회분/연속공정이 결합된 PVC 공정의 단기일정계획

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Scheduling model for a polymerization process with batch and continuous operations

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1. Introduction

Processing with batch and continuous units has received increasing attention as a mode of production for high value added specialty chemicals such as polymers over the last several years. Here a process of the polymer production which contains both batch and continuous units is considered.

Researchers have developed the method for scheduling these batch and continuous plants (Kudva et al., 1994, Djavdan, 1992). Especially, the PVC process scheduling has been considered by Shah et al. (1996). They developed the algorithm by using the STN which is based on the discrete time horizon. The uniform discretization of the time domain was also applied to parallel flowshop scheduling. However, the discrete model is not appropriate for large-scale problems due to unavoidable high computational expense.

In this study, an MILP model is proposed to sequence products to be manufactured in this plant. The continuous time algorithm proposed by Moon and Hrymak (1999) is used to develop the scheduling model for the polymerization process. In order to solve a real size problem, a heuristic method which will be described in Mathematical Model section, is proposed to increase the efficiency for the large-scale problems.

This paper is presented as follows: first, the targeted process is described. And then the mathematical model for scheduling the process is presented. Last, some examples are illustrated to verify its efficiency.

2. Process Description

2.1 Process description

The process consists of two parts based on the blow down tank (BT): A part is operated in batch mode and the other is operated continuously. The batch part of the process consists of a number of batch reactors (RE) in parallel. The continuous part of the process contains the two kinds of storages tank (blow down tank (BT) and slurry tank (ST)) for the effluent from the reactors, a stripper for monomer removal and the second storage tank to store the product from the stripper, the dewaterer and dryer to remove the water from the product. In this study, there are 10 products to be produced and 10 units to be operated. Nine products are manufactured through the production line ①. But one product is produced through the production line ② which is the same as ① except ST-A and stripper units, as shown in Figure 1. Therefore this process can be classified as a sequential multipurpose scheduling problem. Setup time for each reactor in PVC process is required for the case that a different product should be manufactured.

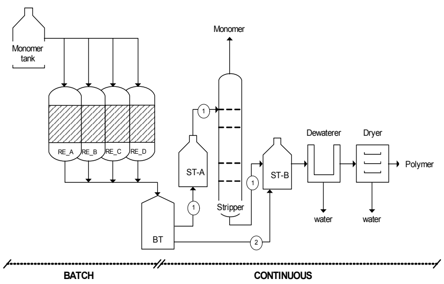

Figure 1. Flow diagram for PVC process

2.2 Assumptions

Some assumptions in this study are the following.

-There is a time delay between the successive continuous operations, for example, a required time to fill up the minimum level of each continuous unit.

-Flow rate of all continuous units is a constant, although each continuous unit has a different capacity from the others and the production yields are not stable for each batch.

-Since the dryer is a bottle-neck of this process, we assume that its processing time is longer than those of the other units included in the continuous parts.

3. Mathematical model

As briefly mentioned earlier, a heuristic method is introduced to schedule the real size polymerization process. The method consists of three steps: The process is separated into two parts based on the batch and the continuous operations and two mathematical models are developed for the parts, respectively. However, the blow down tank should be considered simultaneously in the batch and the continuous parts to combine those models and complete the scheduling for the whole process. In the second step, a scheduling model (MILP) for only the continuous part is performed to determine a production sequence of all products and the production timing for each product. Finally, a mathematical model (LP) for the batch part is solved to decide the production timings for reactors and to control the level of the blow down tank, during the time horizon obtained from the second step.

3.1 MILP model for continuous part

The objective function of the continuous part scheduling is considered to minimize the makespan for the polymerization process as described in Eq. (1). Where, indices *j* and *k* mean unit and time slot.

$$\text{Min.}\ \ ms \ge Te_{jk} \qquad \forall j \in J,\ \forall k \in K \tag{1}$$

In this study, a binary variable X_{ik} is proposed to assign product *i* to time slot *k*. Therefore, variable X_{ik} has to satisfy the following equations.

$$\sum X_{ik} = 1 \qquad \forall k \in K \tag{2}$$

$$\sum_k X_{ik} = 1 \qquad \forall i \in I \tag{3}$$

In the case when product *i* is manufactured in unit *j* at time slot *k*, the operating time of the unit should be equal to the processing time of product *i*, PT_i, as shown in Eq. (4). However, the processing time of Dryer (the last unit) should be longer than the others as discussed earlier. In order to simplify the problem, the processing time of Dryer is assumed to be equal to the product of PT_i and α_i which is greater than 1, as shown in Eq. (5).

$$Te_{jk} - Ts_{jk} = \sum_{i \in I} X_{ik} PT_i \qquad \forall j \in J - j^*, \forall k \in K \tag{4}$$

$$Te_{j^*k} - Ts_{j^*k} = \sum_{i \in I_j} X_{ik} (PT_i \alpha_i) \qquad \forall k \in K \tag{5}$$

