정전기장 내 유체 박막의 모양 변화에 대한 수학적 모델과 관찰

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MATHEMATICAL MODELING AND EXPERIMENTAL OBSERVATION OF THE THIN FILM BEHAVIOR UNDER AN ELECTROSTATIC FIELD

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INTRODUCTION.

Here, we are to consider the surface deformation of static dielectric liquid under a locally electrified field. This is very important for two aspects; one is that the static electricity is easily generated by friction and sometimes harms the units of our industries, and the other is that in the fine process or units the deformation from any causes can be very fatal to the normal operations. Although it is known that the efficiency of heat transfer under an electrostatic field, the rupture of film can hinder it and result in the overheating and finally breakdown of that unit.

For van der Waals effects, the simplest situation to analyze is that involving two hard, flat, effectively infinite surfaces separated by a distance, H , in a certain medium, the forces exerting on mutually faced interfaces is derived as follow;

$$F_{att} = -\frac{\partial G}{\partial H} = -\frac{A_H}{6\pi H^3}$$

where A_H is the Hamaker's constant[1],

Landau[2] derived the boundary condition for the dielectric body from the first law of thermodynamics such that

$$p_0(\rho, T) - p_{env} = \frac{\rho E^2}{8\pi}\left(\frac{\partial \varepsilon}{\partial \rho}\right)_T - \frac{(\varepsilon - 1)}{8\pi}\left(\varepsilon E_n^2 + E_t^2\right)$$

We use the Young-Laplace equation including mean curvature of the interface denoted by film thickness h in order to represent the surface tension,

$$\frac{\rho g}{\sigma}\Delta h = \nabla\cdot\frac{\nabla h}{1+(\nabla h)^2} = \frac{\frac{d^2h}{dx^2}}{1+\left(\frac{dh}{dx}\right)^2}$$

where, $\Delta h = h - h_0$ and h_0 denotes the initial film thickness.

FORMULATION.

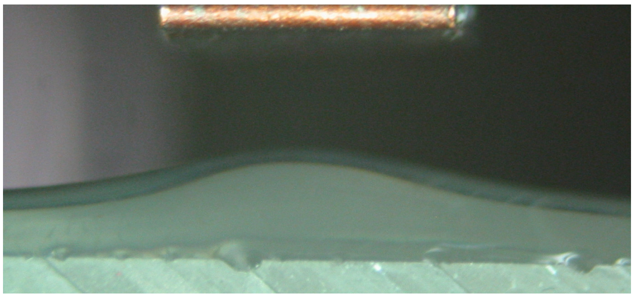

Fig. 1. Camera image using ethylene glycol describes the system of problem. The distance between foil and floor 4.2mm and initial film thickness 0.82mm.

As shown in Fig. 1, the film of ethylene glycol swells up by nearly twice of the initial thickness.

We make Navier-Stoke's equations to be dimensionless, all the notations shown in this procedure will follow R. B. Bird[3]. Table. 1. shows the resultant dimensionless equations and constants.

Table. 1. Dimensionless governing equations with the dimensionless constants.

Dimensionless governing equations	Dimensionless constants	Names
$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$	$Ca=\frac{2\mu U_0}{\sigma}$	Capillary number
$\xi\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\xi\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\xi^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$	$A_H^{'}=\frac{A_H}{12\pi\mu U_0 h_0^2}$	reduced Hamaker's number
$\xi^2\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\xi}{\mathrm{Re}}\left(\xi^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{1}{Fr^2}$	$K_E=\frac{\varepsilon_0 E_D^2}{16\pi\mu U_0 h_0}$	Electrostatic Capillary
$\xi^2\left(\frac{\partial^2 h}{\partial x^2}\right)\Big/ Ca\left(1+\xi^2\left(\frac{\partial h}{\partial x}\right)^2\right)^{\frac{3}{2}}+\frac{A_H^{'}}{h^3}$ $=-\frac{\mathrm{Re}}{2}p+K_E(1-\varepsilon)(\varepsilon E_n^2+\xi^2 E_t^2)$ $+\frac{\xi}{1+\xi^2\left(\frac{\partial h}{\partial x}\right)^2}\left[\xi^2\frac{\partial u}{\partial x}\left(\frac{\partial h}{\partial x}\right)^2-\left(\frac{\partial u}{\partial y}+\xi^2\frac{\partial v}{\partial x}\right)\left(\frac{\partial h}{\partial x}\right)+\frac{\partial v}{\partial y}\right]$	$Fr=\frac{U_0}{\sqrt{gh_0}}$	Froude number
	$\mathrm{Re}=\frac{\rho U_0 h_0}{\mu}$	Reynold's number
	ε_0 $(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})$	permittivity of free space

$\left(\frac{\partial u}{\partial y}+\xi^2\frac{\partial v}{\partial x}\right)\left(\xi^2\left(\frac{\partial h}{\partial x}\right)^2-1\right)+2\xi^2\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)\left(\frac{\partial h}{\partial x}\right)=0$	$\xi=\frac{h_0}{L}<<1$	Slenderness parameter

The power series expansions and the sorting with respect to ξ are applied to the governing equations in order to discern which are significant factors that determine the film behaviors, as follows;

$$u(t,x,y)=u_0(t,x,y)+\xi u_1(t,x,y)+\Lambda\ ,\quad v(t,x,y)=v_0(t,x,y)+\xi v_1(t,x,y)+\Lambda$$
$$p(t,x,y)=p_0(t,x,y)+\xi p_1(t,x,y)+\Lambda$$

The evolution equation is derived by applying the perturbed results of above three solutions into the substantial derivatives of film surface, so called kinematic boundary condition.

