정전기장 내 레이놀즈수가 작은 유체 박막흐름의 비선형 동역학: 고립파의 존재 조건

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Nonlinear dynamics of a small Reynolds number-film flow under an electrostatic field: the existence of the solitary waves

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Introduction

A chopping wave is one of most fearful phenomena on the coast. Once upon a time when a wooden boat met this wave, it was apt to be wrecked. Even if a huge ship made of metal alloys does not be crashed by the wave nowadays, several accidents have occurred annually by the shocks of the chopping waves. In the fluid dynamic point of view, this chopping wave is usually characterized as `solitary wave' which travels with conserving its initial shape. And some one hundred and fifty years ago this wave was recognized and investigated by John Scott Russel for the first time while he was designing a ship in a canal. In an oceanographical methodology, the mathematical models of the solitary waves are linearized by ignoring such effects as the air pressure, the viscosity of the water and etc. Thus those linearized governing equations and the boundary conditions can yield the analytical solution after some proper substitutions, for details see Hsieh[1].

On the contrary, the problems turn to become more complicated than the previous ones when the scales of the flow regimes are reduced into the ones in the chemical process units. For example, the surface tension becomes one of the dominant factors to determine the surface motion. There cannot exist the analytical solution to the displacement of the solitary wave any more, and thus we must solve the nonlinear partial differential equations, so called the evolution equation, only numerically in order to obtain the surface wave. Besides, we will also consider the effect of an electrostatic field in this study. As one of simple geometries, there is a film flow down an inclined plane. The evolution equation of this flow regime was derived by Benjamin[2] and Yih[3] for the first time. They developed the linear stability of the film flow down an inclined plane in different manners but got the same results. On the basis of their works, the supercritical stability was investigated by Gjevik[4]. He expanded the film height by using the Fourier series containing the time dependent coefficients. And from the substitutions of them he reduced the parabolic partial differential equation into some of the ordinary differential equations for the Fourier transforms of the surface wave. A. Pumir et. al. extended these researches to find the conditions at which the solitary waves were formed[5]. They used the moving frames at the same velocities as those of the waves in order to reduce the partial differential equation to the third order ordinary differential one. And then they found the characteristics of the solitary wave using the bifurcation theory.

Evolution equation

We have used the same definitions and equations as used in the paper of Kim[6]. Thus the evolution equation is given as

$$h_t + 3h^2 h_x + \xi \left\{ \frac{6}{5} Re h^6 h_x - h^3 h_x \cot\beta + \frac{2\xi^2}{3Ca} h^3 h_{xxx} + K_E \left(\frac{1}{\epsilon} - 1 \right) \frac{h^3 h_x}{3\left(H + (1/\epsilon - 1)\, h\right)^3} \right\}_x = 0.$$

Linear stability diagram

Linear stability analysis have been performed by using wavenumber, α, versus Reynolds number, Re, and thus we can obtain the following critical Reynolds number diagram with and without the effect of an electrostatic field

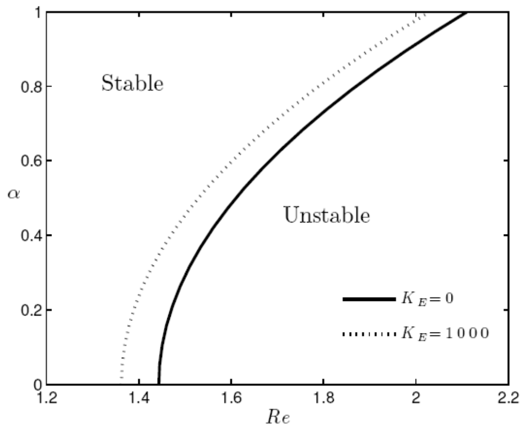

Figure 1. The condition of the existence of the pulse-like solitary wave.

Moving frame

We have employed the moving frame, $z = x - vt$, at a same speed, v, of wave to find the existences of stationary solitary wave. Then we can obtain the homoclinic and heteroclinic trajectories in the three dimensional phase space by adjusting v. These are shown in Figure 2 and 3, respectively.