In the same slot, there is a time delay because of filling up the minimum level for the continuous operating and transfer. Therefore, Eqs. (6) and (7) express that the start time of unit *j'* should be delayed by a small time, 0.5, from the previous unit *j*.

$$Ts_{j'k} - Ts_{jk} \ge DeT - U(1 - X_{ik}) \qquad \forall i,o,k \text{ and } j \in J_{io},\ j' \in J_{i,o+1} \tag{6}$$

$$Ts_{j'k} - Ts_{jk} \le DeT + U(1 - X_{ik}) \qquad \forall i,o,k \text{ and } j \in J_{io},\ j' \in J_{i,o+1} \tag{7}$$

In the successive slots, slot *k* and slot *k*+1, and on the same unit *j*, setup time has to be required to clean up all units for the case when products to be manufactured are changed from product *i* to product *i'*.

$$Ts_{j,k+1} - Te_{jk} \ge 0 \qquad \forall k \in K - k^*, j \in J - j^* \tag{8}$$

$$Ts_{j^*k+1} - Te_{j^*k} \ge S_{iji'} - U(2 - X_{ik} - X_{i',k+1}) \qquad \forall\ k \in K - k^*,\ \forall i,i' \in I_{j^*} \tag{9}$$

Since Dryer's processing time is longer than those of the other units and Dryer is a bottle-neck process, the setup time in only Dryer's operations is considered in this work, as shown in Eq. (9).

3.2 LP model for batch part

The main goal for the scheduling of the batch part is to control the level of the blow down tank during the operating time to be adjusted with the time interval obtained from the MILP model for the scheduling of the continuous part.

$$ms \ge Te_{jk^*} \qquad \forall j \in J - j^* \tag{10}$$

Although the objective function for the batch part scheduling, Eq. (10), is to minimize the whole operating time for the number of batches to manufacture the specific product, it may be meaningless since the level control for the blow down tank can be achieved by determining the release time for a batch from each reactor.

Processing time for each reactor in slot *k* can be defined as the following equation.

$$Te_{jk} - Ts_{jk} = PT \qquad \forall j \in J - j^*, \quad \forall k \in K \tag{11}$$

Since the processing time for blow down tank (the last unit) depends on the flow rate and the remained volume of the product in the unit, the blow down tank is not considered in Eq. (11).

In time slot *k*, we assume that the start time of the blow down tank is equal to the ending time of the reactor, as shown in Eq. (12).

$$Ts_{j^*k} - Te_{jk} = 0 \qquad \forall k \in K,\ j, j^* \in J \tag{12}$$

The following equations show the timing relationship for the batch jobs between slots *k* and *k*', produced in unit *j*, as listed in Eq. (13). However, since the blow down tank is operated in continuous mode, there is no idle time between the two jobs operated in slots *k* and *k*+1, respectively, as represented in Eq. (14).

$$Ts_{jk'} - Te_{jk} \ge 0 \qquad \forall j \in J - j^*,\ \forall k, k' \in K - k^* \tag{13}$$

$$Ts_{j^*,k+1} - Te_{j^*k} = 0 \qquad \forall k \in K - k^* \tag{14}$$

The level control for the blow down tank can be accomplished by the following mass balance equations, Eqs (15)~(22).

In the case when the material is incoming from a reactor to the blow down tank in time slot *k*, Eq. (15) illustrates that the volume of the blow down tank in time slot *k*+1 becomes the sum of the its volume and the volume from the reactor.

$$VBs_{k+1} = VBe_k + BR \qquad VBs_1 = BR,\ \forall k \in K - k^* \tag{15}$$

In the same slot, Te_{j^*k} is determined by Ts_{j^*k} in the above equation (12) and VBe_k under certain limitation. As described earlier, the level control of the blow down tank depends on the level itself and flow rate. At the ending time of the last slot the volume of blow down tank is zero.

$$Te_{j^*k} - Ts_{j^*k} = \frac{VBs_k - VBe_k}{FR} \qquad \forall k \in K - k^* \tag{16}$$

$$Te_{j^*k^*} - Ts_{j^*k^*} = \frac{VBs_{k^*}}{FR} \tag{17}$$

The volume of the blow down tank should satisfy the following upper limits.

$$VBe_k \le VBe_{\max} \qquad \forall k \in K \tag{18}$$

$$VBs_k \le VBe_{max} + BR \qquad \forall k \in K \tag{19}$$

$$VBe_k \le VBs_k \qquad \forall k \in K \tag{20}$$

As mentioned earlier, the start time (ST) and the ending time (ET) can be estimated from the solution of the MILP model for the continuous part scheduling. Therefore, start time of the first slot and the ending time of the last slot in the blow down tank should be satisfied by the following equations.

$$Ts_{j^*1} \ge ST \tag{21}$$

$$Te_{j^*k^*} \le ET \tag{22}$$

4. Examples and Results

In order to verify the efficiency of the scheduling models developed in this study, these models were applied to three examples as below.

