sequence, the first input move Δu_k^t is implemented and the entire procedure 1-7 is repeated at the next sampling time.

Application of NLILC

The objective of property control is to produce copolymers with a uniform copolymer composition and a desired weight average molecular weight (*Mw*). For this purpose, the mol fraction f_1 of MMA in the remaining monomers and *Mw* are chosen as the controlled outputs while the flow rate q_f of the feed, which is composed of monomer 1 (MMA) and initiator (AIBN), and the reaction temperature T_r are taken as the control inputs.

It is aimed to regulate the properties of copolymer product in the presence of an irregular disturbance with respect to the batch index. In a semibatch copolymerization reactor, irregular disturbance can be realized with the heat transfer characteristics. When the reaction temperature is controlled in the jacketed reactor, the temperature control can be involved with aperiodic occurrence of fouling on the reactor wall. In such a case we add the integrated white noise to the reaction temperature computed by the controller. The integrated white noise is calculated by accumulating the random number between 0 and 0.05 with a variance of 1×10^{-4}. With this integrated white noise, the reaction temperature becomes higher than the desired value. Hence, the degree of deviation from the desired value increases as the reaction proceeds.

To prove the effectiveness of the control algorithm itself, we assume that the state is available by feedback in this case study. The sampling time, the prediction horizon and the control horizon are specified as 1 min, 10, and 10, respectively. Weighting matrices are used in diagonal form for convenience and determined by the trial and error method as $\Lambda^y = diag(20, 150)$ and $\Lambda^y = diag(0.01,\ 0.1)$ for outputs and inputs, respectively. The following constraints are taken into consideration:

· Input magnitude: $0 \le q_f \le 10$ [mL/min] & $55 \le T_r \le 90$ [℃]

· Input change w.r.t. the batch index: $-5 \le \Delta q_f \le 5$ [mL/min] & $-4 \le \Delta T_r \le 4$ [℃]

· Input change w.r.t. the time index: $-2 \le \delta q_f \le 2$ [mL/min] & $-2 \le \delta T_r \le 2$ [℃]

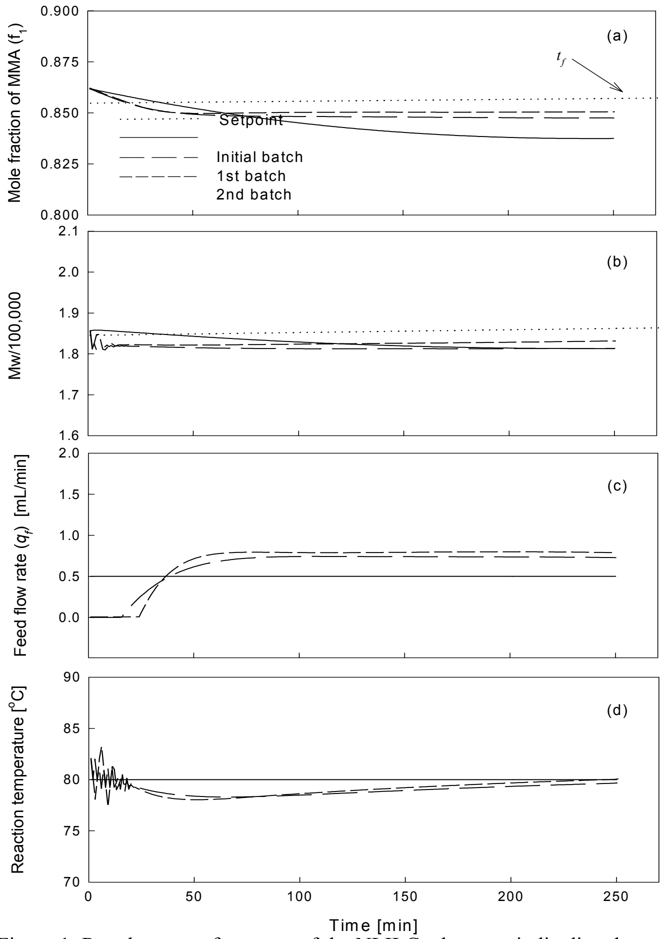

Figure 1. Regulatory performance of the NLILC when aperiodic disturbance is present in the heat transfer characteristics at every batch.

Figure 1 shows the disturbance rejection performance of the NLILC when the setpoints of f_1 and Mw are specified at 0.85 and 182,000, respectively. At the initial batch, the reactor is operated under constant input conditions of q_f=0.5 mL/min and T_r=80℃. At the first iteration, the controller decreases the feed flow rate to its lower bound to drive f_1 to its setpoint quickly. After f_1 reaches its setpoint, the feed flow rate is increased to compensate for the shortage of MMA in the reaction mixture. While the injection of the initiator decreases Mw, the reaction temperature is decreased to increase Mw. During the first iteration, the controlled outputs approach their respective setpoints very closely. The second iteration presents a satisfactory control performance without offset and, though not shown in the figure, further iterations show the same control results. The control inputs after the second iteration do not coincide entirely due to the presence of the irregular disturbance but the divergence problem is not observed in this case.

Conclusions

A nonlinear model-based iterative learning controller (NLILC) is developed by combining the nonlinear model predictive controller (NLMPC) with the iterative learning controller. The main idea of the proposed algorithm is to recast the prediction equation into the form of iterative learning control by linearizing the nonlinear model by using the data of the previous batch. The proposed controller is computationally efficient and the linearized time varying model approximates the nonlinear system as closely as possible because the Jacobians are calculated with the state and input of the previous batch.

A semibatch MMA/MA copolymerization reactor is treated for the quality control of copolymer product under the presence of disturbance in the heat transfer characteristics. The simulation result clearly demonstrates that the NLILC performs remarkably in view of the convergence rate as well as the disturbance rejection capability.

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