정전기장의 영향을 받는 박막 표면 파동의 Fourier series 해법

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Fourier series solutions for the electrohydrodynamic thin-layer surface waves

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Introduction

Free surface wavy motions have possessed the broad attentions of many researchers. Especially, the effect of the electrostatic field on the wave conditions has been one of the most interesting subjects, nevertheless, the undefinitely numerous things remain hard to be determined. The thin liquid layer having initially the static conditions spreads over the horizontal solid plate which is electrically grounded. Upper the layer the electrified body of uniform electric potenial will be positioned at a time, $t=0$. In this case of very thin layer, the evolution equation involving a Hammaker's constant were derived and solved numerically with using the simple implicit method by Kim et al.[1] There one can know that the effect of Van der Waals' attraction and repulsion on the fluid motion does not play a significant role to define the film height as well as all other flow conditions because the reduced Hammaker's constant is much less than the other dimensionless numbers.

Here, as one of the approximations we consider the Fourier series solution applied into the evolution equation with zero Hammaker's constant. In a same way of Gjevik[2], the Fourier series approximation will reduce the evolution equation from the partial differential equation to the system of the ordinary differential equations concerning of only time.

The origin of our system locates in the bottom of fluid $(y=0)$ and the centre of the electrified body $(x=0)$, from which the electrified body, the potential V on the surface is uniform and assumed not to be affected by the perturbation of the liquid, stretches bilaterally by $s/2$ in the horizontal directions. All the variables defining the fluid motion have the static initial conditions; all are not zero because the static pressure takes a function of y . The characteristic horizontal length is L , and the unperturbed film depth is set to be d . The ratio d/L is considered as the slenderness parameter $\xi \ll 1$. See Fig 1.

Governing equations

The governing equations with the electrostatic field effect and the corresponding boundary conditions are as below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\xi\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\xi\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\xi^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\xi^2\left(\frac{\partial v}{\partial t}\right)=-\frac{\partial p}{\partial y}+\frac{\xi}{Re}\left(\xi^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{1}{Fr^2} \tag{3}$$

$$\frac{\xi^2\left(\frac{\partial^2 h}{\partial x^2}\right)}{Ca\left(1+\xi^2\left(\frac{\partial h}{\partial x}^2\right)\right)^{3/2}}=-\frac{Re}{2}p+\frac{\xi}{1+\xi^2\left(\frac{\partial h}{\partial x}\right)^2}\left[\xi^2\frac{\partial u}{\partial x}\left(\frac{\partial h}{\partial x}\right)^2-\left(\frac{\partial u}{\partial y}+\xi^2\frac{\partial v}{\partial x}\right)\left(\frac{\partial h}{\partial x}\right)+\frac{\partial v}{\partial y}\right] \tag{4}$$

$$+K\left(\frac{1}{\epsilon}-1\right)\left(\left(\xi^2\left(\frac{\partial \phi}{\partial x}\right)\left(\frac{\partial h}{\partial x}\right)-\left(\frac{\partial \phi}{\partial y}\right)\right)^2+\epsilon\xi^2\left(\left(\frac{\partial \phi}{\partial x}\right)+\left(\frac{\partial \phi}{\partial y}\right)\left(\frac{\partial h}{\partial x}\right)\right)^2\right) \tag{5}$$

$$\left(\frac{\partial u}{\partial y}+\xi^2\frac{\partial v}{\partial x}\right)\left(\xi^2\left(\frac{\partial h}{\partial x}\right)^2-1\right)+2\xi^2\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)\left(\frac{\partial h}{\partial x}\right)=0 \tag{6}$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}=v \tag{7}$$

$$u=v=0 \tag{8}$$

where, the dimensionless groups of Reynolds number $Re=\frac{\rho U_0 d}{\mu}$, Froude number $Fr=\frac{U_0}{(gd)^{1/2}}$, Capillary number $Ca=\frac{2\mu U_0}{\mathbf{T}}$, and $K=\frac{\epsilon V^2}{4\mu d U_0}$. All other notations are exactly same as those used by Kim [1].

Evolution equation

The finally derived evolution equation is

$$h_t+\xi h_x h^2\left(\frac{2\xi^2}{Ca}h_{xxx}-\frac{Re}{Fr^2}h_x-2K\left(\frac{1}{\epsilon}-1\right)\frac{\partial}{\partial x}{\phi_y}^2\right)$$
$$+\frac{2}{3}h^3\left(\frac{\xi^2}{Ca}h_{xxxx}-\frac{Re}{2Fr^2}h_{xx}-K\left(\frac{1}{\epsilon}-1\right)\frac{\partial^2}{\partial x^2}{\phi_y}^2\right)=0 \tag{9}$$

Fourier series solution

We have the Fourier series expansion describe the film depth such that

$$h(x,t)=1+\sum_{k=1}^{\infty}A_k(t)\,exp(2\pi kxi)+\sum_{k=1}^{\infty}A_k^*(t)\,exp(-2\pi kxi) \tag{10}$$

where, the time-dependent functions, $A_k(t)=a_k(t)+b_k(t)\,i$, are the k th complex coefficients of Fourier series and $A_k^*(t)$ are the complex conjugates of them, and when α is another slenderness factor much less than the unity, A_1 has the order of magnitude of α

and in the same way $A_k's$ have the orders of α^k , respectively. This expansion solution gives us much simplicity not only because by using the Euler's rule, the equation (10) can be equivalent to $h(x,t) = 1+2\sum_{k=1}^{\infty}(a_k(t)\cos(2\pi kx) - b_k\sin(2\pi kx))$, which consists only the real number, but also because the partial differential equation can be easily reduced to the system of the ordinary differential equations by the replacement of the evolution equation (9) by the equation (10).