Example 1

Example 1 consists of 10 products to be manufactured and 6 continuous units. In Example 1, the scheduling of the continuous part in PVC process was decided to minimize the makespan. Table 1 shows the setup times in Dryer in the case when the different product is started to be manufactured. The setup times include the time for shut-down and start-up, clean up time, and/or repair time, etc. Figure 2 illustrates a Gantt chart of the scheduling solution for the continuous part from Blow down Tank to Dryer. The minimum makespan was obtained as 732.4, as shown in this figure, and the optimal production sequence is shown as E-A-

Table 1. Setup time in Dryer

$S_{i,i'}$	A	B	C	D	E	F	G	H	*I*	J
A	0	9	10	8	6	9	10	7	6	7
B	9	0	0	0	8	7	6	9	7	9
C	10	0	0	0	7	8	9	6	8	9
D	8	0	0	0	8	6	10	6	9	7
E	6	8	7	8	0	10	9	11	10	7
F	9	7	8	6	10	0	9	8	11	11
G	10	6	9	10	9	9	0	6	9	7
H	7	9	6	6	11	8	6	0	9	9
I	6	7	8	9	10	11	9	9	0	6
J	7	9	9	7	7	11	7	9	6	0

Table 2. parameters

BR	VBe_{max}	*PT*	*FR*	*ST*	*ET*
20	30	9	8.6	60.2	133.3

I-J-G-H-C-B-D-F. As mentioned earlier, since Product J has a different unit sequence to be used from those of the other products, this scheduling problem is classified as a sequential multipurpose process.

Example 2

In Example 2, the scheduling for the batch part in PVC process is performed by the LP scheduling model, described above. Table 3 shows the data used to schedule the batch part. In order to consider the efficiency of the LP model, timings of batch reactors to manufacture the Product A and the level control for Blow down Tank during the time horizon (60.2~133.3, as shown in Figure 2) obtained from the solution for Example 1 are illustrated in Figure 3. As shown in this figure, the polymerization in the batch reactor and the transportation of the product to Blow down Tank should be finished before the unit (Blow down Tank) in continuous mode becomes empty. In this study, the maximum capacity of the blow down tank is 50 unit volumes and the batch size is 20 unit volumes. Table 3 summarizes the calculation results obtained from the proposed scheduling models for each example. As shown in this table, the number of binary variables for Ex. 1 and Ex. 2 is 100 for assigning 10 products to 10 time slots. However, there is no binary variable in Ex. 3 since the scheduling model for Ex. 2 is formulated as an LP, as explained previously.

Table 3. The calculation results for each example.

	Ex. 1	Ex. 2
# of binary variable	100	-
# of continuous variable	121	190
# of equations	1905	358
computational time (sec)	697.5	0.05
error range (%)	6.6	0.0
objective value	732.4	meaningless

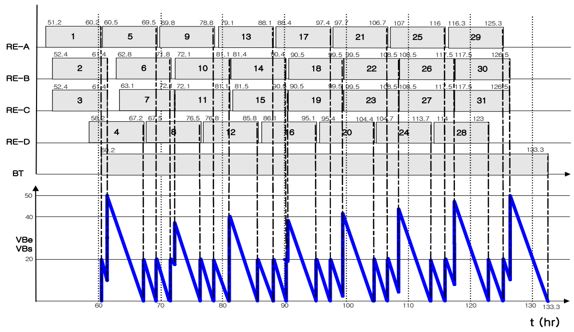

Figure 2. Gantt chart of the scheduling for Example 3 and the level profile for the Blow down Tank

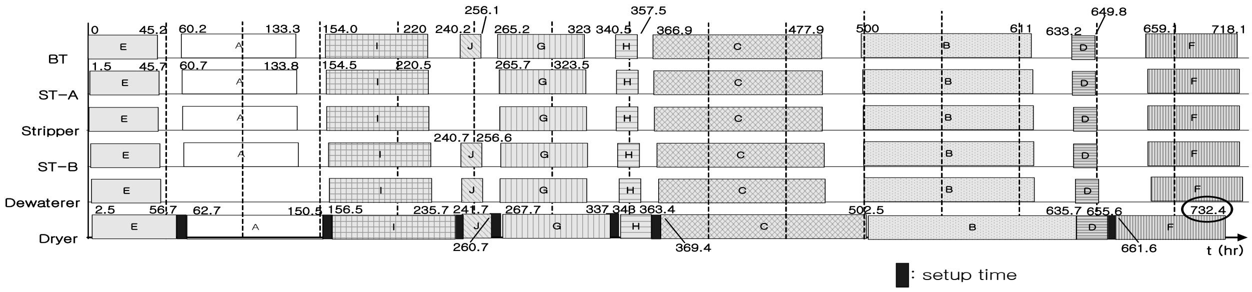

Figure 2. Gantt chart for the scheduling problem to minimize makespan (Example 1)

5. Conclusion

The proposed mathematical models for the scheduling of PVC plant were performed with three examples involving the real scale problem and the reasonable scheduling solutions were derived. These models in this research can help the engineers to decide an optimal production sequence for the plant and the proper operating time for each unit.

Acknowledgement

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