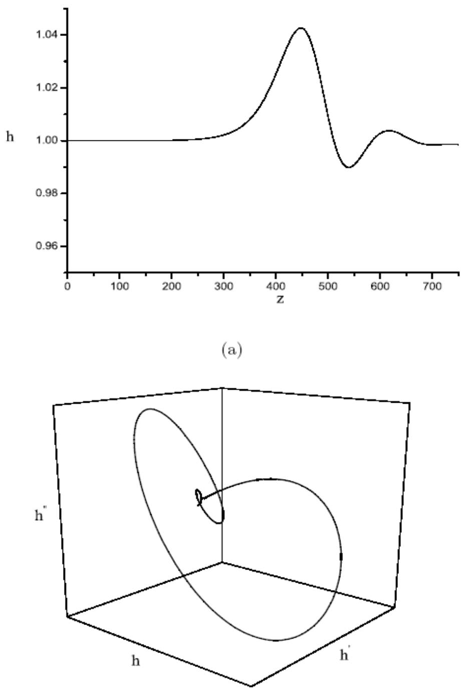

Figure 2. The pulse like solitary wave and its homoclinic trajectory in the phase space.

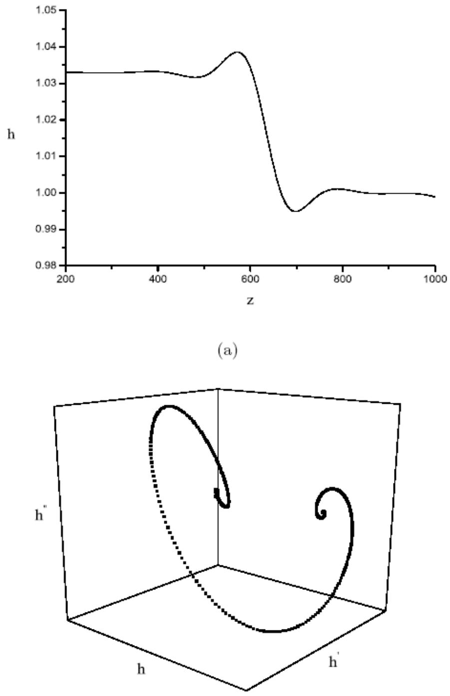

Figure 3. The hydraulic jump and its heteroclinic trajectory in the phase space.

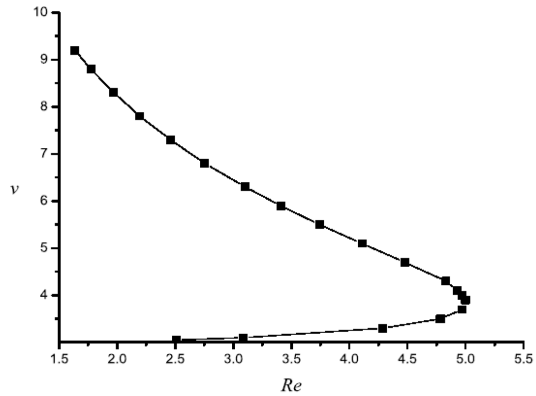

Figure 4. The condition of the existence of the pulse-like solitary wave.

Laboratory frame

We have solve the evolution equation numerically to get a solitary wave. Hamming's predictor-corrector method have been employed with the fourth order Runge-Kutta methods. After long time calculations, we can find the pulse-like solitary waves such as in Figure 5.

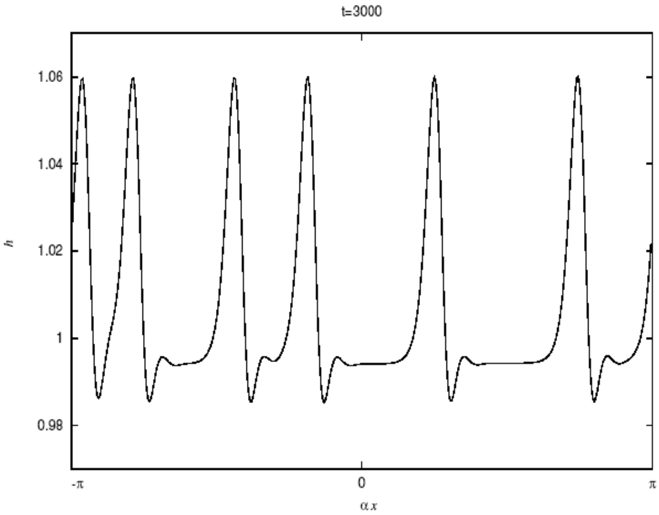

Figure 5. The example of the pulse-like solitary wave.

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