

## Optimal Tuning Combined with BLT and Mp criteria For Multiloop PID Controllers

Truong Nguyen Luan Vu, Kihong Lee, Moonyong Lee\*, Jietae Lee<sup>1</sup>  
 School of Chemical Engineering and Technology, Yeungnam University,  
<sup>1</sup>Department of Chemical Engineering, Kyungpook National University  
 (mynlee@yu.ac.kr\*)

### Introduction

The proposed method was achieved by combining Mp tuning and BLT tuning criteria. Mp tuning criterion will select an initial set of tuning parameter  $\lambda$ . As  $\lambda$  decreases, the closed loop response of loop  $i$  is faster or even unstable while as the  $\lambda$  increases, the closed loop response is more sluggish or stable. Our goal is to find an optimal set of  $\lambda$  to make the closed loop response more fast, stable and robust. Lee et al. (2003) suggested Mp tuning criterion. But since the method does not consider the interaction effect for  $\lambda$  evaluation,  $\lambda$  only by Mp criterion does not always give the optimal enough to minimize interaction and overshoot in the loops and sometimes it does not give the desired degree of damping. The damping coefficient of system is sometimes quite low. For this reason, BLT tuning criterion is used, which satisfies the objective of arriving at reasonable controller setting with only a small amount of engineering and computation effort and the feedback controller is designed to give a maximum resonant peak or hump in closed loop log modulus plot. A commonly used maximum closedloop log modulus specification is 2N dB (N is the order of system). The controller parameters are adjusted to give maximum peak in the closedloop servo log modulus curve of 2N dB. The method gives a reasonable compromise between stability, robustness and speed in multivariable systems.

### Theory

#### 1. Mp tuning criterion for the multiloop system.

The multiloop feedback system can be shown by Fig. 1,

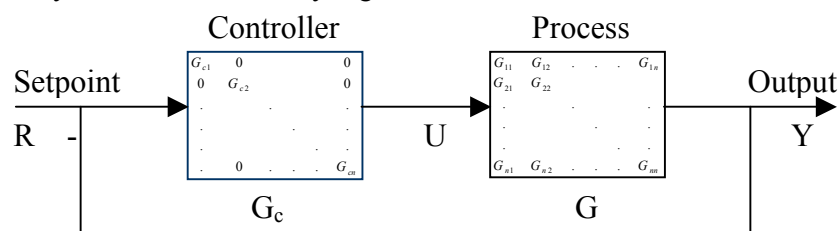


Fig. 1. Block diagram for multiloop control

The criterion is based on closed loop frequency response.

In MIMO system, the closed loop transfer function can be represented by:

$$H(s) = G(s)G_c(s)[I + G(s)G_c(s)]^{-1} \quad (1)$$

It can be expressed by the following matrix,

$$H(s) = \begin{bmatrix} H_{11} & H_{12} & \dots & H_{1n} \\ H_{21} & H_{22} & & \\ \vdots & & \ddots & \vdots \\ H_{n1} & H_{n2} & \dots & H_{nn} \end{bmatrix} \quad (2)$$

The closed loop frequency response can be found by setting  $s = i\omega$ .

$$H(i\omega, \lambda) = G(i\omega)G_c(i\omega, \lambda)[I + G(i\omega)G_c(i\omega, \lambda)]^{-1} \quad (3)$$

- The magnitude of frequency response is called the closed loop amplitude ratio (AR).
- The maximum magnitude over the frequency range is defined as  $M_p$ .

The above frequency domains objectives can be achieved by solve the following optimization problem.

$$\min \left[ \sum_i \sum_{i \neq j} Mp_{ij} + w \sum_i Mp_{ii} \right] \quad (4)$$

$$s.t. Mp_{ii} \geq Mp_{low} \quad (5)$$

Where,  $Mp_{ij} = \max_{\omega} |H_{ij}|$ ;  $Mp_{ii}$  is the function of  $\lambda$ ,  $Mp_{low}$  is lower bound of diagonal  $Mp$ ;  $w$  is weighting factor for the diagonal  $Mp$ .