We also take the electrostatic field to be described by the Fourier series witht the exactly same way as in $h(t)$.

$$\phi_y = 1+\sum_{k=1}^{\infty}(c_k(t)+id_k(t))(t)\,exp(2\pi kxi)+\sum_{k=1}^{\infty}(c_k(t)-id_k(t))(t)\,exp(-2\pi kxi) \tag{11}$$

When we consider up to order α^2 , the system of the ordinary differential equations take the forms of

$$-\frac{1}{\xi}a_1'(t) = \left(\frac{32\pi^4\xi^2}{3Ca}+\frac{4\pi^2Re}{3Fr^2}\right)a_1(t)+\frac{16}{3}K\left(\frac{1}{\epsilon}-1\right)\pi^2c_1(t) \tag{12}$$

$$-\frac{1}{\xi}b_1'(t) = \left(\frac{32\pi^4\xi^2}{3Ca}+\frac{4\pi^2Re}{3Fr^2}\right)b_1(t)+\frac{16}{3}K\left(\frac{1}{\epsilon}-1\right)\pi^2d_1(t) \tag{13}$$

$$\begin{aligned}-\frac{1}{\xi}a_2'(t) = &\left(\frac{64\pi^4\xi^2}{Ca}+\frac{8\pi^2Re}{Fr^2}\right)\left(a_1^2(t)-b_1^2(t)\right)+\left(\frac{512\pi^4\xi^2}{3Ca}+\frac{16\pi^2Re}{3Fr^2}\right)a_2(t)\\ &+32K\left(\frac{1}{\epsilon}-1\right)\pi^2(a_1(t)c_1(t)-b_1(t)d_1(t))\\ &+\frac{32}{3}K\left(\frac{1}{\epsilon}-1\right)\pi^2\left(c_1^2(t)-d_1^2(t)\right)+\frac{64}{3}K\left(\frac{1}{\epsilon}-1\right)\pi^2c_2(t)\end{aligned} \tag{14}$$

$$\begin{aligned}-\frac{1}{\xi}b_2'(t) = &\left(\frac{128\pi^4\xi^2}{Ca}+\frac{16\pi^2Re}{Fr^2}\right)a_1(t)b_1(t)+\left(\frac{512\pi^4\xi^2}{3Ca}+\frac{16\pi^2Re}{3Fr^2}\right)b_2(t)\\ &+32K\left(\frac{1}{\epsilon}-1\right)\pi^2(b_1(t)c_1(t)+a_1(t)d_1(t))\\ &+\frac{64}{3}K\left(\frac{1}{\epsilon}-1\right)\pi^2(c_1(t)d_1(t)+d_2(t))\end{aligned} \tag{15}$$

Conclusion

In order to define the film depth, we can solve the equations through (12) to (15) numerically. On the other hand, before solving these system of ordinary differential equations, we have to define the electrostatic field equation, $c_k(t)$ and $d_k(t)$; however, in our case those work must be such difficult problems that we ought to research them further beyond this paper. Some simple state assumption will be able to be taken to define $c_k(t)$ and $d_k(t)$

Reference

[1] Kwangseok Kim, Hyo Kim, "Mathematical Modeling And Experimen- Tal Observation

Of The Thin Film Behavior Under An Electrostatic Field", Theories and Applications of Chemical Engineering, KIChE, **9**(2), 2655 (2003).

[2] Gjevik, B., "Occurrence of Finite-Amplitude Surface Waves on Falling Fiquid Films", Phys. Fluids, **13**, 1918 (1970).

Figure

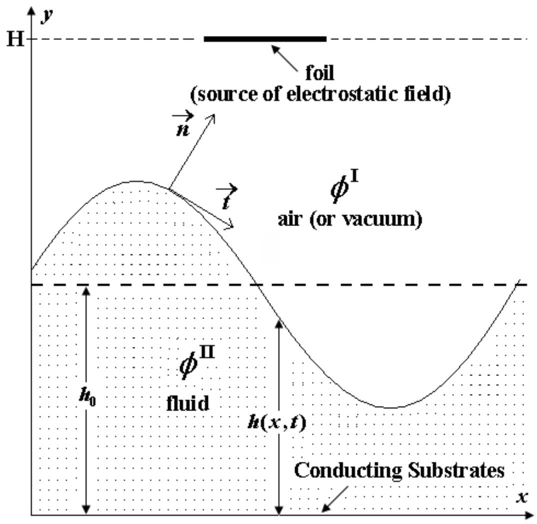

Fig. 1. The system configuration, in which the origin of the coordinates locates in the centre of foil and the bottom of fluid.