정전기 장에서의 박막 표면의 거동

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Free surface behavior of liquid film under an electrostatic field

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Introduction

Electrohydrodynamics(EHD) is one field that combines the electrodynamics and the hydrodynamics. Unlike this simple definition, it gives us large amount of difficulties and confusions when investigating for details because there are too many factors sensitive to the exterior and interior circumstances to be considered, moreover much about them are unveiled until now. For that reason, most of the investigations on electrohydrodynamics, for instance instability, are performed by estimating and jibing with the orders of magnitude of each parameter. The highly ideal and also conceptual assumptions, however, are essential to set up a problem, to simplify it and finally to resolve it. The stability analyses over the global electric field are commonly examined up to now regarding to the diverse situations. In this article, the procedures through which the motion of liquid layer when it comes to a local electric charge source are simulated will be shown.

Governing equations

The free surface of the thin liquid layer, especially as long as the surface tension of it is strong, is unstable for the exterior perturbations and is easily led to rupture. Now consider that two fluids (I and II) having mutually confronted interface spread on the flat plate, and they have different mass density ρ , viscosity $\rho\upsilon$, and electric permittivity ε . The charged foil immerged within the upper fluid is separated by distance η from the interface with down layer of thickness δ . The electric fields and electric potentials of two phases are represented by E_I, ϕ_I for upper fluid and E_{II}, ϕ_{II} for down fluid.

The governing equations of the electrohydrodynamics are

$$\nabla \cdot \vec{\mathbf{u}} = 0$$
$$\rho \frac{\mathbf{D}\vec{\mathbf{u}}}{\mathbf{D}t} = \nabla \cdot \left(\mathbf{T}^m\right) + \rho\vec{\mathbf{g}}$$
$$\nabla \cdot \left(\varepsilon\vec{\mathbf{E}}\right) = q$$

, and the boundary conditions are much familiar to us such that

$$\vec{\mathbf{u}} = 0, \quad \phi_f = 0 \quad @\, y = 0$$
$$\phi_v = 0 \quad @\, y = \mathrm{H}$$
$$\vec{\mathbf{n}} \cdot \left[\mathbf{T}^v\right] \cdot \vec{\mathbf{n}} - [p] - \tfrac{1}{2}\left[\varepsilon(1-a)E^2\right] + \left[\varepsilon E_n^2\right] = \sigma\left(\tfrac{1}{R_1} + \tfrac{1}{R_2}\right)$$
$$\vec{\mathbf{t}} \cdot \left[\mathbf{T}^v\right] \cdot \vec{\mathbf{n}} + q_s \vec{\mathbf{t}} \cdot \vec{\mathbf{E}} = 0 \quad @\, \text{interface}$$

, where, $\mathbf{T}^m = p\vec{\vec{\mathbf{I}}} + \mathbf{T}^v = p\vec{\vec{\mathbf{I}}} + \rho\nu\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^{\mathrm{T}}\right)$, $a = \frac{\rho}{\varepsilon}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T$, and $[A]$ is the jump of A.

Consider the case that the upper fluid is a gas phase, and then all the hydrodynamic terms in the equations are vanished.

Thin film limit

We have the equations and boundary conditions to be dimensionless with slenderness factor $\xi = {}^{h_0}\!/\!{}_L$, and in order to collect the dominant effects we expand the variables such as

$$\begin{cases} u(t,x,y) = u_0(t,x,y) + \xi u_1(t,x,y) + \xi^2 u_2(t,x,y) + \cdots \\ v(t,x,y) = v_0(t,x,y) + \xi v_1(t,x,y) + \xi^2 v_2(t,x,y) + \cdots \\ p(t,x,y) = p_0(t,x,y) + \xi p_1(t,x,y) + \xi^2 p_2(t,x,y) + \cdots \end{cases}$$

Then, we make the terms of order ξ^0, ξ consist of the dominant equations.