## 2. Analytical design of multiloop PID controllers for desired closed-loop responses.

According to Lee et al (2004), the proportional gain and derivative time constant can be obtained by

$$K_{ci} = f'_i(0) \quad ; \quad \tau_{Di} = f''_i(0) / 2K_{ci} \quad (6)$$

Design of integral time constant  $\tau_{Ii}$

$$\tau_{Ii} = - \frac{(G'_{ii+}(0) - n_i \lambda_i) K_{ci}}{(G^{-1}(0))_{ii}} \quad (7)$$

Where  $G(s)$  is open loop stable process,  $G_{ii+}$  is the non part of  $G_{ii}$  and is chosen to be all pass form.  $\lambda$  is an adjustable constant for system performance and stability,  $n_i$  is chosen so as the IMC controller to be realizable. The requirement of  $G_{ii+}(0) = 1$  is necessary for the controlled variable to track its setpoint.

## 3. BLT tuning.

BLT tuning provides such a standard tuning methodology; this method is very easy to use and easily understandable by control engineers.

It includes the following four steps:

- A multiloop PID controller is initially designed based on  $M_p$  tuning method (Lee et al, 2003).
- In Eq. (6) and (7), detune  $\lambda_i$  according to the following BLT tuning procedure.
- According to the guessed value of  $\lambda_{1,2}$  and resulting controller settings, a multivariable Nyquist plot of the scalar function is made as followed.

$$W(i\omega) = -1 + \det(I + G_p G_c) \quad (8)$$

a multivariable closed loop log modulus  $L_{cm}$  is

$$L_{cm} = 20 \log_{10} \left| \frac{W}{W+1} \right| \quad (9)$$

The peak in the plot  $L_{cm}$  over the entire frequency range is biggest log modulus  $L_{cm}^{\max}$ .

- $\lambda_{1,2}$  is varied until  $L_{cm}^{\max}$  is equal to  $2N$ , where  $N$  is order of the system.

The BLT procedure was applied with PI controllers. The method can be extended to include derivative action (PID controllers).

## 4. Simulation study

**Example 1.** The Wood and Berry (WB) column model [Luyben,1986]

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \quad (10)$$

For the WB column  $\lambda$  was selected by 0.276/2.91 as an initial setting. Optimal  $\lambda$  for loop1 and loop2 are found as 0.79/8.38. The resulting tuning values are listed in table1. Fig.2 shows the closed loop responses tuned by various methods such as the proposed, Mp and BLT tuning methods.