$$\frac{\partial h}{\partial t}=\xi\left[\left(\frac{1}{3}h^3\right)\left\{-\frac{2\xi^2}{Ca}\left(\frac{\partial^4 h}{\partial x^4}\right)-\frac{24A_H'}{h^5}\left(\frac{\partial h}{\partial x}\right)^2+\left(\frac{6A_H'}{h^4}+\frac{\mathrm{Re}}{Fr^2}\right)\left(\frac{\partial^2 h}{\partial x^2}\right)\right\}\right.$$
$$+h^2\left(\frac{\partial h}{\partial x}\right)\left\{-\frac{2\xi^2}{Ca}\left(\frac{\partial^3 h}{\partial x^3}\right)+\left(\frac{6A_H'}{h^4}+\frac{\mathrm{Re}}{Fr^2}\right)\left(\frac{\partial h}{\partial x}\right)\right\}-2K_E(1-\varepsilon)\varepsilon\int_0^h\frac{\partial}{\partial x}\int_0^\eta\int_h^\lambda\frac{\partial}{\partial x}\left(E_N^2\right)d\tau d\lambda d\eta$$
$$\left.-2K_E(1-\varepsilon)\varepsilon\left(\frac{\partial h}{\partial x}\right)\int_0^h\int_h^\lambda\frac{\partial}{\partial x}\left(E_N^2\right)d\tau d\lambda\right]$$

THEORETICAL RESULT BY SIMPLE IMPLICIT METHOD

We use a ethylene glycol[4] as a dielectric liquid and the simple implicit method[5] to solve above partial differential equation numerically.

Also, electrostatic field intensity caused by local electric foil are derived using the procedures of Paul Nasar[6] and tabulated in order to apply the numerical solution.

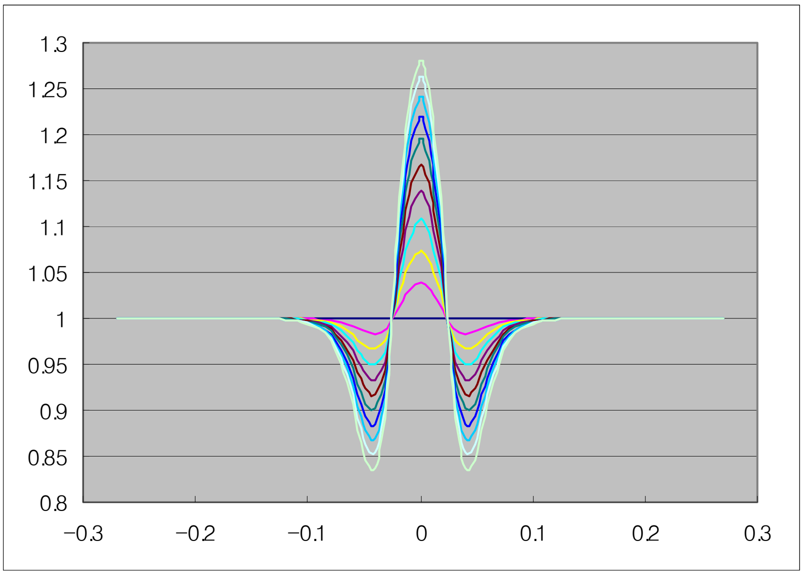

Fig. 2 The result of numerical solution.

The theoretical evolution of film thickness is shown in Fig. 2 for 3 second, and further simulation results in the convergence of the height. The main trends of surface deformation are very similar to that of camera image in Fig. 1, whereas some differences are observed.

First, the sinked parts around the swelled one appear due to the instantaneous electrified foil and, therefore, in order to conserve the total mass. That phenomenon is also observed in Fig. 1, although it is not as serious as in Fig. 2.

Second, the maximum height of film increases remarkably at the beginning and decreases very fast as time passes. Fig. 1 is pictured when the height of film has the maximum value.

Finally, note that the Fig. 2 derived from the evolution equation is dimensionless, that is to say, the ratio of vertical to horizontal scale, ξ, is 0.0082. That means that in order to consider as a real scale we must stretch Fig. 2. by 121.9 times long horizontally.

CONCLUSION

As a matter of fact, the electrical discharges occur so that the swelling does not persist on. But the mathematical model means the ideal condition and the discharge is not considered.

From this investigation, in a tiny system such as a surface of hard disk in computer or LCD, we can find that the electrostatic field can affect physically on the fluid.

This investigation does not include the effects of vaporization or heat transfer, of course it can be desirably neglected in mild condition such as a room temperature. For more detailed or applicable researches it will be very useful of adding their effects.

REFERECES

1. Drew Myers, "Surfaces, Interfaces, and Colloids; Principles and Applications, 2nd edition", WILEY-VCH, New York (1999).
2. Landau L. D. and Lifshitz E. M. and Pitaevskiĭ L. P., "Electrodynamics of Continuous Media 2nd edition", Pergamon Press (1984).
3. R. B. Bird, Warren E. Stewart, Edwin N. Lightfoot, "Transport phenomena, 2nd edition", John Wiley & Sons, Inc. New York (2002)
4. John A. Dean, "Lange's handbook of chemistry, 15th edition", McGraw Hill, New York (1999)
5. Steven C. Chapra, Raymond P. Canale, "Numerical Methods for Engineers; With Programming and Software Applications, 3rd edition", McGRAW-HILL (1998).
6. Clayton R. Paul Syed A. Nasar, "Introduction to Electromagnetic fields, 2nd edition"McGraw-Hill Book Company, New York (1987)