$$O(1): \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0$$

from mass continuity equation

$$O(\xi): \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0$$

$$O(1): \frac{1}{\mathrm{Re}}\frac{\partial^2 u_0}{\partial y^2} = 0$$

from horizontal momentum equation

$$O(\xi): \frac{\partial u_0}{\partial t} + u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y} = -\frac{\partial p_0}{\partial x} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u_1}{\partial y^2}$$

$$O(1): -\frac{\partial p_0}{\partial y} - \frac{1}{Fr^2} = 0$$

from vertical momentum equation

$$O(\xi): -\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_0}{\partial y^2}\right) = 0$$

$$O(1): \frac{A_H'}{h^3} + \frac{\xi^2}{Ca}\left(\frac{\partial^2 h}{\partial x^2}\right) = -\frac{\mathrm{Re}}{2} p_0 + \frac{1}{4} E_V\left[\varepsilon(\phi_y^2)\right]$$

from normal stress condition

$$O(\xi): -\frac{\partial u_0}{\partial y}\left(\frac{\partial h}{\partial x}\right) + \frac{\partial v_0}{\partial y} - \frac{\mathrm{Re}}{2} p_1 = 0$$

$$O(1): \frac{\partial u_0}{\partial y} = 0$$

from tangential stress condition

$$O(\xi): -\frac{\partial u_1}{\partial y} + E_V\left[\varepsilon\phi_y\right]\left(\phi_x + h_x\phi_y\right) = 0$$

$$O(1): u_0 = v_0 = 0$$

no slip condition

$$O(\xi): u_1 = v_1 = 0$$

Numerical solution

We solve the above equations, and get the evolution equation describing the film thickness Solving those equations by numerical method yield the film height shown in Fig 1.

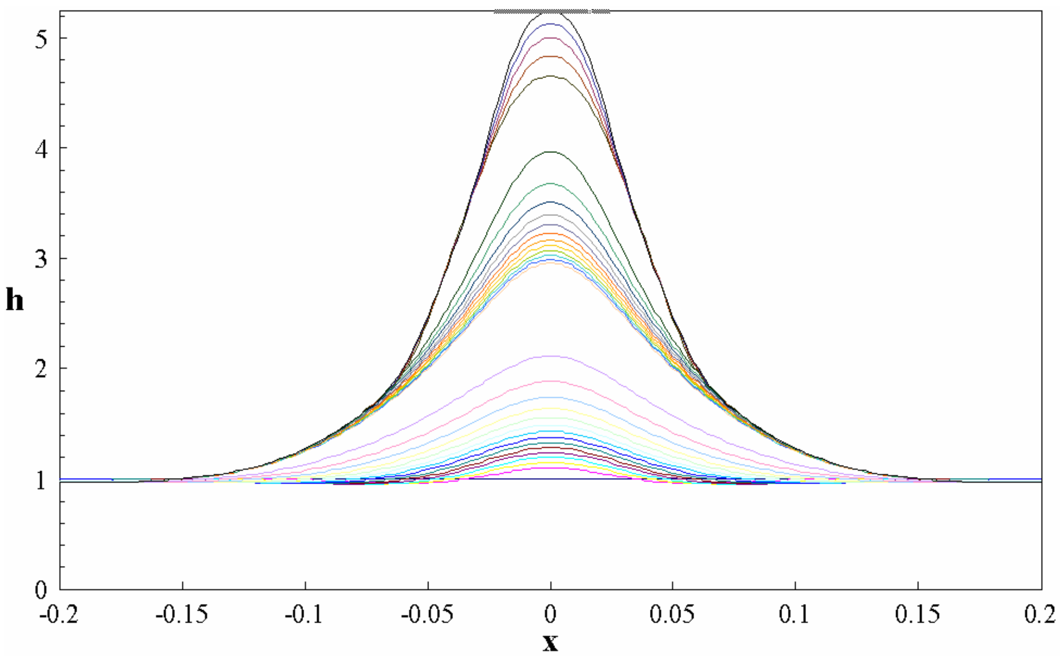

Fig. 1. Numerical solution from the simple implicit method.

In this case, we use the distributions of the electric field from that used before, such as

$$E_n = \frac{1}{2(H-y)}\left\{\frac{D-2x}{\sqrt{(x-D/2)^2+\xi^2(H-y)^2}}+\frac{D+2x}{\sqrt{(x+D/2)^2+\xi^2(H-y)^2}}\right\}$$

This is derived from the approximation of the Coulomb's law.

Conclusion

This work is meaningful because this shows how to approach for obtaining the solution of the evolution equation. Although we get the evolution equation of the film thickness, it must be refined again and again in order to approach toward the actual values, especially about the electrostatic field. Those process are performed now and on. We make them as further work.

References

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