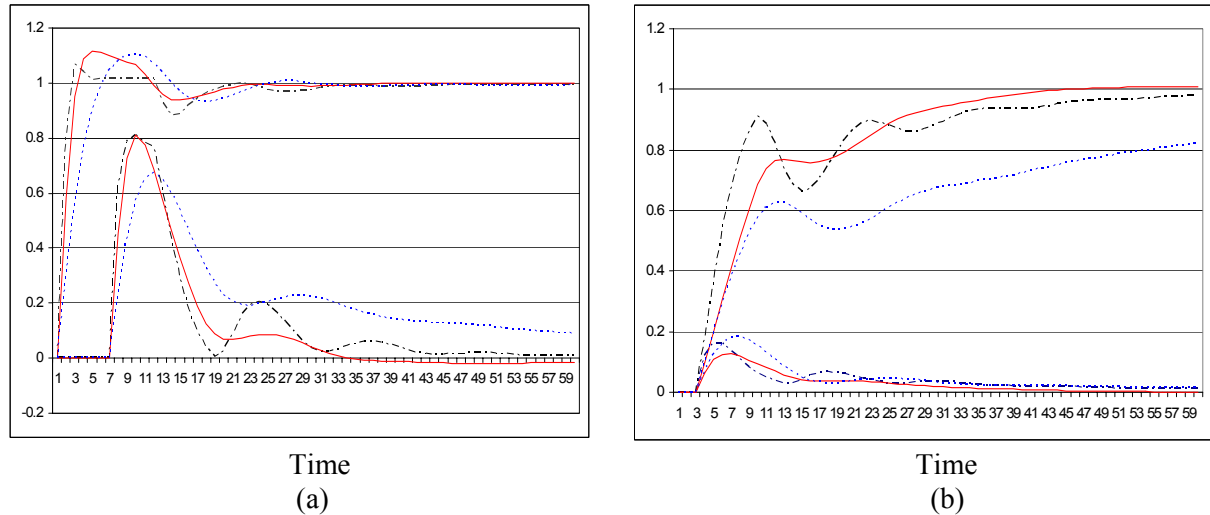


Figure 2. Closed loop time responses of each loop for example 1 (WB). (a) Setpoint change in loop1; (b) setpoint change in loop2; solid line denotes the proposed method; dotted line denotes BLT tuning; dash dotted line denotes Mp tuning.

Table 1. Tuning results by the proposed method, Mp and BLT.

Process model	Model parameters	Proposed method	Mp tuning	BLT tuning
WB	$\lambda_{1,2}$	0.79, 8.38	0.276, 0.91	-
	$k_{1,2}$	0.74, -0.069	1.047, -0.13	0.38, -0.075
	$\tau_{I1,2}$	8.45, 7.67	17.1, 15.2	8.29, 23.4
	$\tau_{D1,2}$	0.27, 0.37	0.37, 0.71	-
	IAE1	10.69, 6.98	11.27, 6.81	12.09, 16.6
	IAE2	1.748, 19.28	2.299, 19.4	3.39, 39.9
T4	$\lambda_{1,2,3}$	0.5, 0.95, 7.65	0.11, 0.21, 1.6	-
	$k_{1,2,3}$	-27.6, -5.2, -0.13	-41, -14, -0.4	-11.3, -3.5, -0.18
	$\tau_{I1,2,3}$	61.2, 27.3, 117	67, 4.93, 11.8	7.09, 14.5, 15.1
	$\tau_{D1,2,3}$	0.2, 0.99, 0.136	0.3, 1.31, 0.38	-
	IAE1	5.94, 2.09, 12.7	7.7, 3.47, 14.2	14.6, 10.3, 37.1
	IAE2	5.14, 8.3, 33.6	7.7, 16.73, 43.3	25.7, 45, 49.3
	IAE3	1.647, 3.98, 24	5.2, 7.25, 24.8	9.3, 14.1, 53.1

The results show the proposed method gives better performance over the other methods.

**Example 2.** The Tyreus case 4 (T4) [Luyben, 1986]

$$G(s) = \begin{bmatrix} \frac{-1.986e^{-0.71s}}{66.67s+1} & \frac{5.24e^{-60s}}{400s+1} & \frac{5.984e^{-2.24s}}{14.29s+1} \\ \frac{0.0204e^{-0.59s}}{(7.14s+1)^2} & \frac{-0.33e^{-0.68s}}{(2.38s+1)^2} & \frac{2.38e^{-0.42s}}{(1.43s+1)^2} \\ \frac{0.37e^{-7.75s}}{22.22s+1} & \frac{-1.13e^{-3.79s}}{(21.74s+1)^2} & \frac{-9.81e^{-1.59s}}{(11.36s+1)} \end{bmatrix} \quad (11)$$

For T4, initial  $\lambda$  was 0.114, 0.214, and 1.63. Optimal  $\lambda$  are -27.6/-5.3/-0.13 for loop1, loop2, loop3, respectively. The result tuning values are also listed in table1. The results show the proposed method gives better performance over the other methods.

**Conclusions** In this paper an combined method for optimal tuning of multiloop PID controller was proposed based on the analytical method by Lee et al. (2004) in which the proportional gain and derivative time constant are designed by neglecting of off-diagonal elements while the integral term is designed by taking the off-diagonal element fully into account. Mp tuning criterion was firstly used to set initial  $\lambda$  and then the BLT criterion was applied to find optimal tuning values which compromise performance and stability in the multiloop system. The resulting multiloop PID controllers have the closed loop responses to meet desired performance and robustness as close as possible.

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